

COORDINATING ENERGY AND RESERVES IN A
WHOLESALE ELECTRICITY MARKET

A THESIS
SUBMITTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR THE DEGREE
OF
DOCTOR OF PHILOSOPHY IN MANAGEMENT SCIENCE
IN THE
UNIVERSITY OF CANTERBURY
BY
GLENN. R. DRAYTON-BRIGHT

UNIVERSITY OF CANTERBURY

1997

:SIS
yl

To Vicki and our boys, Edward James, and Alexander Robert

Acknowledgments

First and foremost, I would like to acknowledge the excellent support and guidance of my supervisor Dr. E. Grant Read of the University of Canterbury. The earlier chapters of this thesis build on some results Grant theorised in his thesis. Our later work together on the integration of reserve aspects into our models was a most enjoyable and stimulating experience.

Brett Graydon of Core Management Systems was involved with the research program where we compared various offer designs. I thank him for his patience during the debugging process for WEMSIM and for his efforts in extracting and reporting on the results. I would also like to thank Andrew Kerr of the University of Canterbury for his contribution to WEMSIM.

Chris Wallace at the University of Canterbury gave valuable advice as I was learning to program in C++. Grant Telfar of Core Management Systems helped with the integration of our optimisation code with Microsoft Excel. Ed Klotz of CPLEX Optimisation always provided fast and accurate technical support.

Abstract

This thesis addresses a number of questions related to the design of a wholesale electricity market and the decentralisation of a mixed hydro and thermal system.

Initially it concentrates on the response to price of a Linear Programming model of a hydro station and existence of a step supply curve consistent with that function. This has implications for the existence of the perfect competition equilibrium in a simplified energy market. An experimental analysis is presented, which attempts to quantify the theorised discrepancy between an 'ideal' centrally coordinated solution and the market's solution.

The latter half of this thesis develops a Linear Programming based representation of the joint energy and reserve capability of a generating unit or station, called the Fan Approximation. This approach is used to develop an offering and market-clearing model for energy and reserves which allows hydro, thermal, and interruptible load participants to compete equally.

Contents

1	INTRODUCTION	1
1.1	PREVIOUS RESEARCH	2
1.2	THE ENERGY MARKET	5
1.3	THE ENERGY AND RESERVES MARKET.....	7
2	A THEORETICAL ENERGY MARKET.....	9
2.1	THE CENTRALISED COORDINATION OF ENERGY	10
2.2	LP-OFFER MARKET DECOMPOSITION.....	16
2.3	EXISTENCE OF EQUILIBRIUM AND CONVERGENCE	19
2.4	IMPLICATIONS FOR THE SIMPLIFIED MARKET IN THIS THESIS	19
3	EXISTENCE OF STEP SUPPLY CURVES.....	25
3.1	FORMING SIMPLE SUPPLY CURVES	26
3.2	EXISTENCE OF FIXED SUPPLY CURVES	28
3.3	EXISTENCE OF SCULPTED SUPPLY CURVES.....	37
3.4	IMPLICATIONS FOR OFFER FORMATION.....	47
4	SUMMARY OF ENERGY MARKET RESULTS	49
4.1	THE SIMPLIFIED MARKET	50
4.2	EXPERIMENTAL METHODOLOGY.....	51
4.2.1	<i>The Day-ahead Market.....</i>	<i>52</i>
4.2.2	<i>The Periodic-update Market</i>	<i>56</i>
4.3	FIXED OFFERS.....	57
4.3.1	<i>Simple Three-band Fixed Offers</i>	<i>57</i>
4.3.2	<i>Fixed Offers Based on Multiple Scenarios.....</i>	<i>58</i>
4.4	SCULPTED OFFERS.....	59
4.4.1	<i>The Deterministic Case</i>	<i>59</i>
4.4.2	<i>The Uncertainty Case.....</i>	<i>60</i>
4.4.3	<i>Sculpted Offers Based on Multiple Scenarios</i>	<i>62</i>
4.5	THE PERIODIC-UPDATE MARKET	65

4.6	CONCLUSIONS.....	67
5	AN ENERGY AND RESERVES MARKET	69
5.1	THE ROLE OF RESERVES IN THE NEW ZEALAND SYSTEM.....	70
5.2	CENTRAL COORDINATION OF RESERVES	73
5.3	PRINCIPLES OF A MARKET FOR RESERVES	74
5.4	INSTANTANEOUS RESERVE CAPABILITY.....	78
5.5	A GENERAL CONSTRUCT FOR OFFERING ENERGY AND RESERVES	81
5.6	MODELLING RISK REQUIREMENTS	85
5.7	ILLUSTRATION	86
6	CONCLUSIONS	91
6.1	THE SIMPLIFIED ENERGY MARKET	91
6.2	A MARKET FOR ENERGY AND RESERVES.....	93
6.3	OPEN RESEARCH QUESTIONS.....	94
	BIBLIOGRAPHY	95
A.	THE REFERENCE MODEL	103
A.1	BACKGROUND	103
A.2	THE FAN APPROXIMATION.....	104
A.2.1	<i>The Unit Flow Function</i>	<i>104</i>
A.2.2	<i>Defining the Station Level Approximation</i>	<i>105</i>
A.2.3	<i>Modelling the Cost of Motoring Plant.....</i>	<i>111</i>
A.2.4	<i>Calculating Delivered Reserve.....</i>	<i>113</i>
A.2.5	<i>Summary.....</i>	<i>114</i>
A.3	STORAGE AND RIVER CHAINS	116
A.3.1	<i>Daily Release Cycle Planning Model.....</i>	<i>120</i>
A.3.2	<i>Block Implementation Planning Model.....</i>	<i>122</i>
A.3.3	<i>Smoothing Storage Trajectories and Release Patterns</i>	<i>124</i>
B.	OVERVIEW OF WEMSIM	127
C.	ENERGY MARKET RESULTS	131
C.1	SCULPTED OFFERS	132
C.1.1	<i>Simple Three-band Sculpted Offers.....</i>	<i>135</i>
C.1.2	<i>Multi-band Sculpted Offers</i>	<i>137</i>

C.2	FIXED OFFERS.....	143
C.2.1	<i>Building a Fixed Offer</i>	144
C.2.2	<i>Calculating a Monotone Supply Curve</i>	147
C.2.3	<i>Handling Inconsistent Observations</i>	151
C.2.4	<i>Three-band Fixed Offers</i>	156
C.2.5	<i>Multi-band Fixed Offers</i>	158
C.3	OFFERS PREPARED FROM MULTIPLE SCENARIOS	160
C.3.1	<i>Sculpted Offers with Multiple Scenarios</i>	160
C.3.2	<i>Averaging Responses over Equi-priced Periods</i>	166
C.3.3	<i>Multi-band Fixed Offers with Multiple Scenarios</i>	170
C.4	THE PERIODIC-UPDATE MARKET	172
C.4.1	<i>Periodic-update versus Day-ahead Market</i>	177
C.4.2	<i>Sequential Days</i>	208
C.5	THE IMPACT OF FORECAST UNCERTAINTY	218
C.5.1	<i>Alternative Optima in Market Clearing</i>	219
C.5.2	<i>Demand Forecast Sensitivity</i>	226
C.5.3	<i>Water Value Balance</i>	233
C.5.4	<i>Tributary Uncertainty</i>	243
C.5.5	<i>Uncertain Tributary Flows in the Periodic-update Market</i>	248
D.	RESERVE OFFERS IN PRACTICE.....	253
D.1	OFFERS FOR THERMAL UNITS	253
D.1.1	<i>Minimum Unit Loading Points</i>	253
D.1.2	<i>Fixed Costs and Declining Marginal Cost Curves</i>	257
D.2	HYDRO STATION OFFERS	262
D.2.1	<i>Station PLSR Capability</i>	262
D.2.2	<i>Combining PLSR and TWD</i>	265
D.3	CALCULATING A RESERVE OFFER	269

Chapter 1

Introduction

Deregulation of electricity markets is an international phenomenon. Gilbert and Kahn [1996] compare regulation in the established and emerging markets of Norway, the UK, Chile, New Zealand, and Australia, with other more regulated industries in the US, and continental Europe. At the time of writing of this thesis¹, the Norwegian and UK markets were fairly well established, but reforms were continuing to be made in other markets, including New Zealand. Once considered the exception, these fledgling markets are now looked upon as role models for further reform.

New Zealand's situation is somewhat unique because of its heavy reliance on hydro generation (up to 80%); and further, the sparsity of the network, and small average loads, make reserve requirements particularly important.

¹ This thesis is set in early 1996. At that time, the wholesale electricity industry was under going significant reform. The existing single generator, ECNZ, was to be split into two competing entities. Plans for a wholesale electricity market were being formed, but the details of this market's operation were yet to be decided. In particular, the form of offers made by generators was a matter of considerable interest. Since that time, a market has been established; the results of the research presented here influenced the offer form used in that market. Further, the market now includes the mechanism proposed here for integrating reserve with the offering and market clearing process—see Grimwood [1996].

Culy *et al* [1995] traces the development of the New Zealand power supply industry from its inception. They describe the improvements gained, from recent reforms, in the context of an on-going process of privatisation, with the logical conclusion being fully deregulated retail and wholesale markets for electricity and reserves.

They describe the New Zealand experience in three phases. Over the period 1945–78 the State Hydroelectric Department controlled development of the supply industry. Hydro developments were viewed as national resources, rather than a commodity to be marketed. Toward the end of this period, electricity began to be viewed as a scarce resource that required coordination on a national level. The Ministry of Energy phase lasted over the period 1979–1987 and saw the industry run to meet the requirements of an Energy Plan. During this time some rationalisation occurred, especially in the retail sector, but increasing pressures on the economy led to a new Government with a will to embrace free market policies. This led to the Electricity Corporation phase, which persists today. During this phase, the retail and wholesale sectors have been deregulated. In particular, responsibility for the transmission system has been separated from generation, and fuel supply has been deregulated and must now be negotiated for. The retail side has been nearly fully deregulated, but reform continues on the wholesale side. The next most significant stage, on which this thesis is focused, is the development of a competitive wholesale electricity market.

1.1 Previous Research

Three major studies of the wholesale generation and transmission sector in the New Zealand have been conducted. The Electricity Task Force 1989 covered both the retail and wholesale sectors, while the WEMS [1992], and the WEMDG [1994] studies concentrated on the wholesale sector. Culy *et al* [1996] said of these, “...all studies have recommended development of neutral market structures, to facilitate efficient coordination, contract trading and entry...”

The need to develop this market structure, out of a knowledge base derived from traditional centralised systems, has opened new areas for research; these areas include:

- 1) Nodal pricing of energy in a deregulated market.
- 2) The impact of generators' contract position on incentives for exploiting monopoly power (gaming) in a deregulated market.
- 3) Generating unit commitment and its effect on the market.
- 4) Mechanisms for dealing with uncertainty.
- 5) The integration of reserves pricing and dispatch into the market framework, and its potential effect on generator incentives.

Recent international, and local work in particular, has begun to address several of these areas. Ring [1995] extended nodal pricing theory to include pricing for reserve capacity. Kaye *et al* [1995] examined a theoretical decentralised environment, in which contingency requirements are communicated to generators via an appropriate set of prices and probabilities, and showed that the optimal reserve allocation is achievable. Scott and Read [1992, 1993] studied regulation of gaming, and contracts for the energy market. Kerr and Read [1996] discussed modelling and management issues related to unit commitment scheduling in a market environment. Johnson *et al* [1996] has also studied the issue of unit commitment and its impact on the market. Locally, Philpott [1996] has examined techniques for handling uncertainty, with applications the electricity sector; recently Waterer [1996] began to extend this work into a 'market context' by the examining the effect of unit commitment and uncertainty at the hydro group level.

As discussed further in Chapter 2, the existing literature offers only some insight into the appropriate design of generators' offers or market clearing models, although some inferences can be drawn from the general economics literature and previous experimental analyses. However, integration of instantaneous reserves into the offering and market clearing process has not occurred in any existing market and is largely overlooked by the literature—although Hirst *et al* [1995] acknowledge that it is one of a number of important issues that need to be addressed. As is typical in existing markets the UK pool

treats reserve costs as one of a number of ancillary services lumped into an 'uplift' payment (Green [1994 (1), (2)]).

Thus, New Zealand enters the current phase of deregulation not knowing all of the answers. Of this phase Culy *et al* [1996] said, "Government and public need to be convinced that the benefits of further reform will outweigh the costs. Successive studies have found this hard to prove, particularly in view of the relatively large overheads required to establish sophisticated market mechanisms in such a small economy. Privatisation is not expected to provide further improvements in operational efficiency or co-ordination, both of which appear to be more than adequately furnished in the status quo..." In relation to these perceived benefits, Read and Scott [1992] said, "It has been suggested that the New Zealand electricity sector may operate more efficiently under a market structure... It is recognised that while a more diversified market may improve performance in areas such as investment and innovation, it may perform less well in other areas such as short term coordination efficiency." Read and Scott [1993] also said, "...privatising one or more hydro reservoirs and creating a wholesale market for electricity could be expected to result in some loss of coordination efficiency compared with the status quo. On the other hand, this should be expected to improve the modelling and management of local catchments as independent groups pay more attention to the local conditions and the needs of their businesses. Privatisation in the UK in 1990-1991 has resulted in a wide range of gaming behaviour by the generators, but most commentators agree that overall efficiency has improved."

According to Culy *et al* [1996] the 'fear' of exposing the existing centrally coordinated system to the effects of gaming has dominated the market design process in New Zealand. During this design phase commentators, when making recommendations, have attempted to draw on lessons learned in other markets, particularly those of the UK and Victoria.

In their 1994 study of the Victorian market, Culy and Read examined gaming incentives implied by various market designs. They characterised designs by their degree of simplification or regulation of generator offers. A high degree of regulation implies a

simpler offer, e.g. a simple supply curve applying for a whole day is a more regulated offer than one which has a different supply curve for each period.

Somewhat contrary to the popular belief that a more regulated market would discourage gaming, they concluded, "...a requirement for players to offer a single price quantity over the week, while designed to limit generators' ability to game, actually seems likely to increase gaming and spot price distortions, particularly if contract coverage is low." However, the simpler the offer and market clearing design the more transparent and accessible the market appears to both participants and auditors. This criterion may be traded off with the potential for distortion due to gaming to achieve an appropriate balance.

1.2 The Energy Market

This thesis assumes a basic market design that uses supply curve offers which hold for a certain period of time—be it one half hour or a whole day. Supply curve offers, by definition, do not allow generators to directly specify intertemporal or other operational constraints. This fact in itself imposes some cost of moving away from optimal centrally coordinated solution. This motivates the first key research question:

Key Research Question A: Can a simple market adequately represent the complexity of a real hydrothermal system?

This thesis focuses on the design of the market used to plan and implement the day-ahead dispatch process, with special attention to the effects that the form of the offers submitted by generators to a market may have on the feasibility, productive efficiency and robustness of the dispatch. It examines a number of possible offer designs, for both energy only, and the combined energy and reserves markets. These designs differ in the complexity of the offer and the frequency with which they are updated.

Previous studies have implied that the cost of moving to a simplified offer, whatever that is, could be overshadowed by the cost incurred from generator gaming. In particular, Culy *et al* [1996] said, "...On the down side, concern has arisen... that a multitude

of generators... may deliberately ‘play games’, in ways which raise prices, increase costs, reduce energy efficiency or cause outages...”

However, in the real market participants will not take their chances on the energy market every day. Rather, they will form contracts in the short term and in the long term; but as Scott and Read [1992, 1993] show, unless contract levels are very high, generators will wish to choose an offering strategy for the spot market that they expect will maximise their profits. A complete analysis would compare incentives for gaming the market when generators submit complex offers versus one in which they must submit simplified offers, but Scott and Read [1992] show that a profit maximising generator faces a complex, non-linear objective function in the presence of contracts—a concept that is difficult to model in an LP framework. This thesis will not attempt to make this comparison. Rather, the experiments presented here concentrate on the existence of ‘efficient offers’, assuming that generators try to do the best for *the system overall*, rather than considering their own agendas.

Chapter 2 describes the Reference Model (RM), which is a model of a centrally coordinated dispatch. To measure the performance of a market its solutions will be compared to that of the RM. The research question being addressed is:

Research Question A.1: What impact does simplification of the generators’ offers have on achieving dispatch feasibility and optimality?

Chapter 2 then decomposes the RM to form a set of sub-models, one for each ‘generating company’ (generator) in the market. A theoretical market, in which these sub-models are used as offers, is discussed. Motivation is then given to the use of a simplified offering and market-clearing model. The issues of existence of solutions and convergence are raised and insights from the literature are discussed.

Chapter 3 introduces some variations of a supply curve offer and examines the nature of the response to price of the LP model of a hydro station; the research question being addressed is:

Research Question A.2: Is a step supply curve consistent with a hydro station's underlying supply model in all cases?

Chapter 4 summarises the results of a series of experiments comparing the relative performance of these offer designs in an energy market. Details of these experiments are provided in the appendices². The questions addressed by the experimental analysis are:

Research Question A.3: What degree of cost distortion can be expected from an energy market in which generators submit a fixed offer (a single supply curve offer for a whole day) versus one in which their offer is sculpted (a different supply curve for each period of the day)?

The performance of the market is explored further by examining the sensitivity of the solutions to random changes in individual circumstances, such as randomly inaccurate estimation of: the market clearing prices; demand patterns; water value estimates; in-flow uncertainty. The research question being addressed here is:

Research Question A.4: How robust are the measured distortions?

1.3 The Energy and Reserves Market

For the New Zealand power system, provision of reserves is the second highest generation cost after meeting of system load. This implies that there is a need for efficient scheduling of reserves, and potentially, a significant market for reserves. Drayton *et al* [1992] described the problem of coordinating reserves in New Zealand in some detail.

The inherent difficulty of producing a centrally coordinated model of the entire New Zealand system for a whole day is demonstrated by the complexity of the solution

² The experimental results are drawn from reports prepared for ECNZ during a related research project. Full details of these experiments can be found in Drayton-Bright *et al* [1995(1)–(6)].

method used in GARSP, which is detailed by Drayton-Bright [1995]. This added weight to the argument for a decentralised coordination process—the key motivations for which are discussed in Chapter 2.

The latter half of this thesis addresses the second key research question:

Key Research Question B: How can reserves be represented in the offering and market clearing process?

Chapter 5 describes the combined energy and reserve capability of a generating unit. (Appendix A, in describing the Reference Model, expands on the concepts developed in Chapter 5 and develops an approach called the Fan Approximation to modelling hydro station releases.). Chapter 5 goes on to describe an integrated offering and market-clearing framework for energy and reserves based on the Fan Approximation, and presents some examples.

Chapter 6 summarises the theoretical and experimental results of the thesis, and points to further research areas it has opened.

Chapter 2

A Theoretical Energy Market

It is well known that the problem of centrally coordinating the energy dispatch can be cast as an Linear Programming (LP) problem. Section 2.1 briefly discusses the LP formulation, used in this thesis as the Reference Model (RM), i.e. a model of the system prior to the decentralisation of the in 1996³. Section 2.2 takes this LP problem and creates a number of sub-problems, one for each generator, using the standard decomposition technique of dualising the nodal energy balance constraints. It then describes a theoretical energy market in which these LP sub-problems are used by generators as offers. Finally, Section 2.3 discusses simplifying the offer form, and theorises about the effect of this simplification on the ‘quality’ of the market-derived solution and the convergence properties of the market coordination process.

³ This formulation was derived from earlier unpublished work of Read *et al* [1994]. Appendix A describes the extensions made to that model for this thesis.

2.1 The Centralised Coordination of Energy

The RM assumes a planning horizon of 24 hours divided into scheduling periods of equal length. The model used is an approximation of the New Zealand system, and is composed of geothermal, thermal, and hydro generators, and a set of forecast loads at the reference nodes in the North and South Islands.

In the simplest case any of these units would be able to produce power at the same efficiency level over a generating range from zero to some maximum, and there would be no uncertainty in load forecasts, transmission losses or constraints, or other constraints implied by system security requirements, etc. In that case, the least cost dispatch could be found easily by committing the cheapest units until the required demand is met in each period. This type of approach has been called the “merit-order dispatch”. It can be formulated as a simple LP problem, however it is far too simplistic for representing the New Zealand system because it does not take account of:

- intertemporal effects such as unit commitment⁴, constraints on storage of water in reservoirs and river flows, and rate-of-change constraints;
- unit efficiency curves, and reserve capability profiles;
- transmission network constraints and losses⁵;
- security constraints, such as system reserve requirements

Culy *et al* [1996] suggests that it is unlikely that the forthcoming NZ market will allow generators to specify unit commitment in their offers—this would overly complicate the offering and market clearing processes, as well as making it difficult to determine ex-post prices. Rather, they support a ‘self commitment’ market, i.e. one in which genera-

⁴ The issue of unit commitment has been widely studied in the hydrothermal coordination literature. Muckstadt and Koenig [1977]; Burton *et al* [1974]; Engles *et al* [1976]; and later Shaw *et al* [1985]; Bertsekas *et al* [1983]; Yang and Chen [1989]; Cohen and Sherkat [1987]; Oliveira *et al* [1993]; Brannland *et al* [1988]; Hobbs *et al* [1988]; use variations of spatial and/or station-wise decomposition of the unit commitment problem with dynamic programming most commonly being used to determine the optimal station unit dispatch. In the New Zealand context, Drayton *et al* [1992] and Kerr and Read [1995] use various heuristic techniques to speed solution times.

tors determine their own unit availability, and structure their energy offers appropriately. Therefore, the model used here, and later decomposed to form a model of a market, will not involve treatment of any integer decisions—although it is certainly possible to formulate these aspects in a linear framework, the only problem being that the solution could not be guaranteed to be integer. Hence the RM excludes treatment of unit commitment, start-up and shutdown costs, lower unit output bounds, ‘rough running’ ranges, and minimum unit up and down times.

The RM assumes that the cost per giga-joule (GJ) of fuel for a thermal unit, and the value of water in storage in reservoirs is constant over any given day. One might assume that thermal units, and hydro units attached to reservoirs, could be treated like generation sources in a merit order dispatch, however it is well known that the water value for hydro units in a river-chain in which storage or other constraints might become active within the modelling horizon cannot be determined *a priori* (see Read [1979]). Moreover, unit efficiency varies with output⁶, i.e. the marginal cost of generation is not necessarily constant over the whole of a unit’s operating range.

The RM addresses these issues by: incorporating a network-flow model with LP side constraints, to represent hydro river-chains; and approximating thermal and hydro unit output curves with piece-wise linear approximations. The RM river-chain model is still quite simplified in that:

- each hydro station can be connected to a head-pond or storage reservoir upstream and/or downstream, but cannot be connected in any other way;
- storage is measured in cubic metres (m^3) and flows in cubic metres per hour (m^3/h) and the assumption is made that generating unit efficiency is not affected by storage volume;

⁵ According to Ring [1995], including a full AC loss model in the central coordination problem is too difficult, but a DC load flow model or basic linear location factors are possible.

⁶ See Kirchmayer [1958] for a general discussion of plant efficiency and Drayton *et al* [1992] for examples in the New Zealand system. Efficiency tends to be poor at low output levels and increases to a peak close to maximum output. It is well understood in the literature that these variations in efficiency could change the nature of the optimal generation configuration, especially when the peak efficiency occurs at some point less than maximum output.

- hydro generators are treated as single stations, rather than individual units

As stated earlier, the dispatch of reserves has received little attention in the past. It is omitted in the RM, but treated in detail in later chapters.

The RM uses a two-bus transmission network with two lines connecting them in each direction (these represent the two poles of the HVDC); losses are approximated by a piece-wise linear function.

The equations below show the essential elements of the RM LP problem; Appendix A expands on this description. Table 2-1 introduces the variables; Table 2-2 describes the model's parameters.

Symbol	Description
$e_{i,j}^t$	Output (MWh) from thermal unit i in efficiency segment j in period t
$q_{h,j}^t$	Productive release (m^3) from hydro station h in efficiency segment j in period t
w_h^t	Spill (m^3) from hydro station h in period t
s_h^t	Water held in storage (m^3) in head-pond or storage reservoir h at the end of period t
$\bar{p}_{x-y,in,l}^t$	Power flow (MWh) received at node y from node x on transmission line l in period t
$\bar{p}_{x-y,in,l}^t$	Power flow (MWh) received at node x from node y on transmission line l in period t
$\bar{p}_{x-y,out,l,k}^t$	Power flow (MWh) sent out from node x to node y in loss tranche k on transmission line l in period t
$\bar{p}_{x-y,out,l,k}^t$	Power flow (MWh) sent out from node y to node x in loss tranche k on transmission line l in period t

Table 2-1: Variables in the Reference Model

Symbol	Description
c_f	Incremental cost (\$/GJ) of fuel f
ψ_h	Value of water (\$/m ³) held in storage in reservoir h
$\sigma_{i,j}$	Conversion factor (GJ/MWh) at thermal unit i in efficiency segment j
$\alpha_{h,j}^t$	Conversion factor (MWh/m ³) at hydro station h in efficiency segment j
L_{x-y}	Number of transmission lines between nodes x and y
$\bar{\lambda}_{x-y,l,k}$	Loss factor (MW/MW) for flows on line l from node x to node y in loss tranche k
$\bar{\lambda}_{x-y,l,k}$	Loss factor (MW/MW) for flows on line l from node x to node y in loss tranche k
$\underline{\bar{p}}_{x-y,l}^t, \bar{\bar{p}}_{x-y,l}^t$	Bounds on power flow (MW) on line l from node x to node y in period t
$\underline{\bar{p}}_{x-y,l}^t, \bar{\bar{p}}_{x-y,l}^t$	Bounds on power flow (MW) on line l from node y to node x in period t
$\underline{\bar{p}}_{x-y}^t, \bar{\bar{p}}_{x-y}^t$	Bounds on the total power flow (MW) from node x to node y in period t
$\underline{\bar{p}}_{x-y}^t, \bar{\bar{p}}_{x-y}^t$	Bounds on the total power flow (MW) from node y to node x in period t
d_n^t	Load (MW) at node n in period t
s_h^0	Initial storage (m ³) in head-pond or reservoir h
δ_{x-y}	Delay (hrs) for flows between hydro station x and y
n_h^t	Natural inflow (m ³ /h) into head-pond or reservoir h in period t
q_h^0	Initial release (m ³ /h) from hydro station h
e_i^0	Initial output (MW) from thermal unit i
$\tilde{q}_h^t, \check{q}_h^t$	Maximum increase/decrease in release (m ³ /h) from hydro station h between period $t-1$ and t
$\tilde{e}_i^t, \check{e}_i^t$	Maximum increase/decrease in output (MW) from thermal unit i between period $t-1$ and t
$\underline{s}_h^t, \bar{s}_h^t$	bounds on end-of-period storage (m ³) for hydro station h in period t

Table 2-2: Parameters in the Reference Model

Variables are defined as quantities of flow or output (m³ or MWh), and parameters are specified as rates (m³/h or MW). This feature is intended to allow the formulation to be adapted easily for any length time period, by multiplying parameters by the period length (Δ hrs). Transmission lines are defined as a pair of directed arcs (denoted in and

out) between the nodes they connect. If more than one line exists between two nodes the lines are indexed by l .

The objective of the RM is to minimise total fuel cost, which includes the ‘cost’ incurred from releases of water from reservoirs, as shown in Equation 2-1. This is subject to nodal power balance (and associated multipliers), Equation 2-2; definition of power flow received, Equation 2-3; bounds on power flow, Equation 2-4; water flow balance for hydro head-ponds and storage reservoirs, Equation 2-5; bounds on hydro station release and thermal unit output, Equation 2-6; bounds on rate-of-change of hydro station release, and thermal unit output, Equation 2-7; and bounds on storage volume, Equation 2-8.

$$\begin{aligned} \text{Minimise} \quad & \sum_{f=1}^F c_f \left(\sum_{t=1}^T \sum_{i \text{ fueled by } f} \sum_{j=1}^J \sigma_{i,j} e_{i,j}^t \right) - \sum_{h=1}^H \psi_h s_h^T \\ & e_{i,j}^t, q_{h,j}^t, w_h^t, s_h^t, \\ & \bar{p}_{x-y,in,l}^t, \bar{p}_{x-y,out,l,k}^t, \\ & \bar{p}_{x-y,out,l,k}^t, \bar{p}_{x-y,out,l,k}^t \geq 0 \end{aligned}$$

Equation 2-1

subject to

$$\begin{aligned} & \sum_{i \text{ injects at } n} \sum_{j=1}^J e_{i,j}^t + \sum_{h \text{ injects at } n} \sum_{j=1}^J \alpha_{h,j} q_{h,j}^t + \\ & \sum_x \sum_{l=1}^{L_{x-n}} \left(\bar{p}_{x-n,in,l}^t - \sum_{k=1}^K \bar{p}_{x-n,out,l,k}^t \right) + \\ & \sum_y \sum_{l=1}^{L_{n-y}} \left(\bar{p}_{n-y,in,l}^t - \sum_{k=1}^K \bar{p}_{n-y,out,l,k}^t \right) = \Delta d_n^t \quad \forall n, t \quad (\beta_n^t) \end{aligned}$$

Equation 2-2

$$\begin{aligned} & \bar{p}_{x-y,in,l}^t - \sum_{k=1}^K \bar{\lambda}_{x-y,l,k} \bar{p}_{x-y,out,l,k}^t = 0 \quad \forall x-y, l, t \\ & \bar{p}_{x-y,in,l}^t - \sum_{k=1}^K \bar{\lambda}_{x-y,l,k} \bar{p}_{x-y,out,l,k}^t = 0 \quad \forall x-y, l, t \end{aligned}$$

Equation 2-3

$$\begin{aligned}
\Delta \bar{p}_{x-y,l}^t &\leq \bar{p}_{x-y,in,l}^t \geq \Delta \bar{p}_{x-y,l}^t \quad \forall x-y, l, t \\
\Delta \bar{p}_{x-y,l}^t &\leq \bar{p}_{x-y,in,l}^t \geq \Delta \bar{p}_{x-y,l}^t \quad \forall x-y, l, t \\
\Delta \bar{p}_{x-y}^t &\leq \sum_{l=1}^{L_{x-y}} \bar{p}_{x-y,in,l}^t \geq \Delta \bar{p}_{x-y}^t \quad \forall x-y, t \\
\Delta \bar{p}_{x-y}^t &\leq \sum_{l=1}^{L_{x-y}} \bar{p}_{x-y,in,l}^t \geq \Delta \bar{p}_{x-y}^t \quad \forall x-y, t
\end{aligned}$$

Equation 2-4

$$\begin{aligned}
&\sum_{k=1}^K q_{h,k}^t + w_h^t - s_h^{t-1} + s_h^t - \\
&\sum_{i \text{ upstream of } h} \left(\sum_{k=1}^K q_{i,k}^{t-\delta_{i-h}/\Delta} + w_i^{t-\delta_{i-h}/\Delta} \right) = \Delta n_h^t \quad \forall h, t
\end{aligned}$$

Equation 2-5

$$\begin{aligned}
\sum_{j=1}^J q_{h,j}^t + w_h^t &\geq \Delta \underline{q}_h^t \quad \forall h, t \\
\sum_{j=1}^J q_{h,j}^t + w_h^t &\leq \Delta \bar{q}_h^t \quad \forall h, t \\
\sum_{j=1}^J e_{i,j}^t &\geq \Delta \underline{e}_i^t \quad \forall i, t \\
\sum_{j=1}^J e_{i,j}^t &\leq \Delta \bar{e}_i^t \quad \forall i, t
\end{aligned}$$

Equation 2-6

$$\begin{aligned}
-\Delta \tilde{q}_h^t &\leq \sum_{j=1}^J (q_{h,j}^t - q_{h,j}^{t-1}) + w_h^t - w_h^{t-1} \leq \Delta \hat{q}_h^t \quad \forall h, t \\
-\Delta \tilde{e}_i^t &\leq \sum_{j=1}^J (e_{i,j}^t - e_{i,j}^{t-1}) \leq \Delta \hat{e}_i^t \quad \forall i, t
\end{aligned}$$

Equation 2-7

$$\underline{s}_i^t \leq s_i^t \leq \bar{s}_i^t \quad \forall h, t$$

Equation 2-8

Although it is fairly simple, the RM formulation deals with the fundamental system constraints, and seems a fair representation of what could be achieved if the dispatch

was centrally optimised. As such it represents a reference point against which various market designs can be compared.

2.2 LP-offer Market Decomposition

This section takes the central coordination model and assumes that:

- the coordination process is to be decentralised;
- the existing generation system is to be grouped into G generating companies (the generators);
- an energy market is to be established into which generators can offer power, and from which loads can purchase power;
- generators will freely exchange their system details and forecast data with each other

The rules for operating this market could be:

- 1) Some time, prior to the real-time dispatch, the generators and loads submit energy offers and bids to the market.
- 2) These offers and bids may have any degree of complexity up to and including that contained in the RM model (for clarity, in the discussion that follows, assume that the fixed nodal demand in the RM formulation corresponds to loads making a simple MW bid at some very high price).
- 3) The market will combine the generator offers and load bids into a market-clearing LP problem, with the objective of maximising net benefit.
- 4) Real-time dispatch follows the solution of this market clearing LP problem.

If generators were motivated to maximise the benefit from their generation to the system as a whole, and the RM model is the best possible model of the system, then they would 'simply' offer their part of the RM LP formulation. This assumption would be valid in reality if the competitiveness of the market encouraged generators to make their marginal cost information available, or the market rules (naively) regulated them to do so, or contractual commitments were high enough that generators would not gain signifi-

cantly by acting as anything other than perfect competitors—see Scott and Read [1992, 1993].

These LP offers can be extracted from the RM formulation if it can be decomposed into separate LP problems, one for each generator⁷. The LP can be partitioned where no physical, contractual, or other connections exist between generators. The only equations that link generators are the power and hydro storage balance equations. The latter can be dealt with ensuring that hydro stations on the same river chain are not split. The former links all generators together, and requires more explicit treatment.

The power flow variables are included in the RM formulation to model transfers between the North and South Island systems. If generators and loads offer and bid at their island reference node, and the market clearing process collates offers and bids at those nodes and explicitly models power flows in the same manner as the RM model, then these links can be removed from consideration in the decomposition. If \hat{d}_n^t represents the net demand, after transfers between nodes have been accounted for, then the nodal power balance equations can be simplified to the form shown in Equation 2-9.

$$\sum_{i \text{ injects at } n} \sum_{j=1}^J e_{i,j}^t + \sum_{h \text{ injects at } n} \sum_{j=1}^J \alpha_{h,j} q_{h,j}^t = \Delta \hat{d}_n^t \quad \forall n, t \quad (\beta_n^t)$$

Equation 2-9

If the optimal values of the dual variables for these equations (β_n^t , the energy prices at each node in each period) were known *a priori*, these equations could be placed in the objective function, as shown in Equation 2-10.

⁷ Decomposition via the system price for energy is a well understood technique that produces optimal coordination signals. Muckstadt and Koenig [1977] used decomposition to aid solution speed in unit commitment problems by breaking the coordination problem into smaller sub-problems, while Read [1979, 1983], Chao-an Li and Streiffert [1990], Rakic *et al* [1990], Luo *et al* [1990], Batut *et al* [1990], Mohan *et al* [1991], Kuppasamy and Abdullah [1992], applied decomposition to hydro thermal coordination problems.

$$\begin{aligned}
& \text{Minimise} && \sum_{f=1}^F c_f \left(\sum_{t=1}^T \sum_{i \text{ fueled by } f} \sum_{j=1}^J \sigma_{i,j} e_{i,j}^t \right) - \sum_{h=1}^H \psi_h s_h^T \\
& e_{i,j}^t, q_{h,j}^t, w_h^t, s_h^t, && - \beta_n^t \left(\sum_{i \text{ injects at } n} \sum_{j=1}^J e_{i,j}^t + \sum_{h \text{ injects at } n} \sum_{j=1}^J \alpha_{h,j} q_{h,j}^t \right) \\
& \bar{p}_{x-y,in,l}^t, \bar{p}_{x-y,in,l}^t, && \\
& \bar{p}_{x-y,out,l,k}^t, \bar{p}_{x-y,out,l,k}^t \geq 0 &&
\end{aligned}$$

Equation 2-10

Duality theory states that for any *optimal* solution to the RM problem there exists corresponding prices (β_n^t) that would replicate that solution, in terms of the objective function value, when placed in the dualised objective above.

With the removal of the connecting energy flows, the RM formulation can be separated into a series of generator sub-problems. A hypothetical market for coordinating these generators in this simple two-bus transmission network could proceed as follows:

- 1) Each generator creates an LP offer at its reference node, and loads bid forecast energy requirements at the reference nodes.
- 2) Combining the LP sub-problems, and the two-bus network model, into an LP problem equivalent to the RM formulation and solving clears the market.
- 3) Generators are dispatched according to this solution.
- 4) The dual variables (β_n^t) determine what price loads pay for consumption, and what price generators receive for their energy output.

By definition, this market-clearing LP problem must reproduce the optimal RM solution and hence satisfy every participant. This LP-offer market design may seem attractive, but LP-offer designs have not been implemented in any real market; and according to Culy *et al* [1996] they are not likely to be implemented in New Zealand. Apart from some practical problems, they suggest that this complexity of generator offer and market-clearing model could be seen as undesirable, because managers not skilled in LP would have difficulty understanding the market, and external auditors may find it difficult to assure investors that the market is fair.

An alternative, they suggest, is a simple step supply curve generator offer, i.e. one with no intertemporal links or other complexities of the RM formulation. Simplification of the offer and the market-clearing model design raises two key questions:

- 1) The Existence Question: Can generators represent their complex underlying supply models adequately with simple supply curves?
- 2) The Convergence Question: Is iteration required between solutions of the (simplified) market-clearing model and the generator offers, and if so, would an iterative process converge?

The approach taken here to answering the Existence Question is:

- review previous studies of markets in the economics and OR/MS literature and discover what answers might be expected;
- analyse components of the generator sub-problem to determine what conditions might give rise to inconsistencies with a supply curve model (Chapter 3);
- produce some empirical results that attempt to quantify any theorised effects (Chapter 4)

2.3 Existence of Equilibrium and Convergence

The objective of this section is to describe previous results in the literature that apply to the solution of the decomposed problem presented above. Recall that this thesis is concerned with the 'distortion' imposed on a dispatch as a result of the *simplification* of the offering and market clearing process, in itself. Note especially that it does not address the use of monopoly power by generators.

Economics provides some general results that form a motivation for the decentralisation of the electricity dispatch. Aliprantis *et al* [1989], in their general economic text on the existence and optimality of competitive equilibrium, state, "The classical intuition, that decentralized competitive markets produce out of the self-interested behaviour of economic agents an optimal distribution of resources, dates back at least to Adam

Smith's 'invisible hand'. This intuition is made precise in the two welfare theorems of K. J. Arrow and G. Debreu." And, with respect to these welfare theorems, "...in this setting prices serve as signals of scarcity and agents interact with the market rather than with each other as in the bargaining, implicit in both the core and Pareto optimality". And further, "...every Pareto optimal allocation can be achieved in a decentralized fashion as a competitive allocation subject to income transfers—the second welfare theorem)."

The Arrow and Debreu model of perfect competition, around which these results are based, has proven equilibrium (existence) properties, and very strong affinities with price decomposition problems of the type described here. The problems of obtaining convergence for such models have received attention in the literature and various practical solution methods have been studied. Read [1979] showed that the first-order derivatives of the hydro and thermal generator sub-problems were non-linear and discontinuous (as thermals switch in and hydro storage bounds are activated). Newton's method (see Daellenbach, George and McNickle [1983]) requires that the objective, first-order derivative, and second-order derivative functions be calculable and continuous. But this is not the case for this type of problem, firstly because of the shape of the functions and secondly because one can only sample the functions because they are derived from the response of linear programming sub-problems.

Uzawa [1958] claimed a price decomposition convergence result, but, as discussed by Read [1979], it requires that the first order derivatives be bounded, and so the result does not hold for problems with discontinuities in the first-order derivative functions. Read [1979] adapted the method used by Uzawa [1958] to explicitly account for these discontinuities, in his decomposition of a multi-reservoir hydro system. Having largely overcome the first-order difficulties, he went on to observe that the problem was complicated by the intertemporal relationships in the hydro sub-problems. These caused there to be off-diagonal terms in the Hessian matrix of second-order dual derivatives, which meant that supply curves (or "response surfaces"), derived purely from the diago-

nal terms, tended to overstate the flexibility of the hydro system to changes in the energy price. This resulted in very slow convergence.

After the fuel crises of the early 1970's, and at the same time that Read [1979] was working with decomposition of hydro models, there emerged a group of large-scale energy planning models. Murphy [1983] and later Murphy *et al* [1988] reviewed these models. They included: the Brookhaven Energy system Optimisation Model (BESOM) which became TESOM (Time-staged Energy System Optimisation Model); FOSSIL1, which became FOSSIL2, from the Department of Energy USA; the Project Independence Evaluation System (PIES), which became MEFS (Midterm Energy Forecasting System); and the SRI-Gulf model, which became GEMS (Generalized Equilibrium Modelling System). A key feature of these models was their size and scope, an aspect that presented serious managerial and computational problems. The most common solution approach was to decompose the model into manageable units. The models took various approaches to this partitioning, but having done so, there remained the problem of coordinating the separate parts; this is much like the hypothetical energy market described here.

The PIES model, described by Hogan [1977], partitioned the energy model in to detailed structural, supply, and demand models (operated as satellite models) to create supply and demand curves (gradient functions). If interactions between model parts, or between time periods, is small then these curves can be derived by fixing all other prices and simply making the demand, or supply a function of the commodity itself, and not of other commodities, or of the commodity in other periods. Ahn and Hogan [1982] describe the requirements for convergence of the PIES algorithm. They state that if the demand elasticities appearing in the Hessian matrix satisfy the Diagonal Dominance Property, i.e. that the 'own elasticity' is significantly greater in absolute value than the sum of the 'cross-price elasticities', and if the supply curves are monotonically non-decreasing, then an iterative process of solving each market in succession will eventually converge. However, this diagonal dominance condition held only for thermal elec-

tricity systems in the PIES model. They, like Read [1979], observed that convergence couldn't be guaranteed for hydro systems.

2.4 Implications for the Simplified Market in this Thesis

This chapter has presented a formulation that could be used to represent the New Zealand hydrothermal system called the Reference Model (RM). It has described a hypothetical market, based on decomposition of the RM, which can be guaranteed to reproduce the solutions of the original model (under the assumptions stated above). It then argued that this type of model is unlikely to be implemented in the real system, and motivated a simplified design that has supply curves for generator offers. Two questions were then asked about the suitability of the simplified design: The Existence Question, and The Convergence Question.

Chapter 3 addresses the Existence Question by analysing a hydro generator's LP sub-problem and drawing conclusions about the existence of a supply curve consistent with the underlying supply model of that generator. Chapter 4 summarises a set of experiments performed on a model of the New Zealand system using variations of this simplified design.

The literature indicates that the convergence of a market type equilibration process may be affected by the way in which the approximate derivatives used by the algorithm, represent the complexity of generators' underlying cost structures. The form of these approximate derivatives may be seen as analogous to the form of participant bid/offer curves in a real market. Thus the results of Ahn and Hogan, and Read indicate that there could be significant problems in gaining convergence of a simplified market for systems with a large hydro component. Historically, in the New Zealand system, up to 80% of generation has been provided by hydro. This implies that there might be significant problems coordinating the New Zealand system with a simplified market; however,

there is an important distinction between these earlier models and the simplified market studied in this thesis.

The aim of this previous work was to produce solution algorithms for decomposed problems, where direct solution of the complete problem was unachievable—due to problem size and the state-of-the-art in solution algorithms. This thesis assumes that competing hydro (and thermal) generators are capable of formulating complete system models (albeit with estimated data about their competitors), and of estimating cleared quantities and nodal energy prices from its solutions. Thus generators are assumed to prepare offers around some system-wide solution that is reasonably close to the optimum, and “slow convergence” will manifest itself in terms of the rate at which the market adjusts to changing situations in real time

In reality, of course, generators would not necessarily be motivated to reproduce the optimum. Rather, those with ‘market power’ might distort the solution away from the optimum for their own benefit (gaming), and it might be suggested that this will introduce distortions which dominate any due to convergence, or approximation, effects. It should be recognised, then, that this thesis has limited goals. Specifically, it does not address gaming, and nor does it attempt to address the Convergence Question. Chapters 3 and 4, and the experimental results presented in Appendix C, merely attempt to determine the distortion inherent in the offer design by performing a single iteration of the market process. It does not attempt to determine how efficient any given design would be at converging from a bootstrap position in an iterative offering and market-clearing scheme.

Further, this thesis assumes that the market will not implement an iterative offering/bidding regime as part of its benefit maximising objective; rather iterative offering/bidding would occur only to allow generators/loads to update their positions in the face of updated system data as the real-time conditions are revealed. Chapter 4 presents some experiments in which generators *are* allowed to update their offers, but this is aimed at measuring the robustness of the market when physical circumstances change, rather than quantifying the convergence properties of any particular design.

All that can be observed here, regarding convergence, is the ‘stickiness’ of the solution close to the optimum. In this respect, ‘poor’ convergence could actually be ‘advantageous’ (from the perspective of these experiments). If a simplified design proves robust, in these experiments, then it might be concluded that the objective ‘surface’ around the optimum is fairly flat; hence convergence from any given starting point, or indeed divergence away from the optimum, could be difficult, and vice versa.

Chapter 3

Existence of Step Supply Curves

In the simplified market, introduced earlier, generator offers would take the form of simple step supply curves. This chapter attempts to characterise the circumstances under which the solutions to generators' LP sub-problems (derived earlier from the Reference Model), can be used to derive step supply curves. In this discussion, a market design with supply curve offers means:

- offers must take the form of simple monotone non-decreasing step functions;
- no 'contingent' offers are allowed, i.e. generators are not allowed to submit a set of supply curves applying to different circumstances, e.g. high, medium and low inflow/demand cases

This chapter considers two main variations of these supply curves:

- 1) Fixed supply curves, i.e. a single supply curve that applies to the whole day.
- 2) Sculpted supply curves, i.e. a set of supply curves, one for each period of the day.

The ‘existence’ of supply curves is studied where generators, prior to the day, know the energy market clearing prices, and where generators, prior to the day, can only estimate the energy prices. It is assumed that generators, in order to account for this uncertainty, solve their LP sub-problems for each of several scenarios. The conjecture is considered that certain operating constraints (particularly intertemporal constraints), may make the LP sub-problems respond to energy prices in such a way that a supply curve cannot be ‘drawn’ through the observed price and response pairs. Focus is given to the hydro station’s sub-problem, as this is the most relevant to the New Zealand context. More specifically, the discussion will consider the conditions under which the following hypotheses hold:

Hypothesis 3.1: The response to price of the LP model of a hydro station is consistent with a fixed supply curve in all cases.

Hypothesis 3.2: The underlying response to price of the LP model of a hydro station is consistent with a sculpted supply curve in all cases.

3.1 Forming Simple Supply Curves

Consider the Reference Model (RM) described in Chapter 2. That discussion showed that the model could be decomposed into a set of LP sub-problems, one for each generator. Further, it described a theoretical market in which these LP sub-problems were used as offers, and combined with other parts of the RM model to clear the market. The nodal energy prices and cleared quantities could be used to describe the optimal solution to that model, and were used in that context to coordinate the generators and settle the market.

It is well known that the dual solution is not necessarily unique for a given primal solution and vice versa. Further, if all input costs are scaled by a factor then prices would be scaled by the same factor without changing the primal solution. In fact, even the relative prices could be scaled, within limits, without affecting the primal solution—

provided this scaling does not change the position of the marginal stations in the 'merit order'.

It is obviously not necessary to use the price and quantity pairs from the optimal primal/dual solution to reproduce the primal or dual solutions individually. However, this thesis is concerned with reproducing the primal and dual solutions exactly because they will be used as signals on the supply and demand sides respectively.

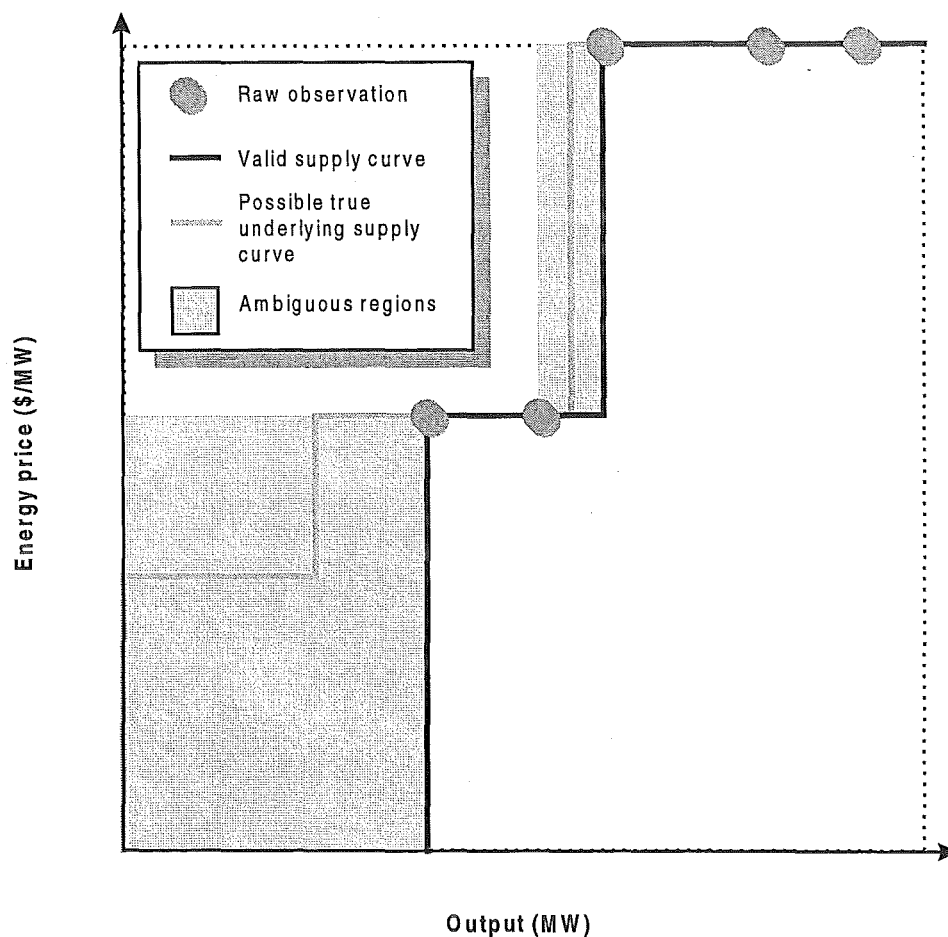


Figure 3-1: Raw observations of energy price and quantity generated

Forming a supply curve could be accomplished by:

- plotting each price and quantity pair from the solution(s) to the LP sub-problem;
- drawing a monotone step function through the points

Figure 3-1 illustrates this process for a set of monotone observations; but even in this case the raw observations won't necessarily define a unique step supply curve. The shaded areas on the graph show the regions in which there is some ambiguity about where the true supply function (if one exists) might lie; but to reproduce the observed pairs all that is required is to draw a single curve through all of the observed points.

Section 3.2 determines under what conditions this can be accomplished for a fixed offer; Section 3.3 extends this to a sculpted offer.

3.2 Existence of Fixed Supply Curves

Figure 3-2, Figure 3-3, and Figure 3-4 illustrate some results from WEMSIM (an implementation of the RM described in Appendix B). They show the energy price and output pairs for a river chain for different system demands. Each point represents one period's price and quantity pair. The raw observations in Figure 3-2 are entirely compatible with a step supply curve. Any such curve passing through the observed points and lying inside the shaded regions would be acceptable.

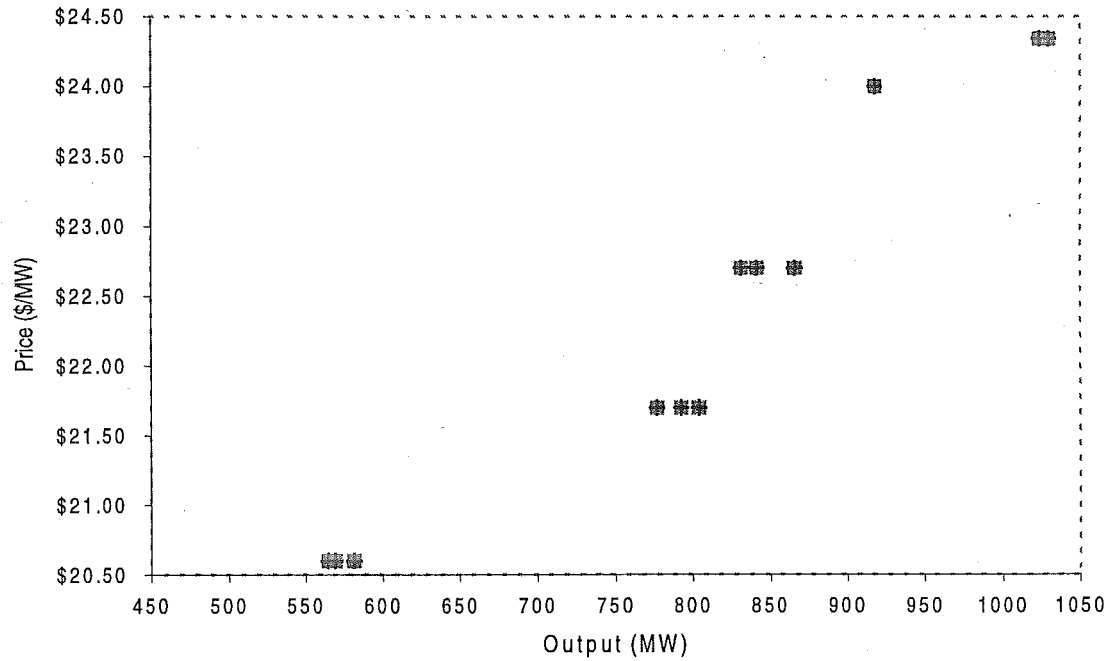


Figure 3-2: Observed response of a hydro river chain for a high (average) demand case

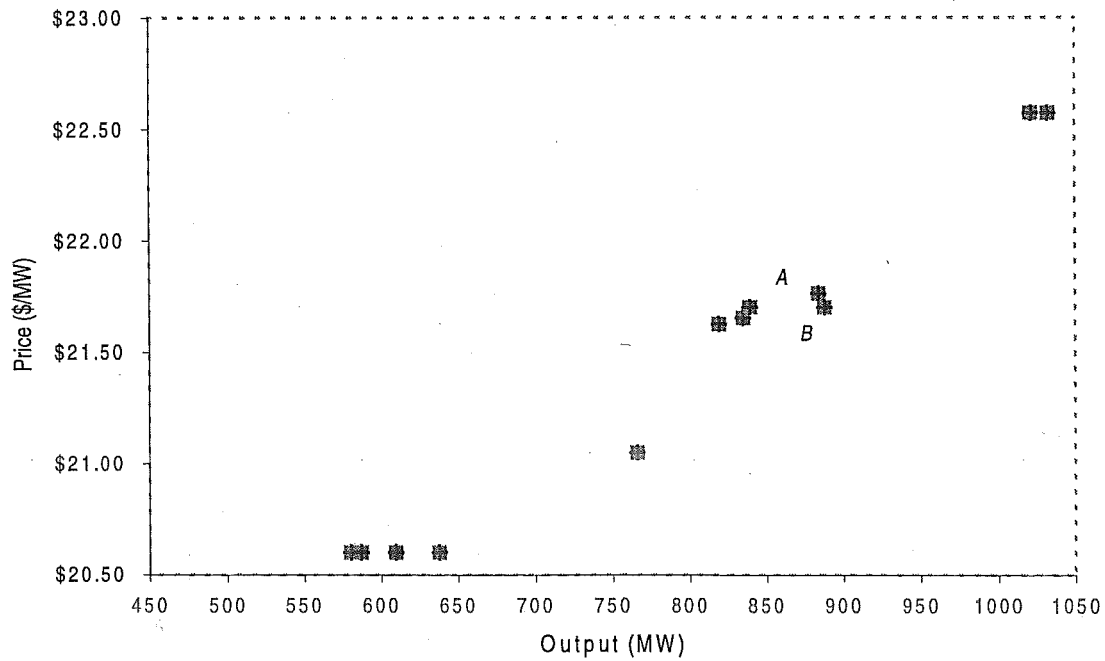


Figure 3-3: Observed response of a hydro river chain for a low (average) demand case

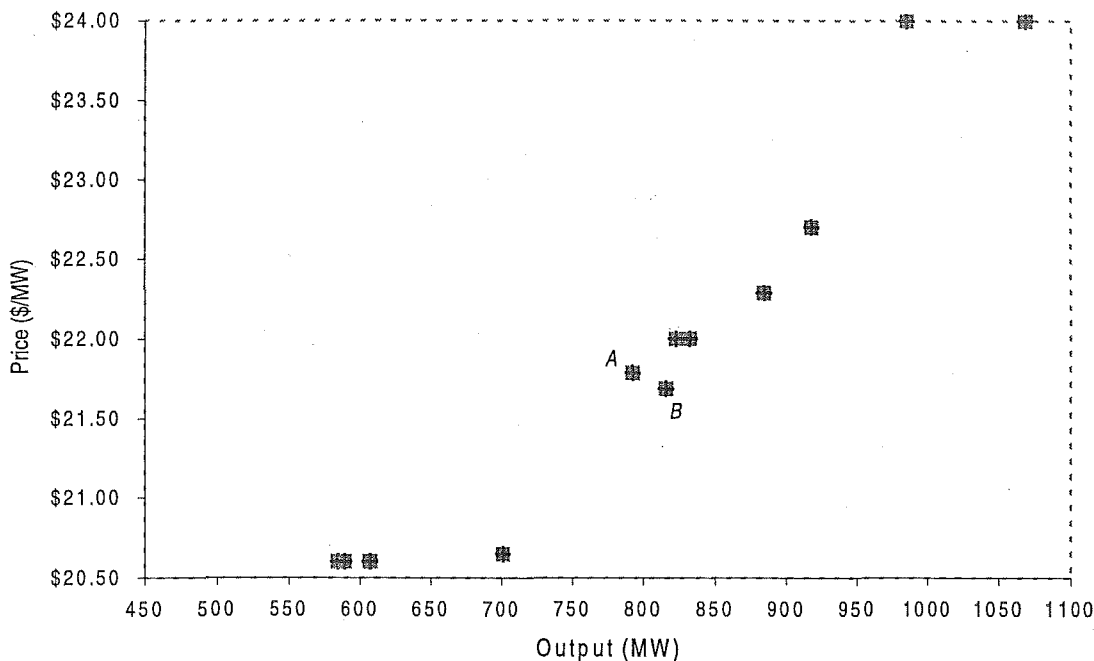


Figure 3-4: Observed response of a hydro river chain for a moderate (average) demand case

Figure 3-3 and Figure 3-4, for the low and moderate demand cases respectively, have some inconsistent responses. In Figure 3-3 point A has a higher output for a lower price than point B, and in Figure 3-3 point A has the same relationship to point B. It is not possible to draw a single step function through these observations. This empirical data contradicts the first hypothesis, i.e. it appears that a fixed supply curve does not exist for a hydro river chain in every case.

A hydro river-chain's underlying supply curve (if one could be proven to exist) would be the aggregation of the supply functions of the stations in the chain. Although it is conceivable that inconsistencies in individual station responses might compensate perfectly, so that the river chain's overall response is always consistent, in general it seems intuitive to say that if any station's response has inconsistencies then the group's response can not be guaranteed to be consistent.

The following discussion examines the hydro station LP sub-problem to determine under what conditions its response to price is consistent with a supply curve. For clarity, the hydro representation in the RM model will be simplified further. Storage volumes

will now be expressed in equivalent MWh, and variations in unit efficiency will be ignored. In this case the capability of a hydro station with an externally specified water value could be expressed with a fairly simple LP sub-problem.

The objective is to maximise revenues received from the market via the energy price plus the value of water held in storage, as in equation Equation 3-1. This is subject to water flow balance, Equation 3-2; and bounds on releases, Equation 3-3.

$$\text{Maximise } \sum_{t=1}^T \beta^t q^t + c s^T$$

$$q^t \geq 0, s^t \text{ free}$$

Equation 3-1

subject to

$$q^t - s^{t-1} + s^t = n^t \quad \forall t \quad (\psi^t)$$

Equation 3-2

$$q^t \leq \bar{q} \quad \forall t \quad (\delta^t)$$

Equation 3-3

An alternative to explicitly stating the water value, c , is to target a storage volume at the end of the day. This would change the objective function of Equation 3-1 to that shown in Equation 3-4, and it would add Equation 3-5.

$$\text{Maximise } \sum_{t=1}^T \beta^t q^t$$

$$q^t \geq 0, s^t \text{ free}$$

Equation 3-4

$$s^T = s^{T*} \quad (\omega)$$

Equation 3-5

There are no bounds on storage in the first model, and none except for the last period in the second. This will be referred to as the 'unconstrained' case, since fuel (water) stocking policy for the station is free of any constraint except that water might be given an exogenous value, or it might be subject to an end-of-day target.

The dual of the first problem has the objective shown in Equation 3-6. Subject to dual rows for the primal storage variables, Equation 3-7; and dual rows for the primal release variables, Equation 3-8.

$$\begin{aligned} &\text{Minimise} \quad \sum_{t=1}^T (n^t \psi^t + \bar{q} \delta^t) \\ &\psi^t \text{ free}, \delta^t \geq 0 \end{aligned}$$

Equation 3-6

subject to

$$\begin{aligned} \psi^t - \psi^{t+1} &= 0 \quad t=1, \dots, T-1 \quad (s^t) \\ \psi^T &= c \quad (s^T) \end{aligned}$$

Equation 3-7

$$\psi^t + \delta^t \geq \beta^t \quad \forall t \quad (q^t)$$

Equation 3-8

The dual variable ψ^t reflects the value of water held in storage in each period, or more simply the implied incremental fuel cost for the station. There is only one feasible solution for this dual, i.e. $\psi^t = c \quad \forall t$. The dual also shows that the release variables, q^t , can be non-zero only when $\beta^t \geq c$. The optimal release policy for the station can be described by the decision rule of Equation 3-9.

$$\begin{aligned} &\text{if } \beta^t > c \text{ then } q^t = \bar{q} \\ &\text{if } \beta^t = c \text{ then } 0 \leq q^t \leq \bar{q} \\ &\text{otherwise } q^t = 0 \end{aligned}$$

Equation 3-9

This rule applies to every period of the day; hence any set of observations of the station's response to price would be consistent with a step supply curve. In fact the decision rule shown describes a supply curve with one step of size \bar{q} offered at a price of c . It is important however to note that a pathological case can be constructed, even for this most simple of models. If the maximum release bounds were allowed to vary across the day then it would be trivial to elicit a set of responses from the model was not consistent with a fixed supply curve. However, no rational generator would deliberately create such inconsistencies if they could avoid them at no cost, hence this analysis will concentrate on inconsistencies that might arise 'naturally' from the normal modelling of a hydro station.

The target-driven primal formulation replaces the exogenous water value, c , with an end of day target. The dual objective of Equation 3-6 changes to Equation 3-10, and Equation 3-7 changes to Equation 3-11.

$$\text{Minimise } \sum_{t=1}^T (n^t \psi^t + \bar{q} \delta^t) + s^T \omega$$

$\psi^t \text{ free}, \delta^t \geq 0, \omega \text{ free}$

Equation 3-10

$$\begin{aligned} \psi^t - \psi^{t+1} &= 0 \quad t = 1, \dots, T-1 \quad (s^t) \\ \psi^T + \omega &= 0 \quad (s^T) \end{aligned}$$

Equation 3-11

The dual states that $\psi^t = \omega$ and $\psi^t = \psi^{t+1}$ $t = 1, \dots, T$, i.e. the water value must be constant over the entire day, as before, but in this case it will equal the shadow price on the end-of-day storage target. Thus the decision rule of Equation 3-9 applies in this case, with $c = \omega$, which is not known *a priori*, but can be observed. Hence, observations of the station's response must be consistent with a fixed supply curve in this case too.

In reality storage capacity is not unlimited over a horizon as long as a day. Introducing a lower bound on storage adds Equation 3-12.

$$s^t \geq \underline{s} \quad \forall t \quad (\sigma^t)$$

Equation 3-12

The dual objective has an additional term, $\sum_{t=1}^T \underline{s}^t \sigma^t$, and Equation 3-7 changes to Equation 3-13.

$$\begin{aligned} \psi^t - \psi^{t+1} + \sigma^t &= 0 \quad t = 1, \dots, T-1 \quad (s^t) \\ \psi^T + \sigma^T &= c \quad (s^T) \end{aligned}$$

Equation 3-13

The new dual variables, σ^t , are associated with greater-than-or-equal-to primal constraints and hence must be non-positive in sign. Rearranging the equation defining the water values in Equation 3-13 gives Equation 3-14.

$$\begin{aligned}\psi^t &= \psi^{t-1} + \sigma^{t-1} \quad t = 1, \dots, T \\ \psi^T &= c + \sigma^T\end{aligned}$$

Equation 3-14

Using the Complementary Slackness Theorem, this may be interpreted as $\psi^t < \psi^{t-1}$ if the period $t - 1$ lower storage bound is binding (with a non-zero multiplier), and $\psi^t = \psi^{t-1}$ otherwise. With an end-of-day storage target $-\omega$ replaces c in Equation 3-14, which leads to the same result. These equations imply that the value of water in storage will fall when a lower storage bound is binding, and remain constant in between times—a well known result, described by Read [1979].

To illustrate this effect consider the example of a hydro station with a maximum output rate of 150 MW, facing the peaked price profile shown in Figure 3-5, with prices ranging from \$1.00-6.00/MWh, constant inflows of 50 MW, and a water value of \$2.00/MWh.

Figure 3-8 shows the optimal storage and release policies with and without the lower storage bound. The unconstrained solution would run the reservoir out of water in period three, and must have a constant water value, $\psi = c = 2$, as shown in Figure 3-7. The lower bound is active during periods four and five in the constrained case thus $\psi^5 < \psi^3$. The final period's lower bound is not active, so $\psi^6 = c = 2$, and it follows that $\psi^5 = \psi^6 = 2 < \psi^3$. Thus the water value is higher leading up to the lower bound becoming active than in the unconstrained case. Replacing the externally set water value with a storage target of 50 m³ replicates this result—the multiplier on the end-of-day storage target would replace c in the equation for ψ^6 .

Figure 3-8 graphs price versus the station's response for the constrained case. The overall station response is non-monotone and hence not consistent with a fixed supply curve.

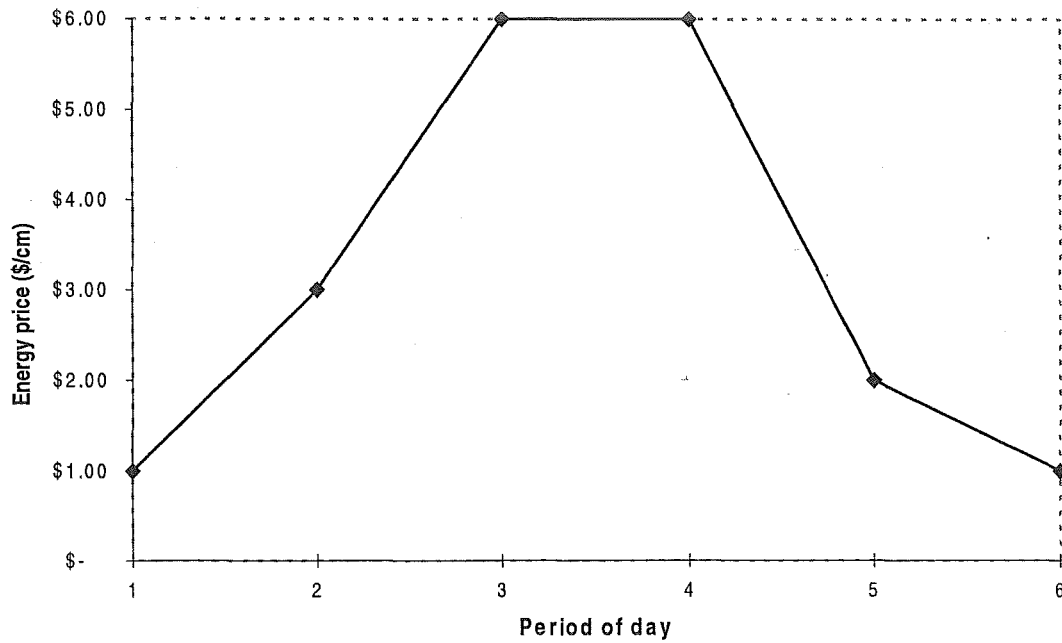


Figure 3-5: Energy price profile

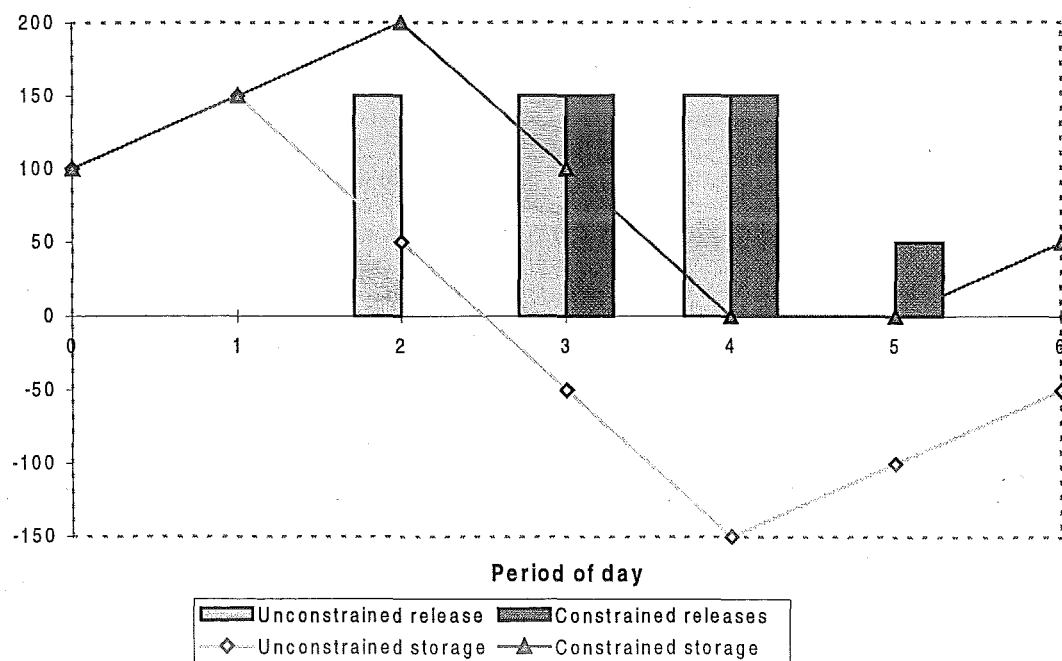


Figure 3-6: Storage and release profiles with and without a lower bound on storage

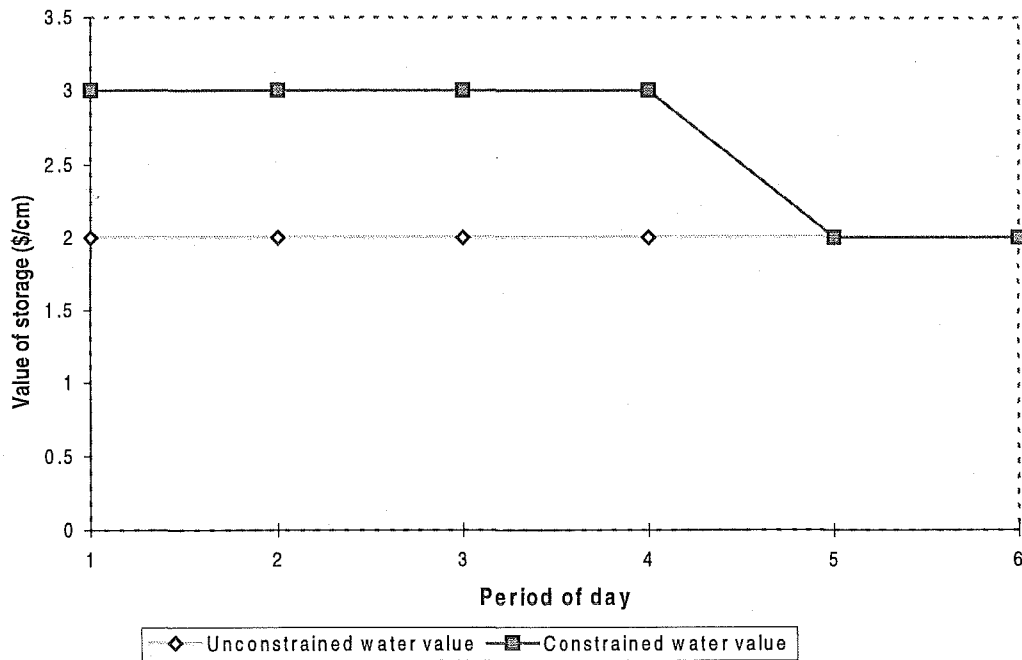


Figure 3-7: Value of water in storage

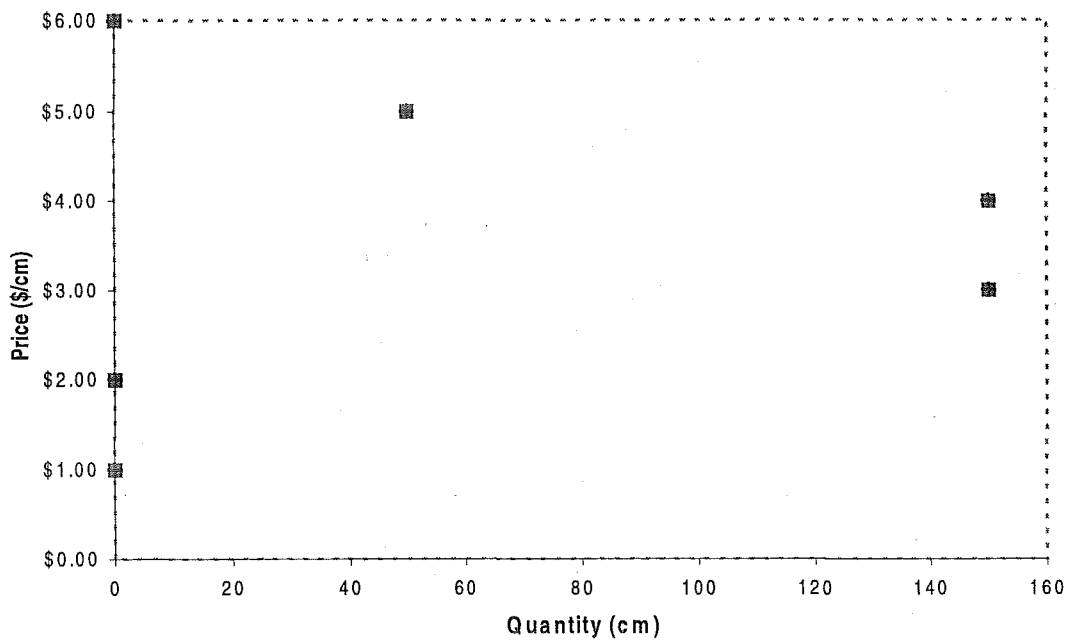


Figure 3-8: Raw price and quantity observations

Consider the addition of an upper bound on storage, as in Equation 3-15.

$$s^t \leq \bar{s} \quad \forall \quad (s^t)$$

Equation 3-15

The dual objective has the new term $\sum_{t=1}^T \bar{s} \tau^t$ and Equation 3-7 changes to Equation 3-16.

$$\begin{aligned} \psi^t - \psi^{t+1} + \sigma^t + \tau^t &= 0 \quad t=1, \dots, T-1 \quad (s^t) \\ \psi^T + \sigma^T + \tau^T &= c \quad (s^T) \end{aligned}$$

Equation 3-16

Because $\tau^t \geq 0$ the water value must rise when the upper bound is binding (with a non-zero multiplier), which mirrors the effect of a binding lower bound—this result too may be found in Read [1979].

From this discussion it appears that the response to price of the LP model is consistent with a fixed supply curve, but only if the water value is constant over the day. The above examples have demonstrated that Hypothesis 3.1 does *not* hold when a lower or upper bound on storage is binding (with a non-zero multiplier) because the water value must change. This causes the station's release policy to be time dependent i.e. there is no underlying fixed supply curve. Hence, Hypothesis 3.1 does not hold in all cases, i.e. *the response to price of the LP model of a hydro station cannot be guaranteed to be consistent with a fixed supply curve in all cases*. Of course it might be possible to observe a set of responses that is consistent with a fixed supply curve even when the underlying supply curve does not exist (e.g. a subset of the points in Figure 3-8), but in general such an 'inferred' supply curve cannot be guaranteed to exist.

3.3 Existence of Sculpted Supply Curves

With sculpted offers, the supply curve is allowed to be different in every period of the day. A hydro station that cannot form a fixed supply curve, like that in Figure 3-8, could form a set of supply curves, one for each period, with each curve based on one observa-

tion. If all system data are deterministic, then the optimal station response in each period can be determined with certainty, and the set of optimal prices and quantities can be used to form a set of sculpted supply curves. Hence, Hypothesis 3.2 holds when all data are deterministic.

In reality, sculpted offers like this might perform very poorly, since real-time requirements will inevitably deviate from the generators' projections. To improve the robustness the supply curve generators could prepare their curves using a number of scenarios. For Hypothesis 3.2 to hold under certainty the station's observed response *within a period and across scenarios* must be consistent with a step supply curve.

Figure 3-9, Figure 3-10 and Figure 3-11, which were produced by WEMSIM, show the price and output pairs for five demand cases in a single period. Figure 3-9 shows a monotone response because the observed price is the same in each case and only the quantity generated varies. Figure 3-10 and Figure 3-11 show highly non-monotone responses to price over the five cases. So it would appear from these empirical results that the response to price of the LP model of a hydro station is not consistent with a sculpted supply curve in all cases.

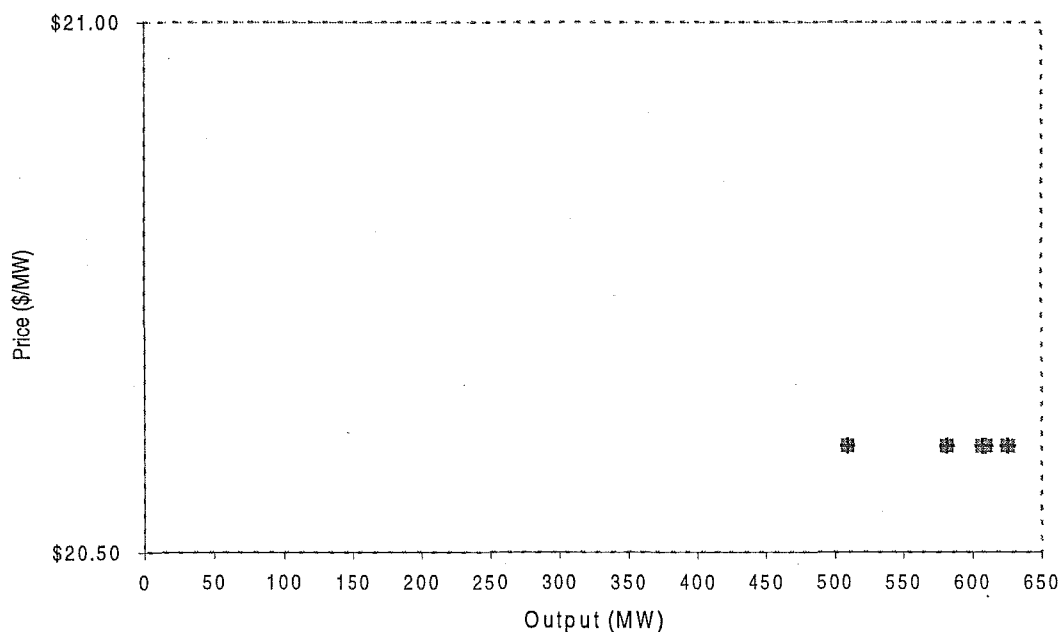


Figure 3-9: Observed response of a hydro river chain, period 3

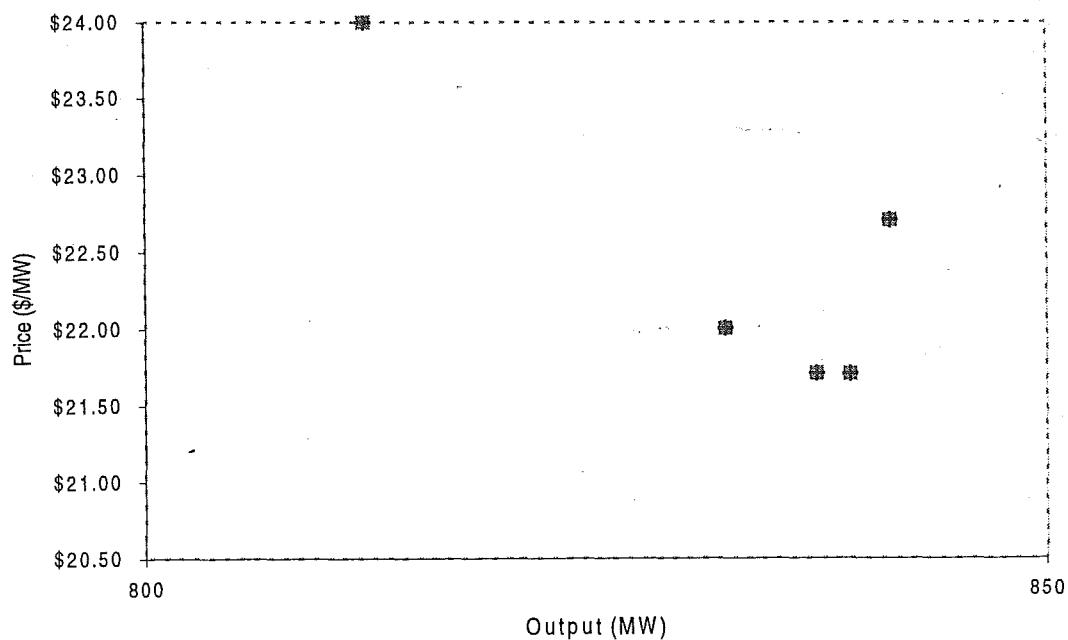


Figure 3-10: Observed response of a hydro river chain (period 6)

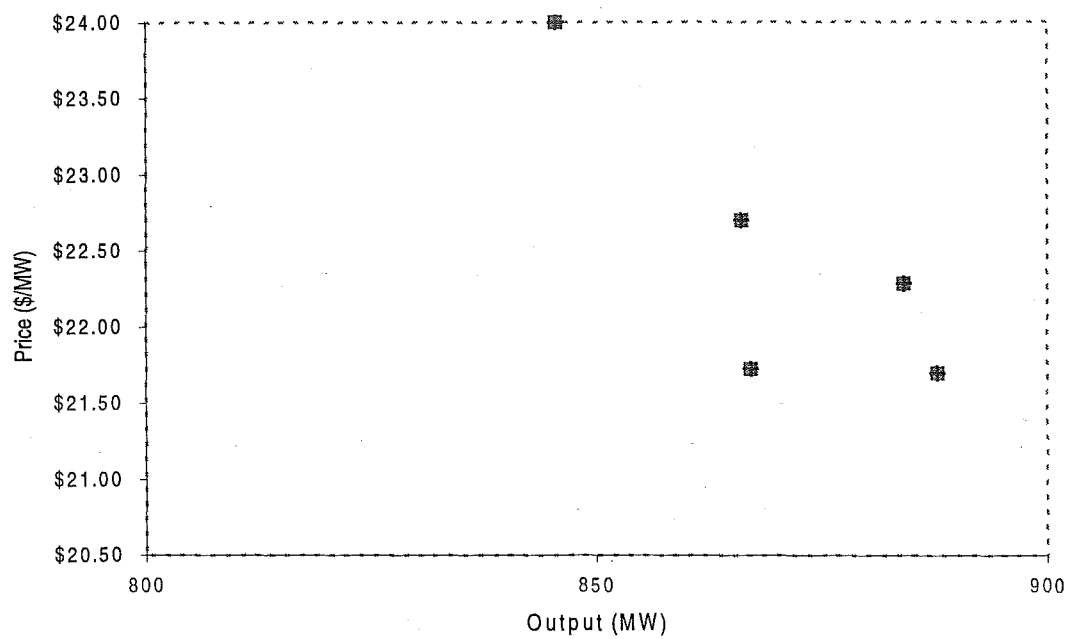


Figure 3-11: Observed response of a hydro river chain (period 10)

The earlier discussion showed that when a lower or upper bound on storage is binding the water value is time-dependent. The observations in Figure 3-10 and Figure 3-11 suggest that the water value of a hydro station in any single period is dependent on the water values in all other periods.

Consider Equation 3-8 of the dual LP problem, shown earlier, which is associated with the release variables in the primal LP problem. Rearranging this gives Equation 3-17.

$$\psi^t \geq \beta^t - \delta^t \quad \forall t$$

Equation 3-17

If the upper release limit is slack, i.e. $0 \leq q^t < \bar{q}$, then $\delta^t = 0$ and the constraint is changed to Equation 3-18.

$$\psi^t \geq \beta^t$$

Equation 3-18

Equation 3-18 can be interpreted as saying that if the station is 'marginal' in any given period the water value must be greater than or equal to the energy price in that period. If the upper release limit is strictly active, i.e. $q^t \geq \bar{q}$ and $\delta^t > 0$, the constraint may be interpreted as saying that when the station is releasing at maximum in any given period the water value must be strictly greater than the energy price in that period. These are well known results, used by Ring [1995] in the context of nodal spot pricing.

These equations show that the water value in any given period is related to the energy price in that period. The earlier discussion showed that, in the presence of a binding lower or upper bound on storage, the water value is time dependent, i.e. the water value in any given period depends on the water values in other periods. The discussion in this section has shown that the water value in any given period is also a function of the energy price in that period. It follows that, in the presence of binding lower or upper storage bounds, the water value in any given period is dependent on the energy price in all other periods. In fact, with an end-of-horizon target, total release must be the same in each scenario, and the average water value must match the average price for that sce-

nario, with the release pattern being driven by the relative water values and prices. This implies that Hypothesis 3.2 does not hold, and a hydro station's responses across multiple scenarios may not be consistent with a set of sculpted supply curves in every case.

Consider the previous example and the additional price scenario shown with it in Figure 3-12. The optimal storage trajectories and release policies are shown in Figure 3-13 and the optimal water values in Figure 3-14. Figure 3-15 shows the prices and responses for the third period. Even though the price is higher in the second scenario the station's response falls because this price is lower relative to those in other periods, i.e. the observed inter-scenario response in period three is non-monotone and a supply curve cannot be formed for that period.

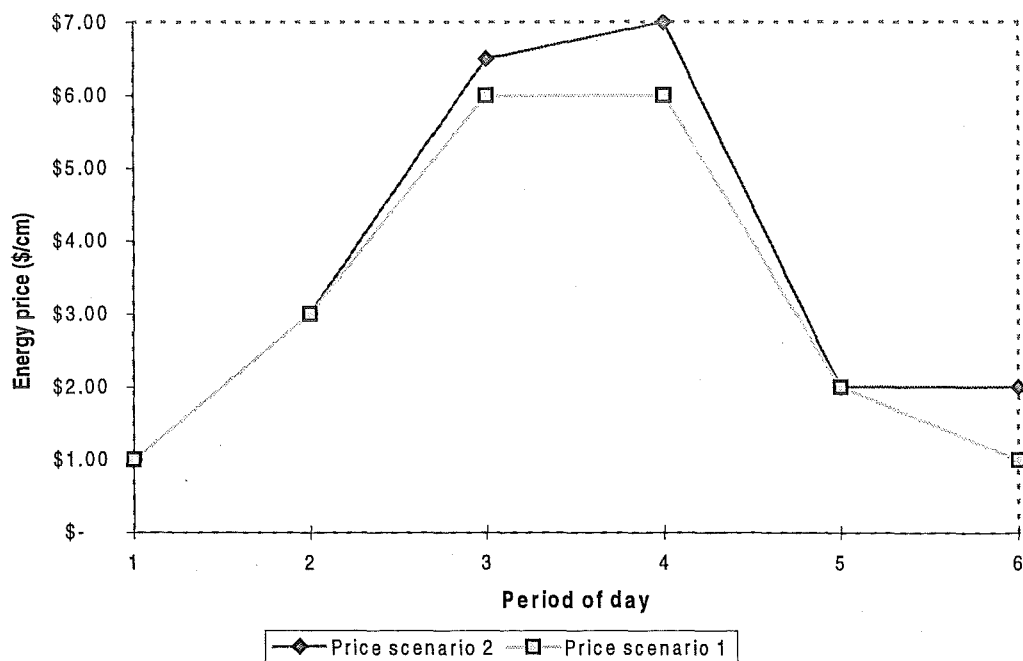


Figure 3-12: Price scenarios

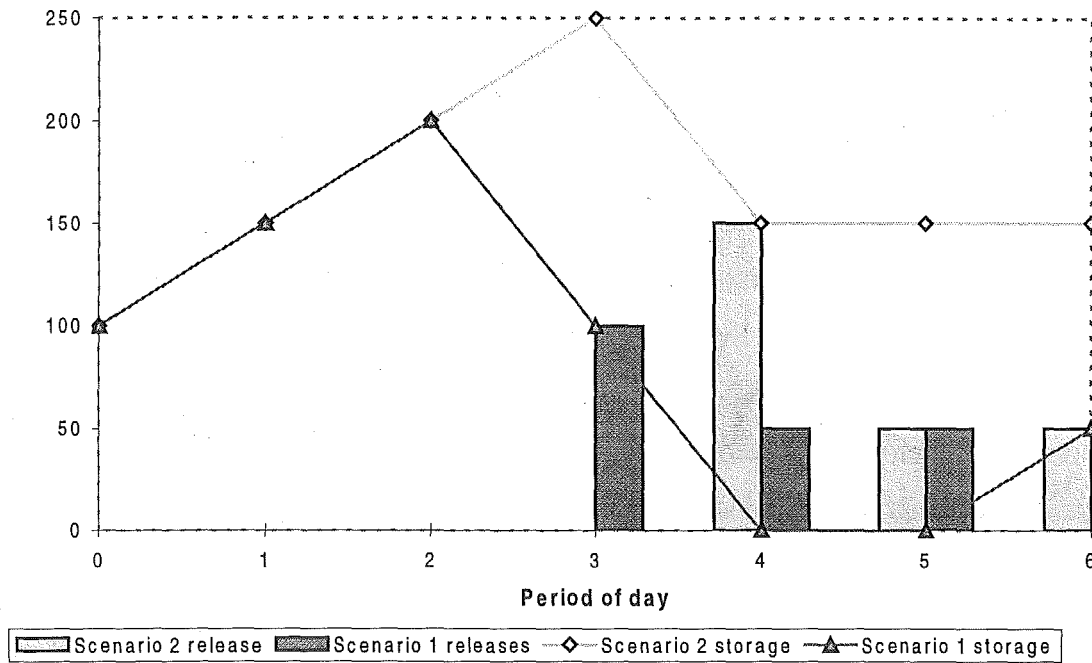


Figure 3-13: Optimal storage trajectories and release policies for two price scenarios

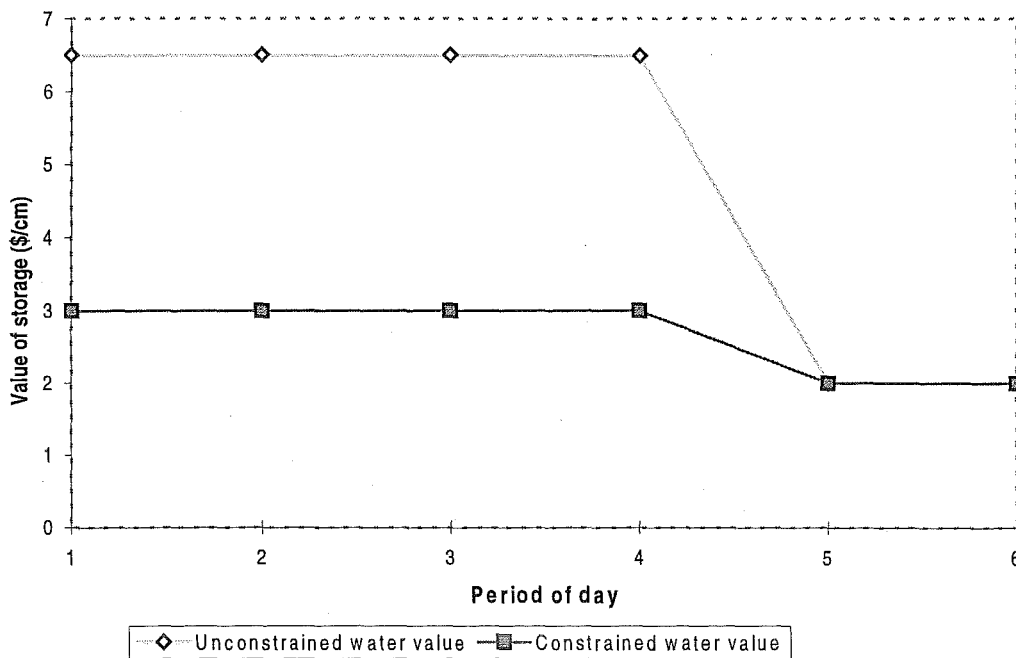


Figure 3-14: Water values for two price scenarios

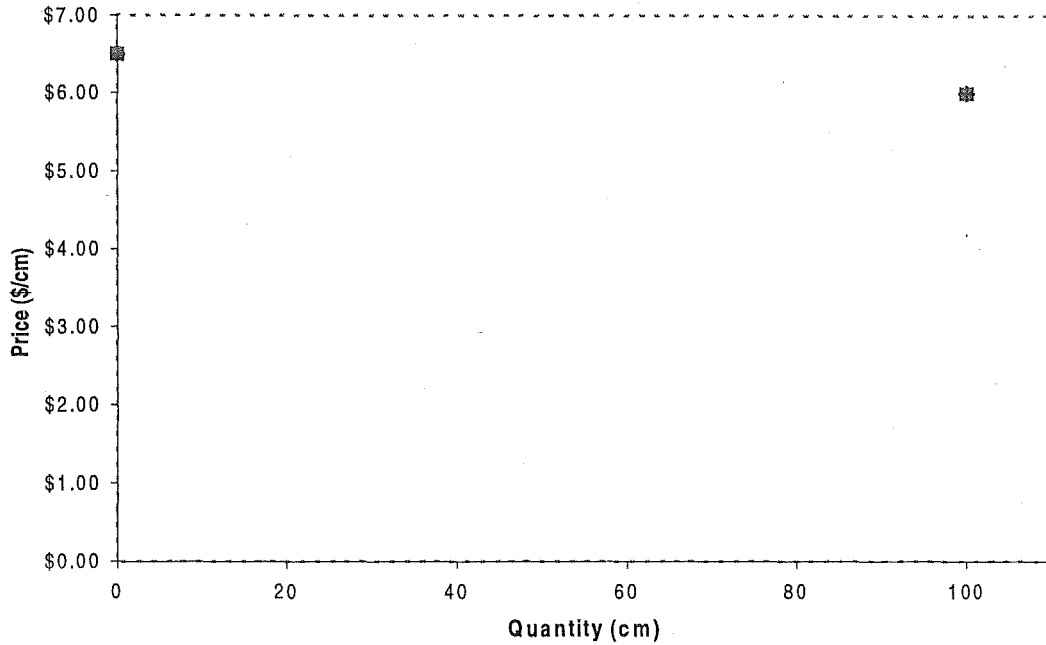


Figure 3-15: Period three prices and responses for two scenarios

Energy limits, like those implied by bounds on storage, are only one type of intertemporal constraint. To further test the observed effects consider the addition of rate-of-change of release constraints to the primal LP problem, as in Equation 3-19.

$$\begin{aligned} q^t - q^{t-1} &\leq \bar{q} \quad \forall t \quad (\theta^t) \\ q^{t-1} - q^t &\leq \tilde{q} \quad \forall t \quad (\phi^t) \end{aligned}$$

Equation 3-19

The non-negative dual variables θ^t and ϕ^t appear in the dual objective with the term $\sum_{t=1}^T (\bar{q}\theta^t + \tilde{q}\phi^t)$ and Equation 3-8 becomes Equation 3-20. If $\theta^t > 0$ and all other multipliers are zero then this becomes Equation 3-21.

$$\psi^t + \delta^t + \theta^t - \theta^{t+1} + \phi^t - \phi^{t+1} \geq \beta^t \quad \forall t \quad (q^t)$$

Equation 3-20

$$\psi^{t-1} \geq \beta^{t-1} + \theta^t$$

Equation 3-21

Equation 3-21 can be interpreted as saying that the value of release is lowered in period t and raised in period $t - 1$ when a rate-of-increase of release constraint is binding in

period t . If the energy prices in those periods have the relationship $\beta^{t-1} < \beta^t$ then the effect of this constraint is to equalise the marginal value of release (MVR). This seems intuitive, since to relieve an active rate-of-increase of release constraint in period t one should seek to equalise releases in periods t and $t - 1$, i.e. increase release in period $t - 1$ (which implies a higher MVR) and decrease it in period t (which implies a MVR). The opposite effect can be observed for rate-of-decrease of release constraints.

In summary, rate-of-change constraints have the effect of altering the release policy by adjusting the marginal value of release. It follows that these constraints when binding can cause time or scenario dependence in water values.

As an illustration assume that the example station described earlier can change its release up or down by no more than 50 MW. Multipliers on these constraints will adjust the 'apparent' energy price in each period. Specifically, the 'apparent' energy price (and hence lower bound on the water value) in each period is given by Equation 3-22.

$$\psi^t \geq \beta^t - \theta^t + \theta^{t+1} + \phi^t - \phi^{t+1}$$

Equation 3-22

Figure 3-16 shows the original energy prices, β^t , and those prices adjusted by the multipliers according to Equation 3-22. When this is combined with the water value graph of Figure 3-18, it is clear that the effect of the multipliers is to equilibrate the MVR and the price in periods when the rate-of-change constraints are active. Figure 3-17 shows the storage and release profiles with and without the rate-of-change constraints active. Figure 3-19 plots the prices and quantities that result, which are once again non-monotone.

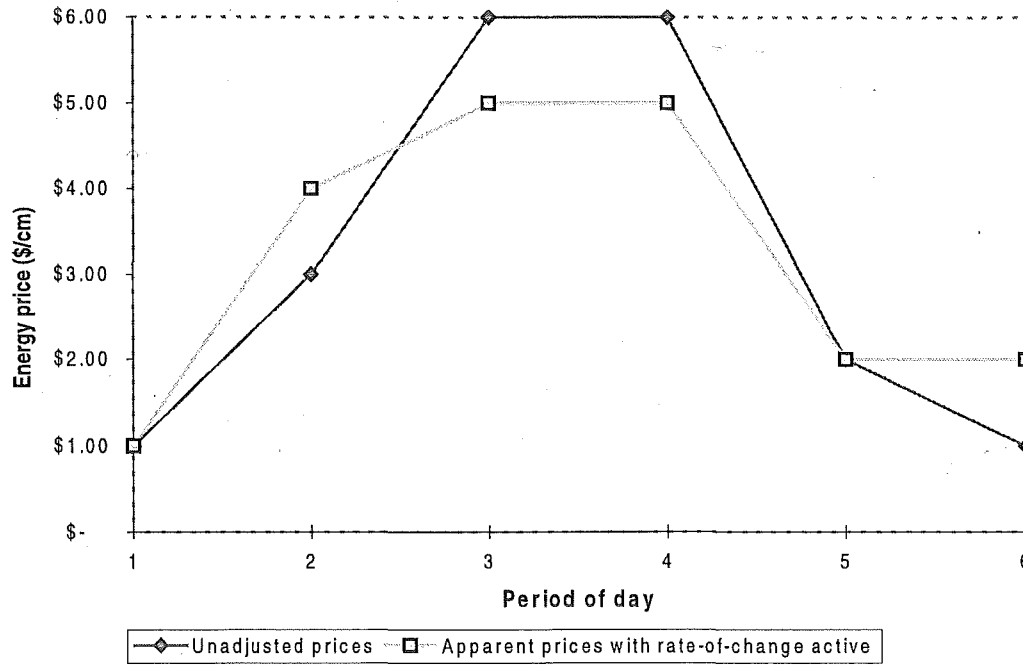


Figure 3-16: Price profile given as input versus that apparent to the station

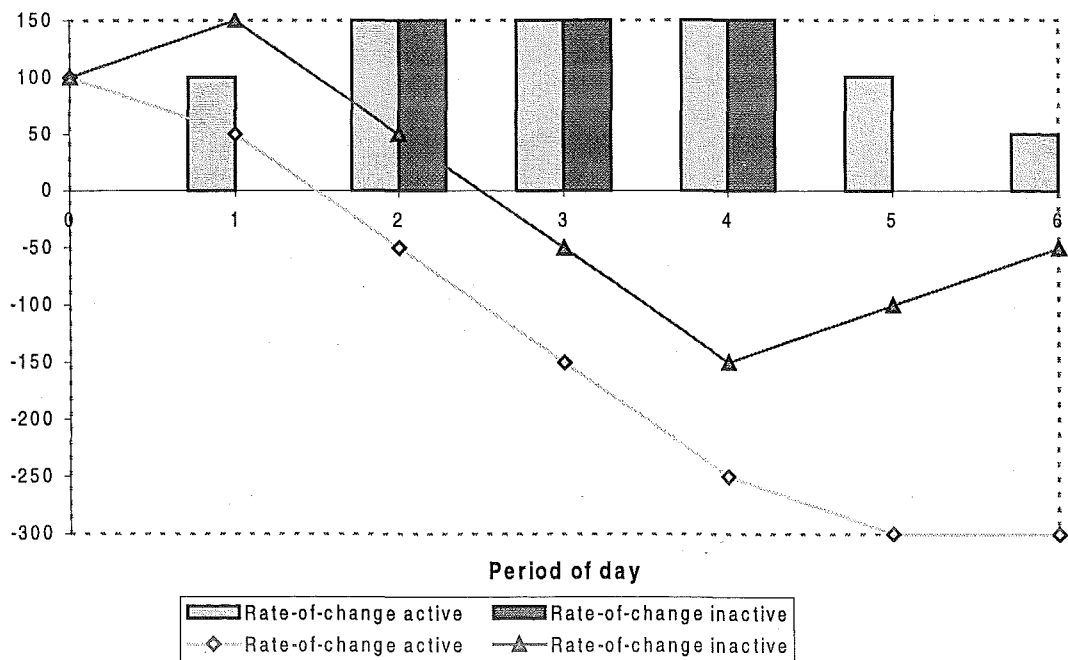


Figure 3-17: Storage and release profiles with and without the rate-of-change constraints

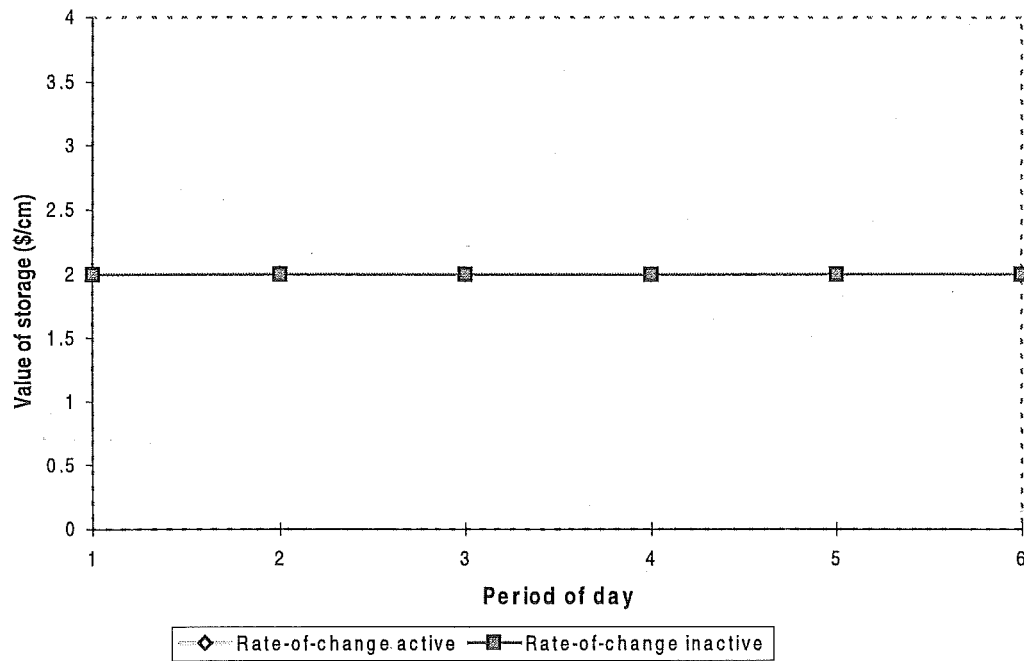


Figure 3-18: Water value profiles with and without the rate-of-change constraints

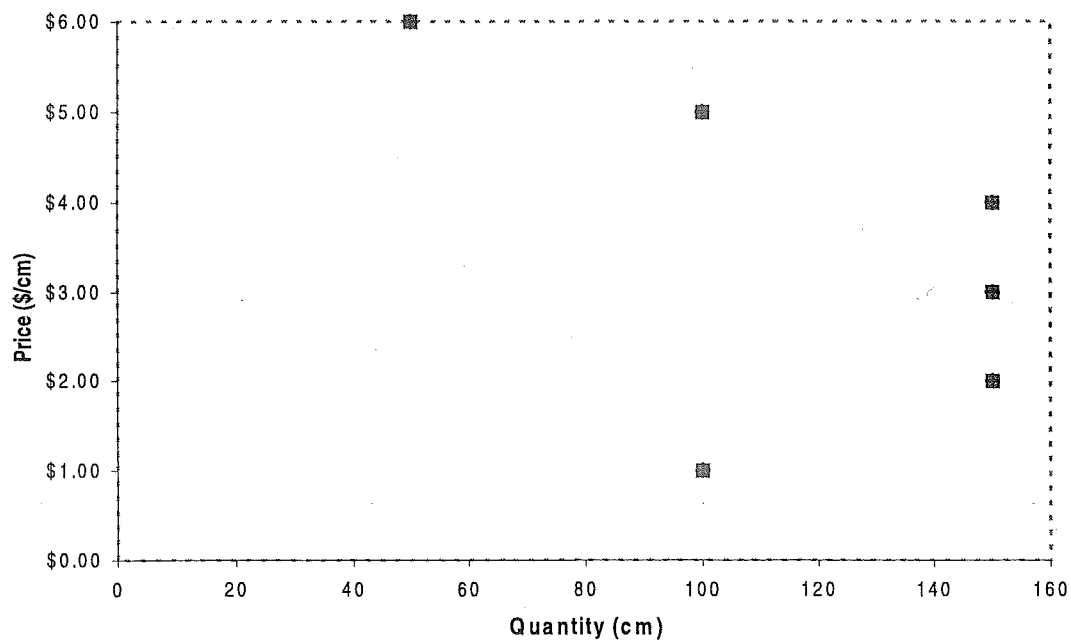


Figure 3-19: Price and quantity observations

3.4 Implications for Offer Formation

This chapter has examined the form of the response to price of the LP model of a hydro station and compared it to a fixed offer and a sculpted offer. Observations made of the dual of the hydro station LP sub-problem support the empirical evidence that this response cannot be guaranteed to be consistent with either of these offer designs in all cases—these conclusions are summarised in Table 3-1.

	<i>All data are deterministic</i>	<i>Data are uncertain</i>
<i>Hypothesis 3.1</i>	does not hold	Does not hold
<i>Hypothesis 3.2</i>	holds	Does not hold

Table 3-1: Conclusions about the existence of supply curve offers

Energy limits implied by binding constraints (of the type investigated above) alter the underlying cost structure of the hydro station, by altering the water values across the day and across scenarios. Often no single supply curve could be formed for the whole day even if the prices are known for certain. However, if sculpted offers are allowed and all data are deterministic then generators can always form an offer in every period using one point per curve.

When data are uncertain, and generators attempt to account for that uncertainty by forming their offers from a number of scenarios, it cannot be guaranteed that a station's observed responses *within* a period will be consistent with a step supply curve.

This chapter has shown that storage bounds and rate-of-change of release constraints can cause non-monotone price response across and within periods by acting on the water values and on the marginal value of release. In general the results of this chapter support the rather intuitive notion that if a generator's actual supply model is more complex than the offer form allowed in the market, then some loss of information may occur—and potentially a loss of efficiency. If simple supply curves are to be used as offers in a real market, the following questions need to be addressed:

- 1) What is the magnitude of the effects theorised here, and demonstrated on small problems, when the problem is scaled up to a realistic size?

- 2) How should non-monotone observations be translated in to supply curves?
- 3) What degree of distortion can be expected from a market using such offers?

The experimental analysis detailed in the appendices and summarised in Chapter 4 attempts to address these questions for the situation of the forthcoming New Zealand energy market.

Chapter 4

Summary of Energy Market Results

The analysis presented in Chapter 3 suggested that the observations of the response of a relatively simple LP-based model a hydro station can not be guaranteed to be consistent with a supply curve. Appendix C shows how approximate supply curves may be derived in ambiguous cases. This chapter measures the distortion that the use of such approximate supply curves might be expected to have on a real market, assuming that generators do not deliberately mislead the market. Frequent reference is made to Appendix C, which contains details of the experimental results. The model used to perform these experiments, called WEMSIM, is described in Appendix B. Appendix A describes the LP formulation used in WEMSIM.

Sections 4.1 and 4.2 describe rules defining the two types of market used: the Day-ahead Market, and the Periodic-update Market. For each experiment, a 'direct cost' distortion and 'transaction cost inclusive' distortion are calculated. In addition, the amount to which generators are forced to default on cleared offers and/or spill water is meas-

ured. The first set of experimental results, in Sections 4.3 and 4.4, compare various offer designs in the Day-ahead Market, Section 4.5 summarises experiments with the Periodic-update Market.

4.1 The Simplified Market

The degree of simplification of a market design can be characterised by the way it sets rules for:

- how complicated an offer the market will accept, e.g. the market may only accept supply curve offers;
- how much the offer is allowed to vary over a day—it might be fixed for a day (*fixed offer*) or be allowed to vary throughout the day (*sculpted offer*);
- how often generators are allowed to update their offers—this could be daily (*Day-ahead Market*) or every few hours (*Periodic-update Market*);

A fixed offer must represent a system using the same cost structure throughout the day. Thus, it would seem appropriate that it be used to represent collections of generating units as there is potentially more scope for absorbing cost variations at that level. The same argument in reverse can be applied to using sculpted offers at a lower level. Unit level offers have been proposed in the New Zealand system for thermal units but not for hydros. For convenience, this discussion concentrates on station level offers for thermal and offers for hydro river-chains (referred to as ‘hydro groups’), and leave it as a further exercise to test the effect of offering in larger or smaller units.

A Day-ahead Market could consist of three phases, which repeat each day:

- 1) **The Offer Formation Phase** (up to 24 hours before the actual day): Generators prepare their offers based on their expectation of the day ahead and submit them to the market.
- 2) **The Ex-ante Market Clearing Phase** (up to 24 hours before the actual day): Taking the offers and adding them together to form an aggregate energy supply curve clears the market. The expected demand is met from that curve,

producing a set of prices in each island for every period of the day ahead.

Generators are signaled the market clearing prices and form contracts with the 'coordinator' at those prices.

- 3) **The Real-time Dispatch Phase (0–24 hours):** Generators operate their systems to fulfil contracts made in the ex-ante market. (Even if contracts were formed they would probably be financial so that the penalty for contract deviation would be implicit in there implied exposure to spot prices and the adverse input of any contract deviation on those prices.)

In a Periodic-update Market generators would update their offers as the real day is revealed. This market would have the same three phases as the Day-ahead Market but they would be repeated every few hours. In such a market, market-clearing prices might only be firm for those few hours and provisional beyond that.

Although this framework is more flexible, because it allows generators to alter their offers, as circumstances require, it creates additional difficulties for offer formation. In particular the daily cycle is being rolled over continually, which complicates the setting of storage targets. The extent to which this is a problem depends on how flexible the offer form is.

4.2 Experimental Methodology

The experimental section of this thesis used an LP-based simulation model called the Wholesale Electricity Market Simulation Model (WEMSIM). WEMSIM was developed specifically for the purposes of studying the New Zealand electricity market. Technical aspects of the model are described in Appendix B. The model is an implementation of the formulation in Appendix A, and includes:

- a simulated day divided into 12 two-hour periods;
- a model of system demand and HVDC transfer and losses (the demand profile used was taken from a typical winter's day in the current system; this is deliberately higher than the average, on the assumption that demand will increase

between the time this study is completed and the energy market is under-way);

- all major reservoirs and most head ponds, with delay times down river chains and water values on major reservoirs;
- two efficiency segments for each hydro station, one for each thermal station;
- minimum release and rate-of-change constraints for hydros;
- rate-of-change of output constraints for thermal;
- uncontrolled inflows

It also includes models of the offering, market clearing and real-time dispatch processes.

4.2.1 The Day-ahead Market

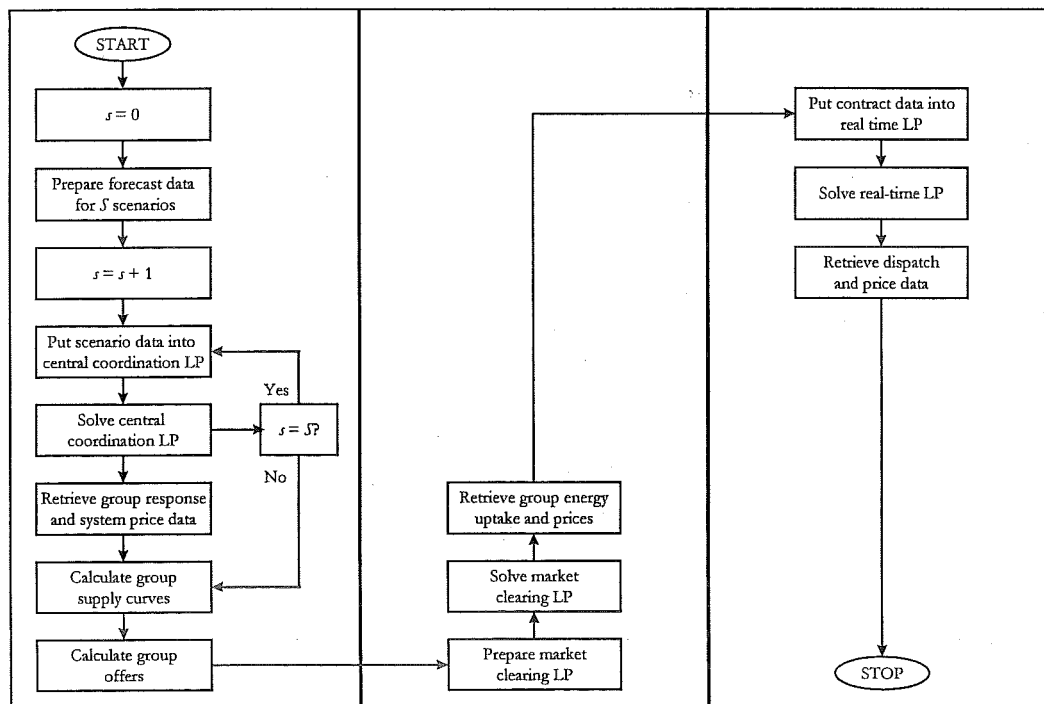


Figure 4-1: Day-ahead market simulation flow chart

The process of conducting one experiment to evaluate the performance of an offer design in the day-ahead market is illustrated in Figure 4-1. It shows three LP problems in operation:

- 1) A centrally coordinated LP problem.
- 2) A market clearing LP problem.
- 3) A decentralised coordination LP problem.

The first step in simulating the market is to solve the centrally coordinated problem. The optimal objective function value for this model, Z_c^* , is recorded. The offer formation phase follows:

- 1) A set of five scenarios based around the actual day's demand is created.
- 2) The centrally coordinated LP problem is solved for each scenario. (It is assumed that the generators will either forecast the energy market clearing prices for the day ahead or simulate the national demand and the effect of competitors to derive price or quantity targets. If all generators acted as perfect competitors, and they accurately forecast demand and prices, the result would be to replicate the central coordinated solution. Hence, the use of the solutions of the central coordination solution as a proxy to the simulations of individual generators.)
- 3) The prices in each period and quantities each generator produced at those prices are recorded, and called 'the scenario data'.
- 4) Supply curves are derived from the scenario data using algorithms described in Appendix C.

The ex-ante market-clearing phase consists of combining the generators' offers (J bands per offer per generator specified by e MW at a price of $\$/\text{MW}$) into a simple market clearing LP for each period. This LP problem has the variables $e_{g,j}$ for the cleared part of band j of the supply curve offer from generator g . The objective is to minimise the total cost of the cleared offers, as shown in Equation 4-1. This is subject to power balance (and associated multipliers), Equation 4-2; definition of power flow received,

Equation 4-3; bounds on power flow, Equation 4-4; and bounds on bands of the supply curve offer, Equation 4-5. (Refer to Section 2.1 for a description of the variables and parameters defining the power-flow equations.)

$$\begin{aligned} &\text{Minimise} \quad \sum_{g=1}^G \sum_{j=1}^J c_{g,j} e_{g,j} \\ &e_{g,j} \geq 0 \end{aligned}$$

Equation 4-1

subject to

$$\begin{aligned} &\sum_{g \text{ injects at } n} \sum_{j=1}^J e_{g,j} + \\ &\sum_x \sum_{l=1}^{L_{x-n}} \left(\bar{p}_{x-n,in,l} - \sum_{k=1}^K \bar{p}_{x-n,out,l,k} \right) + \\ &\sum_y \sum_{l=1}^{L_{n-y}} \left(\bar{p}_{n-y,in,l} - \sum_{k=1}^K \bar{p}_{n-y,out,l,k} \right) = \Delta d_n \quad \forall n \quad (\beta_n) \end{aligned}$$

Equation 4-2

$$\begin{aligned} &\bar{p}_{x-y,in,l} - \sum_{k=1}^K \bar{\lambda}_{x-y,l,k} \bar{p}_{x-y,out,l,k} = 0 \quad \forall x-y, l \\ &\bar{p}_{x-y,in,l} - \sum_{k=1}^K \bar{\lambda}_{x-y,l,k} \bar{p}_{x-y,out,l,k} = 0 \quad \forall x-y, l \end{aligned}$$

Equation 4-3

$$\begin{aligned} &\Delta \bar{p}_{x-y,l} \leq \bar{p}_{x-y,in,l} \leq \Delta \bar{p}_{x-y,l} \quad \forall x-y, l \\ &\Delta \bar{p}_{x-y,l} \leq \bar{p}_{x-y,in,l} \leq \Delta \bar{p}_{x-y,l} \quad \forall x-y, l \\ &\Delta \bar{p}_{x-y} \leq \sum_{l=1}^{L_{x-y}} \bar{p}_{x-y,in,l} \leq \Delta \bar{p}_{x-y} \quad \forall x-y \\ &\Delta \bar{p}_{x-y} \leq \sum_{l=1}^{L_{x-y}} \bar{p}_{x-y,in,l} \leq \Delta \bar{p}_{x-y} \quad \forall x-y \end{aligned}$$

Equation 4-4

$$e_{g,j} \leq \bar{e}_{g,j} \quad \forall g, j$$

Equation 4-5

The optimal dual variable values are the ex-ante market clearing prices at which the generators form contracts for their cleared generation with the ‘coordinator’.

The final phase in the day-ahead market simulation is the real-time dispatch. It is assumed that each generator will attempt to meet the ex-ante quantities, but that there is a facility for inter-generator trading that allows them to under or over-run these quantities at a certain 'transaction cost' levied on a per MWh basis.

The real-time dispatch model is essentially the same as the central coordination model except for the treatment of hydro storage targets and end-effects, which requires a special approach to deal with possible deviations from the 'ideal' storage trajectories, see Appendix A. The main addition to this model is that of output targets for the generators of the general form shown in Equation 4-6. In that equation e_g^t and e_g^t are the generator's target under and over runs respectively. These variables enter the objective function with the term $\Omega \sum_{g=1}^G (e_g^t + e_g^t)$ where Ω is the penalty cost for failing to meet a target.

$$\sum_{i \in g} \sum_{j=1}^J e_{i,j}^t + \sum_{h \in g} \sum_{j=1}^J \alpha_{h,j} q_{h,j}^t - e_g^t + e_g^t = D_g^t$$

Equation 4-6

The optimal value of the objective for this LP problem, Z_M^* , will include the 'transaction costs' and so is not directly comparable with Z_C^* . Thus two measures are calculated⁸: the Percentage Total Direct Cost Distortion, Equation 4-7; and the Percentage Total Transaction Cost Inclusive Distortion Equation 4-8. (Recall that Z_C^* is the objective function value from the central coordination model. It is formulated to measure the amount of 'fuel' used by thermal and hydro sources in the 'ideal' case.)

$$100 \times \frac{Z_M^* - \Omega \sum_{t=1}^T \sum_{g=1}^G (e_g^t + e_g^t) - Z_C^*}{Z_C^*}$$

Equation 4-7

⁸ According to Williamson [1986] this direct cost could be interpreted as a cost of transactions *not made* by generators. In this discussion, these direct costs are calculated separately and only refer to the second component as the transaction cost.

$$100 \times \frac{Z_M^* - Z_C^*}{Z_C^*}$$

Equation 4-8

These results plus the total MWh target defaults form the basis for the comparison of various offers with the Day-ahead Market.

Treating the ex-ante contracts with a target constraint in this manner implies that the generators will try to find a feasible dispatch to meet their target if they can for a lower cost penalty than Ω . Otherwise they will trade their way out of their obligations for an additional total transaction cost fee of Ω per MWh. Because the validity of the results produced depends significantly on the setting of Ω , emphasis is given in the experiments to the impact of variations in Ω , with markets requiring higher values of Ω representing cases in which defaults on targets are heavily penalised and/or trading is difficult.

4.2.2 The Periodic-update Market

The Periodic-update Market is essentially just the Day-ahead Market with a 'roll-over' of the planning horizon every x hours (x is the number of hours between offer updates), except that:

- 1) Some extra effort is required in setting end-of-horizon storage targets. (In particular it is not clear what the storage targets should be, especially since generators knowledge of the day is improving as it is revealed.)
- 2) The penalty cost, Ω , is assumed to be high over the first x hours and starts to fall after that.

At the conclusion of a simulation run, the distortions are calculated in the same manner as the day-ahead market, allowing for comparison to be made between the same types of offers in these markets.

4.3 Fixed Offers

The following aspects of fixed offers were investigated:

- 1) Offers with narrow bands versus offers with wide bands.
- 2) Three-band offers based on a single scenario versus five-band offers based on multiple scenarios.
- 3) The deterministic versus uncertain demand cases.

4.3.1 Simple Three-band Fixed Offers

Section C.2 presents an exploratory analysis of very basic three-band fixed offers formed from a single forecast of the energy market, when the generators accurately forecasts prices. The results were:

- direct cost distortions for fixed offers were estimated to be quite high at between 2–5%;
- and transaction cost inclusive distortions were estimated at 3.0–7.0%

On inspection of the results, this appears to be due to the amount of flexibility the market is given in setting targets across necessarily wide central bands. This is especially so when the central band is wide enough for hydro group's optimal storage trajectory to be achievable. The setting of poor output targets, as often occurred in this case, required generators to default on targets to a large extent, and in sometimes spill is required to avoid overrunning unrealistic targets.

To spread the implicit risk involved with a fixed offer, sub-bands were added to the offers—up to a total of five bands. Scenarios in which generators correctly forecast demand were combined with ones in which they misestimate demand by $\pm 5.0\%$. The results were:

- a drop to 1.34–2.93% in direct cost distortions over all cases;
- and a drop in transaction cost inclusive distortions to 1.64–2.93%

However, the performance of three and five-band fixed offers was inadequate overall. The high level of target defaults indicates this, and consequent high penalty costs were

incurred, and increased spill. For example, setting a 'floor' of a 2% direct cost distortion required transaction cost to be more than \$10.00 per MWh defaulted on (or around 50% of the energy price).

4.3.2 Fixed Offers Based on Multiple Scenarios

Section C.3 examined fixed offers formed from three forecasts of the energy market. The motivation being that more information may improve the performance of the offer under uncertainty. The results here were higher than fixed offers formed out of one scenario with:

- direct cost distortions between 2.1–6%;
- and transaction cost inclusive distortions between 3.3–6.8%

The reason for the increased distortion appears to be that the fixed offer formation process tends to become dominated by the highest demand scenario. This actually exposes the generators to more risk instead of reducing risk as might have been expected. This is because the generators tended to create offers with wider bands to reflect the range of scenarios they evaluated. This lead to the market having more freedom to set unrealistic targets, and hence to a worse performance than fixed offers formed from one scenario.

Section C.5 examines the sensitivity of fixed offers to alternative optima in the market clearing process, water value balance, demand forecasts and tributary forecasts. The results show:

- the best multi-band fixed offers direct cost distortion of 2.1% falls within a range of results from 1.8–2.7% for random perturbations in the price components of the offers;
- fixed offers are highly sensitive to random changes in the price component of the offer;
- when water values become unbalanced, direct cost distortions remain stable but penalty cost inclusive distortions rise as unsuitable targets are rejected more readily

Overall, it can be concluded that, for a market in which generators submit fixed offers, the best results are achieved when hydro groups ‘target’ a trajectory and form their offers around that trajectory, allowing the market little range to distort the dispatch. Direct cost distortions in the range 3–6% can be expected on average. This regime also results in a high proportion of output targets being ignored, resulting in the high penalty cost inclusive distortions of an additional 3–4%. Increase in spill can also be expected for high penalty rate regimes.

4.4 Sculpted Offers

Section C.1 introduces sculpted offers formed by generators using one forecast of the energy market. The purpose of this type of offer is to allow hydro groups with active inter-temporal constraints to follow a desired schedule closely. The following variations were investigated:

- 1) Offers with narrow bands versus offers with wide bands.
- 2) The deterministic versus uncertain demand cases.

4.4.1 The Deterministic Case

The best results were obtained when generators correctly predicted the day-ahead market conditions and created their offers around that prediction using a narrow width central band. The results in this case were:

- direct cost distortions of around 2%;
- and additional transaction cost inclusive distortions of 0–1%

Offers with large bands performed worse on average in both the deterministic and uncertain demand cases. This is because large bands gave the market more opportunity to push hydro generators away from their desired releases and hence from their desired trajectories and the wider the bands the more exaggerated the effect. Figure 4-2 shows a typical result for three-band sculpted offers in the deterministic case.

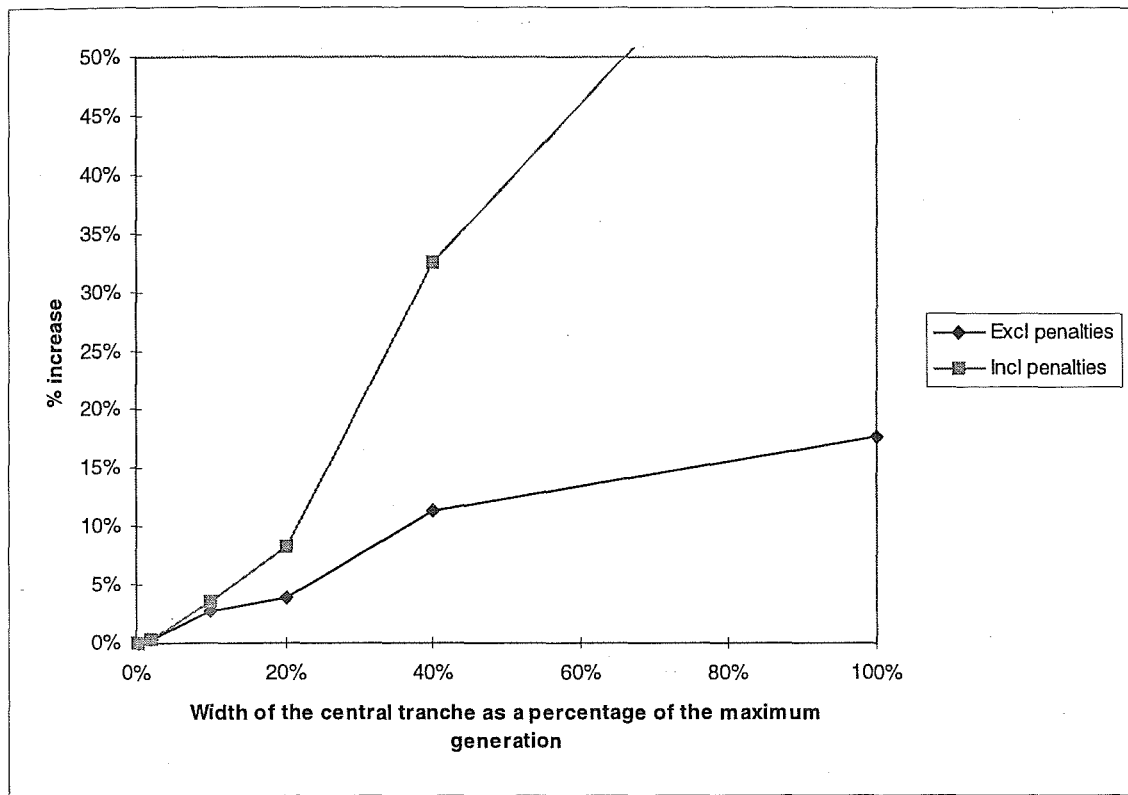


Figure 4-2: Three-band sculpted offers: measured distortions with deterministic demand

4.4.2 The Uncertainty Case

Figure 4-3 and Figure 4-4 show the results for the case in which demand is uncertain. Each line represents a different uncertainty scenario, e.g. 'all high' is the cases in which all generators over-estimate demand (and hence prices).

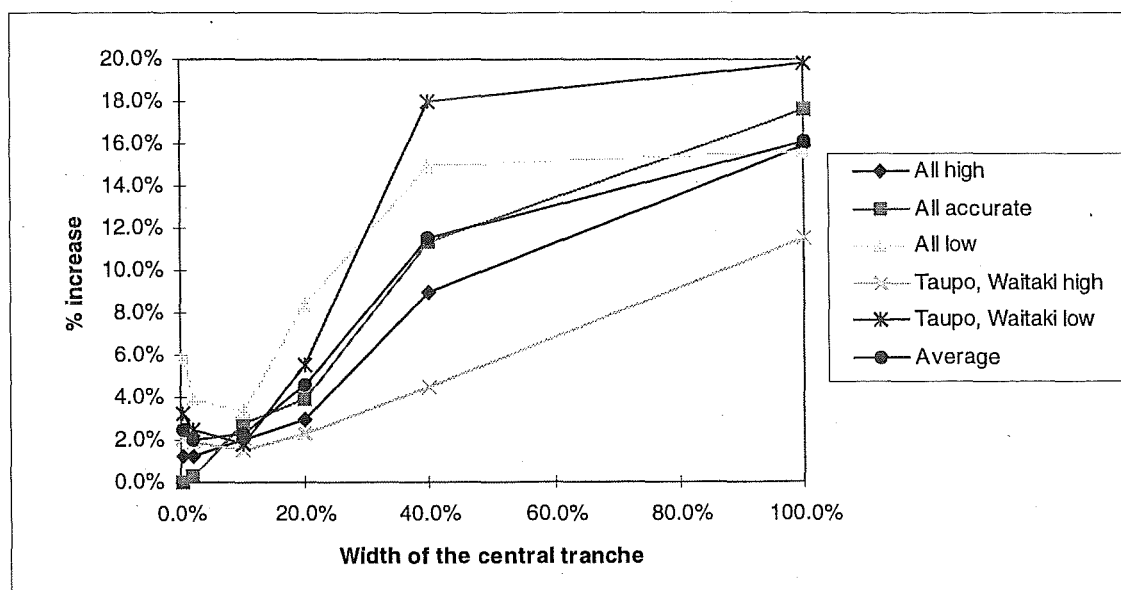


Figure 4-3: Three-band sculpted offers: direct cost distortions with demand uncertainty

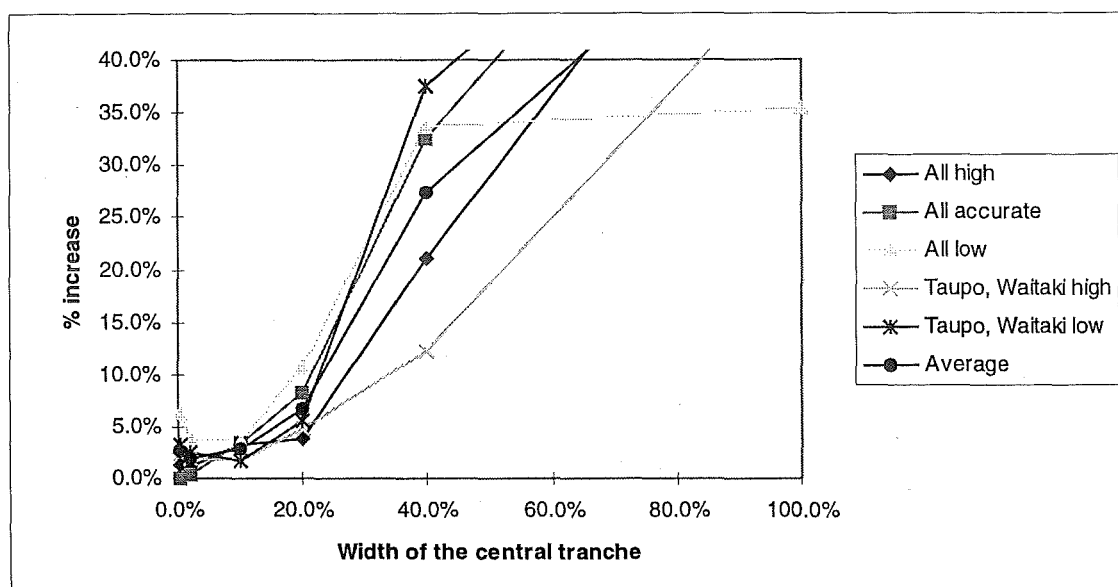


Figure 4-4: Three-band sculpted offers: transaction cost inclusive distortions with demand uncertainty

Even in the uncertainty case, the market is shown to be 'better off' if generators attempt to follow the 'expected' trajectory, than allowing the market freedom to adjust their schedules in the light of better information. Although as Figure 4-4 shows the best performed width of the central band is around 5%, not zero, as this general result might

suggest. These results also showed that the performance of three-band sculpted offers depends heavily on the forecast accuracy.

The final part of Section C.5 adds more bands to the offers in an attempt to improve their performance in cases where the day's demand is uncertain. For higher penalty rates, these five-band offers show reduced cost distortions over the three-band sculpted offers with:

- improvements in the range 0.01–1% (relative to three-band offers) for direct costs;
- and 0–0.85% in transaction cost inclusive distortions

With lower penalty costs, though, these offers show little improvement over the simple three-band offers.

4.4.3 Sculpted Offers Based on Multiple Scenarios

Section C.3 explores the creation of sculpted offers from multiple forecast scenarios. The conjecture tested is that, by including more feasible trajectories in the data used to form offers, generators should produce an offer that returns targets that are more likely to be able to be achieved in real-time.

These five-band sculpted offers do in fact show significant improvement over those formed from only one forecast, with:

- direct cost distortions in the range 0.05–2.4%;
- and penalty cost inclusive distortions in the range 0.3–3.8%

Distortions are under 1% for all but the widest central bands and highest transaction costs. Target defaults and spills are also greatly reduced. Figure 4-5 and Figure 4-6 illustrates some of these results for various penalty cost settings and widths of the central band.

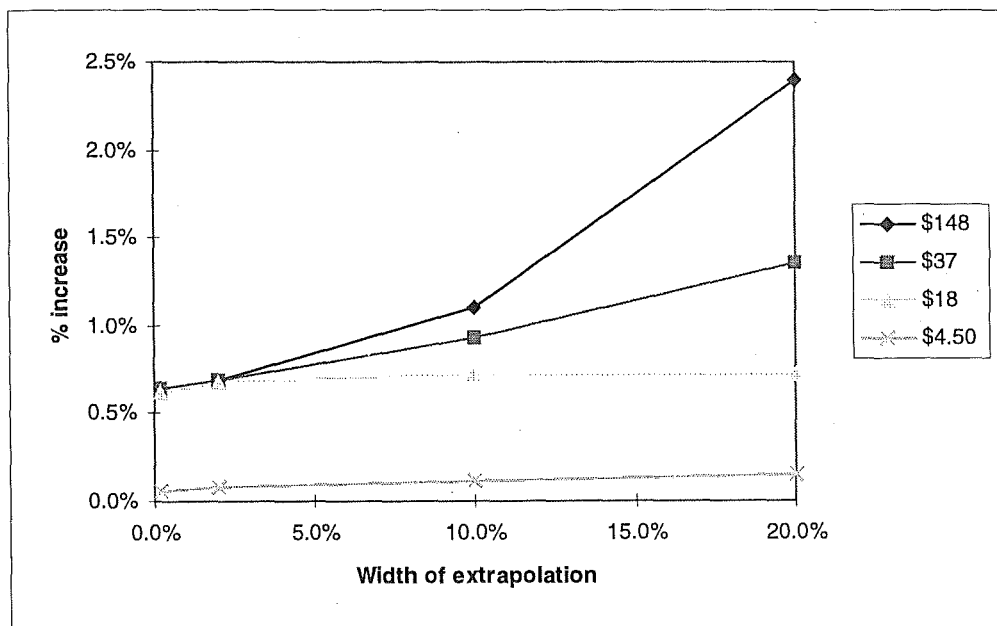


Figure 4-5: Multi-band sculpted offers: direct cost distortions

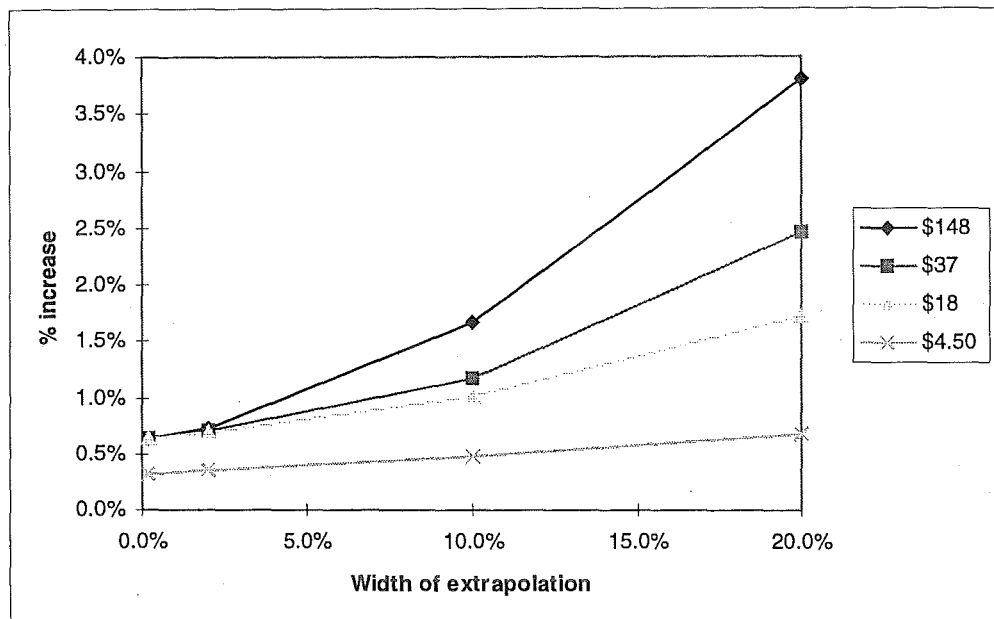


Figure 4-6: Multi-band sculpted offers: transaction cost inclusive distortions

The later part of Section C.3 attempts to improve on these results by reducing the level of arbitrary smoothing of non-monotone observations in the offer formation phase. Two sets of experiments were performed, combining observations from all equi-priced and equi-priced contiguous periods. It is shown that generators are not indifferent about

swapping generation between equi-priced periods—this result seems consistent with the results of Chapter 3.

In the sensitivity analysis of Section C.5 measured distortions for sculpted offers are shown to be more stable than for fixed offers. A test of the ‘moderation factor’ shows that distortions can be reduced further when storage trajectories are smoothed by the method shown in Appendix A. The results were:

- direct cost distortions are reduced to 0.04–0.45%;
- and transaction cost inclusive distortions are reduced to 0.12–0.45%

The best widths for the central band were 0.2–2.0% of the generator’s total capacity. These results were achieved with penalty costs in the range \$4.50–\$148.00. In these ‘smoothed’ cases, generators rarely defaulted on targets and spills remained close to the ideal.

These five-band sculpted offers, formed from multiple scenarios, are shown to be quite robust under uncertainty about demand profile shapes and magnitude. Generators who forecast the correct shape of the demand profile will perform better, in terms of the extent of distortion attributable to them, than those who forecast only the correct magnitude of demand. Overall, for sculpted offers:

- 1) Sculpted offers provided superior performance to the fixed offers in terms of measured distortion over the centralised dispatch.
- 2) Direct cost distortions depend on the transaction cost used, with a range of 0.12–0.45% over a range of \$4.50–148.00 penalty costs.
- 3) Transaction cost inclusive distortions also depend on the penalty rate used, with a range of 0.20–2% within the range \$4.50–148.00 of penalties.
- 4) Offers formed from multiple forecast scenarios performed significantly better than those formed from single forecasts, especially in the uncertain demand case.
- 5) In creating offers, generators should concentrate on correctly forecasting the shape of the day-ahead demand profile as well as the magnitude of the de-

mand. This is due to the importance of rate-of-change and other intertemporal constraints on the offer formation process.

4.5 The Periodic-update Market

The effort required to simulate this type of market is much greater than the day-ahead market. Further, it is not obvious how the market's 'distortion' figures should be calculated.

The day-ahead market was tested by comparing the solution of the 'equilibrium' model, in which storage is recycled (as per Appendix A), with the solution of a decentralised model in which the generators attempt to meet targets while also trying to meet equilibrium storage levels for the next day. When considering the periodic-update market, the equilibrium solution, in which intermediate reservoirs are assumed to recycle every 24 hours, would seem a rather arbitrary benchmark. Hence, it is not clear which solution the periodic-update market's solution should be compared with. This analysis used two methods to gain a measurement of the efficiency of the solutions:

- 1) Allow the model to run through a day but set final storage targets based on the equilibrium day-ahead model.
- 2) Run the model over several sequential days to allow it to reach equilibrium and run a 'clean-up day' at the end to ensure that storage levels are tractable.

The experiments in Section C.4 and Section C.5 examine:

- 1) The periodic-update market versus the best results obtained from the day-ahead market.
- 2) The periodic-update market, run over several sequential days, to examine the longer-term behaviour of the system.
- 3) The effect of uncertainty in forecast data on the performance of the periodic-update market.

Initial experiments with the periodic-update market showed an improvement compared with results from the day-ahead market. The results in the demand uncertainty case were:

- direct cost distortions of averaging 0.15% for the periodic-update market, compared with 0.18% cost distortion for a day-ahead model with sculpted offers formed from multiple scenarios;
- and all targets set from the market were met, and hence there were no additional transaction costs

The periodic-update market was simulated for a number of sequential days, to determine how close the initial experimental solutions were to equilibrium. These experiments showed that:

- the scale of cost distortions was reduced after several days probably reflecting a more suitable equilibrium having been found;
- an estimate for the direct cost distortion, compared with the centrally coordinated solution and using the results from the later days, is about 0.12%;
- the storage, prices, and generation profiles were generally stable, indicating sustainable solutions

Finally, experiments were performed to investigate the effects of uncertainty about the level of tributary inflows into river chains. These experiments focussed on the Clyde system, which showed the greatest inefficiencies in previous experiments in the day-ahead market. The results were:

- when the hydro group initially underestimate the tributary inflows, the periodic-update market, in which they can make some adjustment, shows a significant reduction in the cost of the dispatch, compared with the day-ahead market;
- overestimating tributary inflows did not seem to cause significant problems in these experiments

4.6 Conclusions

This experimental analysis considered various designs of fixed and sculpted offers in a day-ahead market and then used the best offer design to test the periodic-update market.

Results from the day-ahead market showed that:

- 1) When generators submitted fixed offers, the market often set generation targets that resulted in high target defaults, high transaction costs and even increased spill.
- 2) Fixed offers performed poorly when generators attempted to account for uncertainty by preparing their offers from multiple scenarios.
- 3) The best results with fixed offers were achieved when generators ‘targeted’ their expected release/storage trajectories, and allowed the market little flexibility to deviate from that trajectory.
- 4) When generators submitted sculpted offers, prepared from one accurate forecast, the market produced excellent results, with near zero target defaults.
- 5) In the uncertainty case, sculpted offers formed from multiple scenarios performed well and proved robust when input data was uncertain, and generators rarely defaulted on targets set by the market.

Results from the periodic-update market showed that:

- 1) The periodic-update market with sculpted offers produced the best results of all the experiments, with no observed target defaults, even under uncertainty.
- 2) A market which allows generators to update their offers as the day progresses can react far more flexibly and efficiently to unexpected events, compared with a day-ahead market, in which generators are unable to alter offers and hence output targets.

Hence it may be concluded that an energy market in which generators (including hydro stations in river chains) offer simple supply curve offers is feasible. Under the assumptions made here, the day-ahead market performed well when offers were allowed to be

different in each period, but the periodic-update market produced the best results, especially when circumstances differ from the generators' forecasts.

Chapter 5

An Energy and Reserves Market

This chapter introduces a methodology for a competitive spot market in which a single Linear Programming model is used to coordinate energy, and reserve provision, and to determine nodal energy and regional reserve prices, from energy and reserve offers. Section 5.1 describes the role of reserves in the New Zealand system. Section 5.2 describes how reserves were dispatched prior to deregulation. Section 5.3 motivates the integrated approach taken to the design of an energy and reserves market (this discussion is drawn from Read *et al* [1997]). The remainder of the chapter describes a Linear Programming based offering and market-clearing framework for the joint dispatch of energy and reserves. Appendix D provides more details of how the framework can be applied in practice to modelling thermal units and multi-unit hydro stations.

5.1 The Role of Reserves in the New Zealand System

Since this chapter concentrates on a framework for reserve coordination it is convenient to first describe the reserve problem faced in the New Zealand system. The power system consists of two AC sub-systems, for the North and South Islands, connected by a 1200 MW submarine HVDC link. The South Island system is entirely hydro, with moderate sized reservoirs allowing storage of Spring/Summer flows to meet winter peaks, but relatively small inter-annual carry-over. On average, this meets South Island requirements and allows export to the North, where there is a mixed hydro/thermal system. On average 75% of requirements are met from hydro, 7% from geothermal, and the remainder from a variety of thermal plants, mainly burning gas.

There are a number of constraints imposed on the dispatch by the physical operating limits of the generating units and the transmission network. Among these constraints is the requirement that the system operates within an acceptable range of voltages, frequencies, and power flows even if a major generator or transmission link fails. In New Zealand it is the under-frequency constraints which are the most limiting, particularly following a failure of the HVDC link, which is often the largest single supplier in the receiving island. In fact, these constraints are so severe that, on occasion, there has been more capacity committed to reserve provision than to generation in the receiving island.

In defining ‘instantaneous reserve’, the ‘single contingency’ situation is defined, in which there is a failure of the largest unit generating, or a major transmission line such as the HVDC link. Immediately following the disruption the system frequency will fall from the nominal value of 50 Hz and reach some minimum before the remaining generators meet the load imbalance. To meet the imbalance between the reduced generation and the load, three events may occur:

- governors in the automatic generation control systems may react to the falling system frequency, increasing output from generators that are already operating—this is known as partially loaded spinning reserves (PLSR);
- some “tail-water depressed” (TWD) hydro units, synchronised to system frequency but carrying no load, may be brought up to generate;
- the load taken by selected customers may be reduced—this facility, called interruptible load is only available where suitable contracts, and automatic signaling mechanisms have been put in place

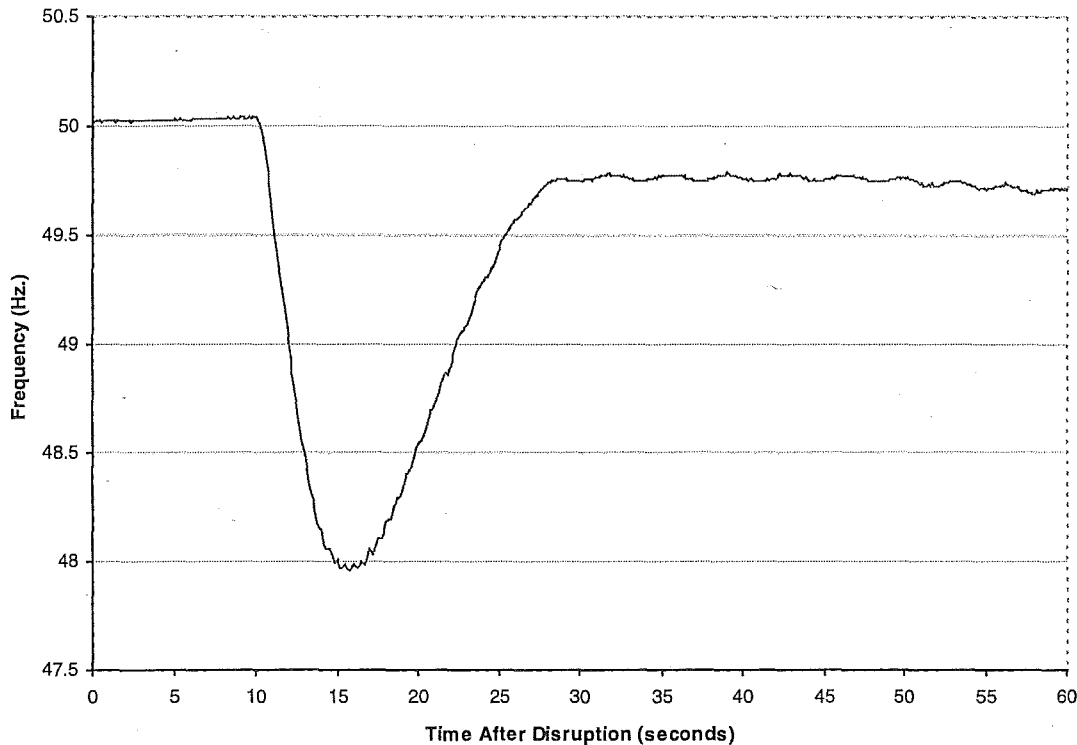


Figure 5-1: System frequency following the sudden loss of a 250 MW unit.

Figure 5-1 is a trace of the system frequency following the loss of a 250 MW unit. The system frequency will generally reach a minimum within five to seven seconds. It is important that the frequency remains above 48 Hz, a requirement referred to as the *minimum frequency constraint*. In addition, there must be sufficient surplus generation capacity available to return the system close to 50 Hz within 60 seconds of the contingency, a requirement referred to as the *surplus reserve constraint*. In the formulation discussed below, these are referred to as the *fast* and *sustained*, or 6 and 60-second reserve constraints.

These requirements imply that the choice of generating units, and their level of generation depends not only on unit availability and cost, but also on the impact each unit can have on these two aspects of the system frequency response during the critical period immediately following a contingency. Some plant provides a rapid initial response but then 'runs out of steam', other plant can only provide delayed response, while yet other plant provides a steady increase in output. The approach outlined below is appli-

cable to either, or both, of the above constraints, provided they could be expressed in terms of a megawatt reserve requirement.

5.2 Central Coordination of Reserves

The central coordination regime, as it existed prior to deregulation, was challenged by the sheer size of the combined energy and reserves scheduling problem needing to be solved centrally on a half-hourly basis; the result was a rather ad hoc treatment of reserves. In fact, prior to 1991, little effort was made to optimise the dispatch of reserves. Control centre dispatchers were responsible for ‘adjusting’ the energy dispatch to meet estimated reserve requirements. Millar and Turner [1992] produced a spreadsheet decision support model for control centre staff. It used a goal-seeking methodology in attempting to minimise the cost of a single half-hour’s energy and reserves dispatch given certain starting conditions as input from the dispatcher.

Experimentation with this simple model led to a greater understanding of the real costs involved in setting aside generating capacity for reserve purposes. This led to further research efforts, with the goal being to produce a model that could be used to find the optimal joint energy and reserves dispatch. Drayton *et al* [1992] detailed the results of part of that research effort. The model that resulted, called Generation and Reserves Scheduling Program (GARSP), optimised the dispatch of energy and reserves in a single half-hour using an aggregated hydro system representation and detailed (non-linear) functions for calculating total system cost and the status of two critical reserve classes—fast and sustained reserves.

The grid operators, Trans Power, have recently redeveloped the Turner and Miller model into an operational tool called FCALC. It is intended that this tool will be used to check that reserve levels in the market clearing solution satisfy the frequency constraints, and that a ‘risk adjustment factor’ will be manipulated in the market clearing model’s reserve requirement constraints, to ensure that they are.

5.3 Principles of a Market for Reserves

Given the strong emphasis worldwide on developing spot markets to achieve real time coordination of electricity production, one might expect that consideration would also be given to developing similar approaches to the coordination of what have traditionally been known as 'ancillary services'. These include reactive power production and various forms of reserves. Indeed the AC pricing model originally developed by Hogan [1992] and described by Hogan et al [1997] provides a theoretical basis for the inclusion of reactive power with active power in a single integrated dispatch. Read and Ring [1996] show, for example, that if reserve constraints are imposed on that formulation, the model will automatically produce prices for the corresponding 'commodities'. In practice, though, ancillary services are generally still dealt with by imposing requirements on participants, or by some form of contracting outside of the spot market.

In New Zealand, spinning reserves are a significant issue, constituting the second most important limitation on dispatch after the requirement that loads are met, and a better approach is considered highly desirable. It may be suggested that reserves could be dealt with by setting aside plant prior to the dispatch of energy, or subsequently perhaps choosing from a portfolio of plant chosen to cover load plus a reserves planning margin. Neither alternative is very satisfactory, though, because the same facilities are required to produce both energy and reserves, while differing widely in their relative ability to perform each task. Thus the energy and reserves dispatch should really be jointly optimised and integrated in a single pre-dispatch, dispatch, and pricing process.

Apart from a desire to improve real-time coordination, this approach is attractive in that it provides an appropriate environment for market-led innovation in the reserves area, in trading off traditional 'spinning reserves' against interruptible load for example. It is also important to provide appropriate price signals for maintenance of and investment in reserves, just as for energy. Before proceeding to describe the market framework, though, it is important to consider carefully exactly what commodity is being traded in this market.

At one extreme, one could imagine a market in which there was no specific 'reserve' commodity, but energy prices were allowed to rise to very high levels during any contingency. Potential reserve providers would be motivated to back-off generation in order to hold capacity free to respond, and so collect high prices during such contingencies. Various parties have advocated such an approach but there are several technical problems.

First, the energy market must use an accounting period short enough to allow the price spike to be represented. The New Zealand system uses half-hourly periods in its dispatch, and any sustained response will be rewarded by the higher prices arising from re-dispatch after a contingency, since these would be reflected in half-hourly prices. The contingency events themselves, though, must be dealt with in a 6–60 second timeframe and real-time pricing on that time scale is hardly plausible at present.

Second, because a practical formulation which includes contingency responses in an integrated dispatch optimisation can only model a small set of contingency events, Ring [1996] argues that such an optimisation is likely to show prices *falling* during most contingency events. This occurs because the plant that was held out of the energy dispatch for reserve purposes virtually by definition has a lower offer price than the marginal unit in the pre-contingency dispatch. This plant is then released to generate as soon as a contingency occurs, so that the dispatch determined to apply during almost any contingency is actually less constrained than the pre-contingent dispatch. Thus the price is lower, although the energy delivered during that period is of lower 'quality', being less secure.

This does not overcome the final problem that high prices for energy would apply to all generation during the contingency period. Thus the potential provider of reserves can do no better than to generate at full output during the contingency. Withholding generation in the pre-contingent dispatch does not enhance generation during the contingency and the potential reserve provider can only lose revenue by doing so. The problem here is that there is no combination of pre-contingent and post-contingent energy prices that could selectively induce just those generators capable of providing rapid response during a contingency to back off.

Thus it seems more appropriate to consider a market in 'reserves', or perhaps in 'reserves capability'. In the latter case one could imagine a market in which a reserve market manager was responsible for entering into contracts that secured sufficient reserve capacity to meet some contingency standard. A tendering regime could be developed to make such a market competitive but the problem is that the market manager can not know in advance which facilities will actually be the most efficient reserve providers in any particular scheduling period and hence which parties to contract with. Nor does the reserve manager know in advance what risk has to be covered in each period. In fact there will be times when it is more economic to back off the largest unit operating rather than to bring on more reserve capacity just to cover the increased risk implied.

Thus the approach described below includes both risk assessment and the constraint that sufficient reserves be available to meet that risk explicitly in an integrated half-hourly energy and reserves dispatch. Generators submit integrated energy and reserves offers into that dispatch process as described in a later section. These arrangements do not preclude elements of the other two approaches being employed as appropriate though. There could be a reserve manager who is free to enter into whatever contracts are deemed appropriate to secure reserve capacity. Those contracts must obviously include arrangements covering the way in which reserve will be called upon to meet contingencies. In principle, contracts could also allow for supplementary payments to be made to a particular reserve source if it is called upon; in which case the reserve source would probably be offered into the dispatch by the reserve manager at a price which reflected the expected cost of utilising it. In principle too, contracts with particular participants might involve longer-term elements under which they might agree to offer their plant in particular ways or to provide a financial guarantee of future availability in the form of hedges on the reserve price. Such arrangements might be appropriate to deal with a situation in which some particular plant would otherwise be able to exercise too much market power, perhaps because of some localised problem. In practice most if not all contracts simply provide for payment of the spot reserve price to those sources placed on reserve duty, as described below.

Finally, one must address the mechanism by which reserve provision is to be funded. Ultimately, reserve provision is part of the cost of delivering a reliable electricity supply to consumers and should be paid for by them. This only provides a useful signal at the macro level though, because demand reduction (as opposed to load interruptibility) would not affect the reserve situation. On the other hand, it is important that generators be given signals to build and maintain plant that does not place excessive burden by being too large or unreliable. Thus, reserves are paid for in the first instance by generators who must then recover their costs via spot or contract energy prices. Since this cost is actually largely independent of energy consumption, it would actually seem preferable to have it recovered via some form of fixed charge, independent of consumption. Such charges are contentious though, as noted by Read [1997].

In principle each LP solution provides a price that is applicable to a notional transaction under which the providers of reserves would be paid by the generator(s) and/or transmission link(s) that define 'the risk' in that period. Mathematically the allocation of charges between those parties would be arbitrary and it could be argued that each one should pay the full amount because each alone would require the full quantum of reserves. On the other hand, it would seem reasonable to divide the charge equally between them. Taking a broader perspective, it also seems arbitrary to single out the very largest unit(s) to cover the entire reserve charge while charging nothing at all to those who may be only very slightly smaller. Such a regime would provide perverse incentives for unit sizing and takes no account of plant reliability. This results from the essentially arbitrary use of a deterministic single contingency standard to reflect what is really a stochastic multi-contingency situation in which all participants require some reserves and impose some costs. Thus, in practice, generators might reasonably be expected to pay for reserves under some regime such as the following (This is known as a 'runway' pricing regime, because it is used to price landing rights for different aircraft types.):

- All unit capacity is divided into bands of say 50 MW each, and each unit is assigned a weight in each band according to its probability of failure and MW in that band.

- The cost of meeting the first 50 MW of the reserves requirement is then apportioned among all units, using the weights for the first band, with the cost of meeting the incremental reserve requirements of each incremental band being apportioned using the weights for that band.

An issue arises as to how the HVDC should be treated, and this depends on the nature of the arrangements between the transmission company, the generators, and the consumers who 'use' the link.

5.4 Instantaneous Reserve Capability

The reserve potential of a hydro station or thermal unit may depend on both the amount of generation and the amount of spare generation capacity. Reserve capability may increase as more energy is produced but is ultimately limited by spare capacity, which falls as the unit approaches full loading. Ignoring factors such as minimum loading levels, and efficiency effects, a continuous approximation to a unit's set of feasible operating points for instantaneous reserve can be plotted in the space of all combinations of output and generation capacity, as in Figure 5-2.

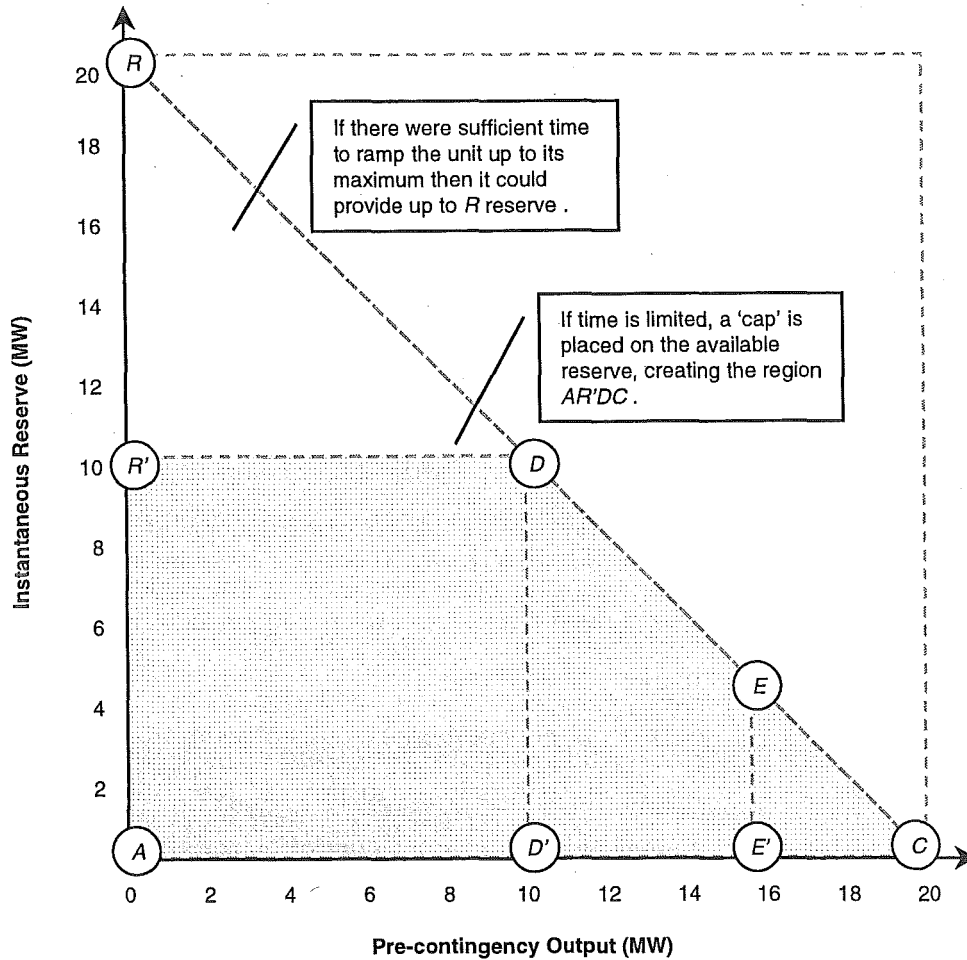


Figure 5-2: Simplified unit reserve capability.

The x -axis in Figure 5-2 represents the unit's energy output prior to the contingency and the y -axis represents the reserve it could provide following a contingency. Point A represents zero output and reserve, while C represents maximum energy output with zero reserves. Energy plus reserve must be less than or equal to the unit's capacity. The 45-degree line RC in Figure 5-2 represents this constraint. If the unit can ramp to its maximum within the required timeframe from any initial output level then up to R MW could be provided as reserves. If the response time is very short then this limits the extent to which the unit can ramp up and hence the reserves it can contribute. This is rep-

resented in Figure 5-2 by a 'cap' on the reserves provided which creates the region $AR'DC$.

In the absence of incentives to provide reserves, one can visualise the unit as normally operating along the line AC . When requirements for reserves are considered the optimal trade-off between energy and reserve provision must be determined. If the unit would have been operating between A and D' it could provide reserves up to the $R'D$ level but no more reserves can be supplied so the unit's operating point would only change if the energy price changed. But, if the unit would have been operating between D' and C , at E' say, it could supply some reserves at E but might need to be 'constrained off' towards D to provide more reserves.

In the first case, the changing energy price would compensate for any change in output. In the second case, the market would at least have to compensate the unit for the profit foregone on the energy market; otherwise the potential provider will have insufficient incentive to provide reserves. If no additional compensation were required to cover, say the cost of operating at lower efficiency to provide reserves, the required compensation is given by the 'opportunity cost' of backing off generation to provide reserves. This is implied by the interaction of the unit's energy offers, the energy price and the reserve capability curve. In the formulation discussed below this compensation will be automatically embodied in the 'reserves price', which is equal to the dual variable associated with the constraint defining the required quantity of reserves. Every provider of reserves should receive the same price for every unit of reserve and by definition this will at least compensate any individual provider for the cost of their reserves as stated in their offers.

If the energy price were fixed then, as in the energy market, there would be a single 'marginal provider' of reserves who would set the price of reserves for the system and for whom the reserves price would exactly compensate for the cost of the most expensive accepted reserves offer. In reality, the reserves and energy dispatches are determined jointly. Ignoring any other constraints, there will be two 'truly marginal' nodes at which the combined energy and reserves price pair leaves the producer indifferent between

producing more or less energy and/or reserves in any combination, as discussed by Read and Ring [1996].

5.5 A General Construct for Offering Energy and Reserves

The simple framework described above assumes that each unit of generation foregone provides an extra unit of reserve, subject to some simple limitations. A more general framework must be able to deal with thermal units, multi-unit hydro stations providing PLSR and TWD, and interruptible loads, in a consistent manner.

Assume that the energy offer for a unit or station is defined by a set of bands of increasing price i.e. by a step supply curve. The reserve offer needs to allow for a number of factors. For example, different modes of reserve provision, notably TWD and PLSR, utilise the same plant to provide differing amounts of reserve; while on some units the combined energy and reserve capacity can exceed the generation capacity by way of a short-term overload capacity. If such constraints were the only concern, the reserve offer could be allowed to take the form of a general piece-wise linear convex capability curve. But the fact that there are costs involved in reserve provision must also be modelled, over and above the opportunity cost discussed previously. In particular, backing off plant often involves generating at lower efficiency. This can be modelled by allowing the offer to define various zones under the reserve capacity curve and to specify fees for operation in those zones. A general offer form consisting of reserve 'wedges' can be used to handle these aspects. A 'wedge' is defined by three parameters:

- 1) A 'cap' on reserve in the wedge.
- 2) The Incremental Reserve Proportion of generation (*IRP*) which allows the specification of an upward sloping boundary to the reserve region, limiting reserves scheduled in that wedge to some proportion of the scheduled energy output.

- 3) The Capacity Utilisation Factor (*CUF*) which allows a combination of wedges for alternative reserve modes in one representation, by defining how much of the potential generation capacity each uses in order to deliver one unit of instantaneous reserve.

Symbol	Description
c^j	The price of band j of a generator's energy offer indexed $j = 1, \dots, J$
e^j	The MW size of band j of a generator's energy offer
f^w	The price of wedge w of a generator's reserve offer indexed $w = 1, \dots, W$
IRP^w	The Incremental Reserve Proportion of generation for wedge w of a generator's reserve offer
CUF^w	The Capacity Utilisation Factor for wedge w of a generator's reserve offer
r^w	The MW cap on wedge w of a generator's reserve offer
CRP	A generator's capacity for reserve purposes

Table 5-1: Symbols in the reserves offer formulation

The mathematical description of this offer uses the notation shown in Table 5-1. In addition, let e^j and r^w represent the MWh quantity accepted by the market from band j of an energy offer and wedge w of a reserve offer respectively. To model a single generator's energy and reserves offer, the market clearing LP problem includes the objective components shown in Equation 5-1; a generator capacity constraint, Equation 5-2; reference ray constraints, Equation 5-3; energy band bounds, Equation 5-4; and reserve wedge bounds, Equation 5-5.

$$\begin{aligned} \text{Maximise} \quad & \dots - \sum_{j=1}^J c^j e^j - \sum_{w=1}^W f^w r^w + \dots \\ & \dots, e^j, r^w, \dots \geq 0 \end{aligned}$$

Equation 5-1

subject to

$$\sum_{j=1}^J e^j + \sum_{w=1}^W CUF^w r^w \leq CRP$$

Equation 5-2

$$r^w \leq IRP^w \sum_{j=1}^J e^j \quad w = 1, \dots, W$$

Equation 5-3

$$e^j \leq \bar{e}^j \quad j = 1, \dots, J$$

Equation 5-4

$$r^w \leq \bar{r}^w \quad w = 1, \dots, W$$

Equation 5-5

In combination, these parameters can be used to express the geometry and economics of reserve provision for a unit or multi-unit station in a variety of circumstances, as described in the following paragraphs, and demonstrated in the example below.

Many generating units can provide little in the way of reserves when operating at low generation levels. This can be represented by specifying an upwards sloping section (using *IRP*) from zero output, limiting the unit's reserve contribution to some proportion of energy output. In such a case, the market may decide that the unit will actually generate at a higher level, in order to provide more reserves, and the reserves price would be set to at least compensate the unit for the loss incurred by such generation. Further, many generating units can not operate below some minimum level. Unfortunately an LP can not deal directly with this 'unit commitment' problem, or with any other non-convexity in a unit's reserve capability curve. But the LP can be discouraged from scheduling a unit at a low generation level in order to produce more reserves by specifying an upwards sloping ray which, at the minimum loading level, restricts reserve output to that which can actually be provided at that loading level.

In most cases there is also a fixed cost incurred by operating a unit, so that the most efficient loading point (MELP) occurs at some output level greater than zero, where average costs are minimised. The MELP for most thermal units occurs at maximum loading, while hydro units typically have a MELP around 85% loading. Ignoring incentives to provide reserve, the unit would not normally operate economically at prices below its minimum average cost or at output levels below the MELP. This is just a particular case of the simple economic result that a producer will not enter the market unless it can

cover its average cost at the entry point, MELP in this case. Once the energy price is raised beyond the minimum average cost, the optimal response will increase until, eventually, the energy price exceeds the marginal cost of operating at full capacity. This suggests that the unit's energy offer would have a single band representing generation up to the MELP. This would be offered at a price at least high enough to recover the cost of operation at the MELP. It would then have consecutive higher priced bands representing falling efficiency between the MELP and full output capacity. If operation at MELP also provides maximum reserve there would be no incentive to operate below this level in order to provide more reserve. Otherwise, the reserve price may correctly provide incentives to operate the unit less efficiently in order to provide more reserve.

This can be represented by using wedges to divide the feasible region into several zones and specifying additional fees to be paid to compensate for operating in an inefficient region to provide additional reserve. This formulation may lead to plant being scheduled to produce at points within the feasible region, in which case their reserve contribution will be understated. It is not clear that any better convex approximation can be produced in what is fundamentally a non-convex situation. The situation is further complicated when, as for New Zealand hydro stations, it is proposed that a single offer be used for a multi-unit station. In that case, any energy output level could imply a range of reserve levels depending on the unit commitment. Assuming identical units and accepting the validity of a linear approximation to an integer situation, increasing unit commitment at a constant per unit loading level corresponds to moving along a ray from the origin in the energy/reserve diagram. This will yield a constant average cost, and proportional reserve contribution (*IRP*). This is naturally represented by the wedge approximation above.

Finally, the example below shows how the Capacity Utilisation Factors (*CUF*) for the various wedges can be set to define a convex piece-wise linear boundary to the feasible reserve region. In particular, it can be used to represent an overall cap on reserve produced by all wedges. Unlike that illustrated in the simple example discussed earlier, it properly represents the impact of a joint limit on the capacity utilised by two alternative

modes of reserve provision, such as TWD and PLSR, for the same plant. It may also be seen that both *CUF* and *IRP* may be derived from such a diagrammatic representation of a unit or station's reserve capability.

5.6 Modelling Risk Requirements

In addition to modelling reserves providers, the market LP model must ensure that sufficient reserves are carried to cover the loss of any of the largest single operating units or links, called 'the units at risk'. Further, the size of the risk should be optimised within the model. It is legitimate and often necessary to carry reserves on a unit that is itself 'at risk'. However, any reserves carried by a unit at risk cannot be counted when considering the reserves needed to cover the outage of that unit. For example, if there were only four equally sized units in the system then the maximum secure output level will be produced by running each at 75% of capacity, so that the failure of any one unit can be covered by ramping up the other three.

This type of situation can be modelled as follows. Let r_i represent the total reserve taken up from all wedges offered by unit i . The LP problem requires the constraint shown in Equation 5-6 for each 'at risk' unit, k . Adding the reserve from unit k to both the left and right hand sides gives Equation 5-7.

$$\sum_{k \neq i} r_i \geq e_k$$

Equation 5-6

$$\sum_{i=1}^I r_i \geq e_k + r_k$$

Equation 5-7

Equation 5-7 states that if an 'at risk' unit is placed on reserve duty that unit's reserves contribution is 'at risk' along with its energy output. This form of the constraint is convenient because the left-hand side represents a unique 'nodal reserve requirement'

independent of any particular risk source. One such constraint is required per 'at risk' unit.

The LP chooses the best set of units to have as the risk, optimally trading the risk off against the cost of providing reserve and the incentives to generate and only the constraints for the units defining the risk will be binding.

5.7 Illustration

Rather than demonstrate the solution of the full LP formulation, this section will show how the representation of a single station's energy and reserves offer, when embedded in such an LP, implies a monotone increasing supply curve for reserves. Consider a hydro station that has submitted an offer to the energy market, in five bands, as shown in Table 5-2.

Energy Offer			
Band	Price (\$/MW)	Limit (MW)	Max. Output for Reserve Purposes (MW)
1	\$ 22.50	20	—
2	\$ 23.00	20	—
3	\$ 23.50	20	—
4	\$ 24.00	20	—
5	\$ 24.50	10	—
Total	—	90.0	100.0

Table 5-2: Example energy offer

A reserves offer for this station is shown in Table 5-3. With the first wedge the generator is offering to provide PLSR up to a ratio of output (*IRP*) of 25% at no more than the natural compensation required in backing the station off (a zero fee). Potential generation capacity created in doing so yields reserve on a one-to-one basis (as stated by a *CUF* of unity). Subsequent PLSR, up to a ratio of output of $25+55+120 = 200\%$, yields less reserve (*CUF* values greater than unity) and requires payment of a fee per MW of reserve as additional compensation for efficiency losses. The final wedge represents operation of TWD units. These units yield one MW of reserve for every MW committed and cost \$3.00/MW of reserve.

Instantaneous Reserve Offer					
Wedge	Price (\$/MW)	IRP (MW/MW)	CUF (MW/MW)	MW	
1	\$0.00	25.0 %	1.0	—	—
2	\$0.68	55.0 %	1.3636	—	—
3	\$2.50	120.0 %	2.5	—	—
4	\$3.00	—	3.0	20.0	—

Table 5-3: Example reserves offer

The feasible region implied by the energy and reserve offers is shown in Figure 5-3. The rays from the origin represent the three PLSR wedges. The first wedge has a boundary along the 45-degree capacity line, i.e. the line that represents the fact that energy plus reserve must be less than or equal to capacity. The other PLSR wedges end below the 45-degree line as defined by the *CUF* values in the offer. The TWD wedge runs vertically from the origin. Its upper boundaries are defined by its *CUF* and MW limit values and the maximum PLSR level.

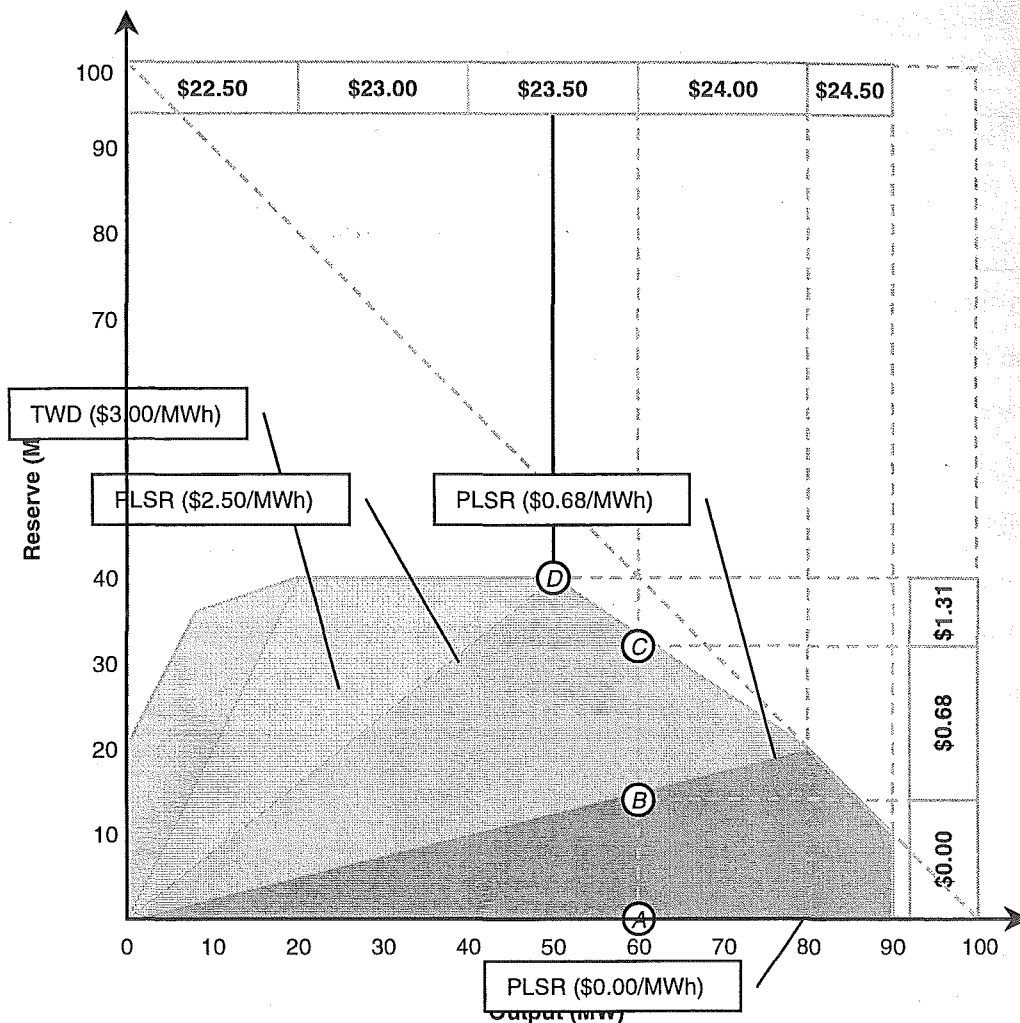


Figure 5-3: Example unit reserve capability

The prices along the top of Figure 5-3 represent the energy price bands, while those on the right hand side represent the effective reserve offer price bands, as determined by the solution of the LP below. If the market clearing energy and reserve prices were β and Γ respectively then the situation faced by this particular station can be represented by a LP with the objective function shown in Equation 5-8. This is subject to the capacity constraint, Equation 5-9; energy offer band upper bounds, Equation 5-10; reference ray constraints, Equation 5-11; and reserve caps, Equation 5-12.

$$\begin{aligned} \text{Maximise} \quad & \beta \sum_{j=1}^J e^j + \Gamma \sum_{w=1}^W r^w - 22.5e^1 - 23e^2 - 23.5e^3 - 24e^4 \\ & - 24.5e^5 - 0.68r^2 - 2.5r^3 - 3r^4 \\ & e^j, r^w \geq 0 \end{aligned}$$

Equation 5-8

$$e^1 + e^2 + e^3 + e^4 + e^5 + r^1 + 1.3636r^2 + 2.5r^3 + 3r^4 \leq 100$$

Equation 5-9

$$e^1 \leq 20$$

$$e^2 \leq 20$$

$$e^3 \leq 20$$

$$e^4 \leq 20$$

$$e^5 \leq 10$$

Equation 5-10

$$r^1 \leq 0.25(e^1 + e^2 + e^3 + e^4 + e^5)$$

$$r^2 \leq 0.55(e^1 + e^2 + e^3 + e^4 + e^5)$$

$$r^3 \leq 1.2(e^1 + e^2 + e^3 + e^4 + e^5)$$

Equation 5-11

$$r^4 \leq 20$$

Equation 5-12

If a constant energy price of \$23.75 is assumed, varying Γ can produce a reserve supply curve for this station, as shown in Figure 5-4. Table 5-4 details the solutions, payments and profits for each point on the curve. The total profits, which must increase monotonically as Γ rises, are determined by the sum of energy and reserve revenue minus the production cost.

Point	Energy	Reserve	Incremental Cost of Reserve	Total Cost	Energy Payment	Reserve Payment	Profit
A	60.0	0.0	\$ 0.00	\$ 1,380.00	\$ 1,425.00	\$ 0.00	\$ 45.00
B	60.0	15.0	\$ 0.00	\$ 1,380.00	\$ 1,425.00	\$ 0.00	\$ 45.00
C	60.0	33.3	\$ 0.68	\$ 1,392.50	\$ 1,425.00	\$ 22.73	\$ 55.23
D	50.0	40.0	\$ 1.31	\$ 1,163.75	\$ 1,187.50	\$ 52.40	\$ 76.15

Table 5-4: Prices and profits

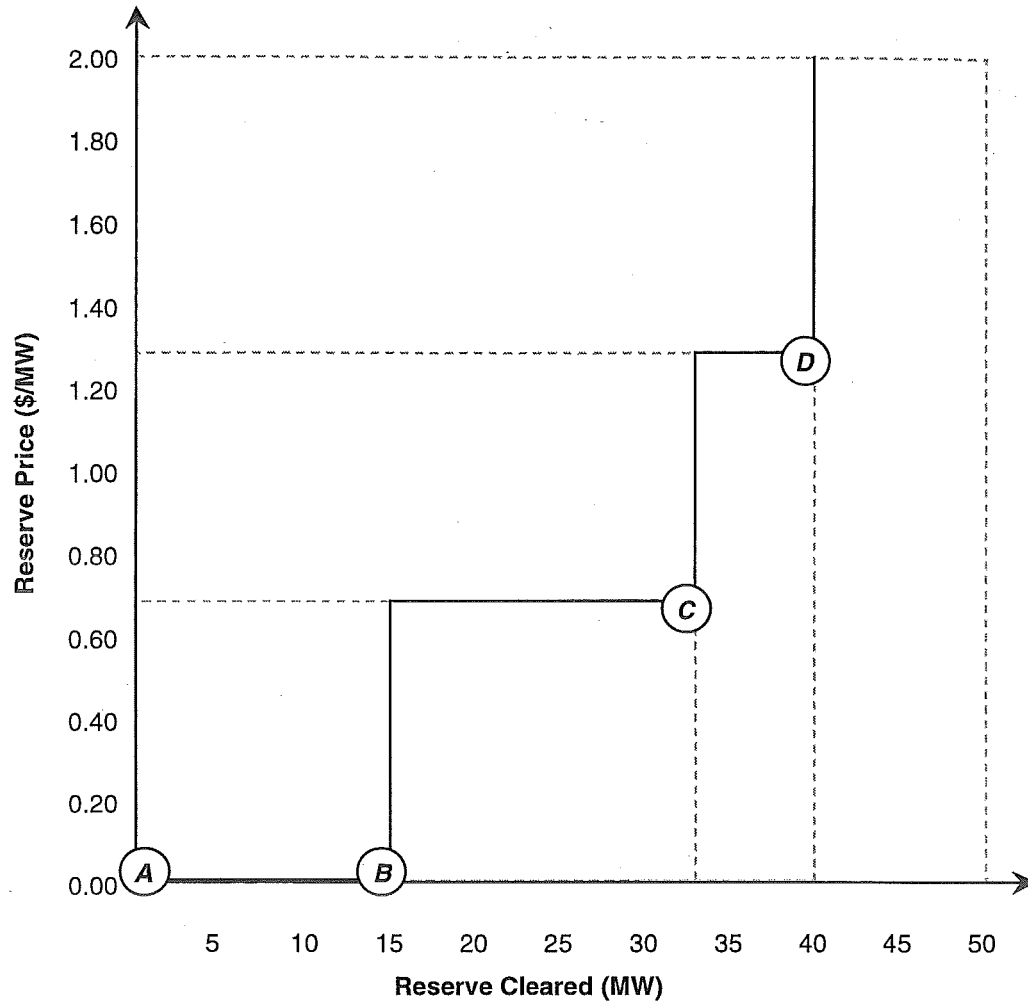


Figure 5-4: Supply curve for reserves

Chapter 6

Conclusions

This thesis has sought to answer two main research questions. This chapter summarises the conclusions of the thesis and discusses some open research questions.

6.1 The Simplified Energy Market

The first key research question was, “Can a simple market adequately represent the complexities of a real hydrothermal system?” Chapter 1 split this question into the following sub-questions. “What impact does simplification of the generators’ offers have on achieving dispatch feasibility and optimality?” and, “Is a step supply curve consistent with a hydro station’s underlying supply model in all cases?” and, “What degree of cost distortion can be expected from an energy market in which generators submit a fixed offer (a single supply curve offer for a whole day) versus one in which their offer is sculpted” and finally, “How robust are the measured distortions?”

The strategy taken in this thesis to answer these questions was:

- 1) Define the centrally coordinated dispatch problem as an LP problem and use its solution as reference point.

- 2) Define a theoretical market, via decomposition of the centrally coordinated model that could reproduce the solutions of the centrally coordinated model.
- 3) Define the proposed 'simplified market' and examine the problem of forming supply curve offers for this market by examining the economics of the hydro station LP problem.
- 4) Produce empirical results to determine how significantly the performance of a real market might be compromised by the simplification of the offer.

Chapter 2 and Appendix A presented a centrally coordinated LP problem for a single day's dispatch. Solutions from this model provided a basis for comparison with those of a market. Chapter 2 went on to decompose this model and form a theoretical framework for an energy market. The literature shows that this market can, at least in theory, reproduce the centrally coordinated dispatch in every case. But existing research suggested that simplifying the offer form could cause problems, especially for systems with a large hydro component.

Chapter 3 showed some empirical results taken from the WEMSIM simulation model. These results showed that the observations of a group's LP problem's response to energy prices are not consistent with a supply curve in every case.

Examples showed that, in the presence of energy limits implied by active storage bounds or rate-of-change constraints, the response to price of a hydro station LP problem could be non-monotonic across a day. It was shown by examination of primal and dual aspects of the hydro station LP problem that this is due to the effect of active intertemporal constraints on the optimal release policy. This discussion went further, to show that these energy limits also imply that the response inside a single period across a number of price scenarios may also be non-monotonic.

In attempting to place a supply curve through inconsistent observations some information must be lost. This implies that, when these supply curve offers are compiled in a market clearing process, the resulting coordination signals, be they prices or targets, might not replicate the centrally coordinated dispatch. Thus, it is concluded that *the*

simplified market cannot replicate the centrally coordinated dispatch in the presence of energy limits that imply changes in the system's underlying cost structure.

Further, the simplified market might not produce feasible or optimal coordination signals in the presence of energy limits that imply changes in the system's underlying cost structure.

The extent to which these feasibility and/or optimality distortions cause problems was tested in the experimental analysis. Chapter 4 summarised some experiments with various designs for the energy market, and Appendix C provided more detail. Overall it was concluded that:

- 1) An energy market in which hydro and thermal generators submit supply curve offers is feasible.
- 2) The theorised effects of energy constraints do appear in practice but can be largely overcome if the market design is 'flexible'.
- 3) A market design in which generators are allowed to vary the shape of their supply curve over a day provides significantly better performance than one in which they must have the same supply throughout the day.
- 4) The best results were achieved when generators were permitted to update their offers as the real day was revealed.

6.2 A Market for Energy and Reserves

The second part of this thesis attempted to answer the question: "How can reserves be represented in the offering and market clearing process?". Appendix A presented an approach called the Fan Approximation which explicitly models a unit or station's reserve capability and energy efficiency in an integrated Linear Programming framework. This approach was later implemented in the WEMSIM model (described in Appendix B).

In Chapter 5, the Fan Approximation was used to develop a framework for an integrated energy and reserves market. It began by examining the economics of reserve provision for a unit. A general construct was described that uses a series of 'wedges' to rep-

resent reserve capability in a linear offer, and an example was presented. Appendix D provides more details of how this offer format can be used in practice to represent particular aspects of reserve provision for thermal units and hydro stations.

6.3 Open Research Questions

In answering the key research questions, a number of other areas of interest have arisen that could lead to further research.

Throughout this analysis the key assumption was made that generators act as perfect competitors. Thus the primary unanswered question is what impact gaming and contracts would have on the distortions measured in the experiments presented here. As discussed in Chapter 1, the affect of gaming and contracts may swap the effects measured here; however, there is no reason why the conclusions with respect to the impact of simplifying the offer form, or of integrating energy and reserves should not hold, even given gaming and contracts.

Other related areas of interest are:

- 1) The effect of more/less vertical integration in the decentralised system. This thesis has examined offers made at the group level for hydros and station level for thermals; however, one could imagine a market in which offers are made at the station or island/national level. It remains an open issue what implications these various degrees of vertical integration have on the distortions measured here.
- 2) The effect of unit commitment decisions on the formation of offers and the implementation of its solutions.
- 3) The impact of nodal effects, including transmission constraints, on the formation of offers and the market clearing process.

Bibliography

- [1] B. Ahn and W. W. Hogan. On convergence of the PIES algorithm for computing equilibrium. In *Operations Research*. 30(2), March–April 1982.
- [2] C. D. Aliprantis, D. J. Brown, and O. Burkinshaw. *Existence and Optimality of Competitive Equilibrium*. Springer-Verlag, 1989.
- [3] K. J. Arrow. An extension of the basic theorems of classical welfare economics. In *the Second Berkeley Symposium on Mathematical Statistics and Probability*. University of California Press, Berkeley and Los Angeles, 1951.
- [4] J. Batut and A. Renaud. Daily generation scheduling optimisation with transmission constraints: a new class of algorithms. *IEEE Transactions on Power Systems*, 7(3), August 1992.
- [5] J. Batut, A. Renaud, and P. Sandrin. Daily scheduling optimisation for power generation units. Optimisation des plannings journaliers des groupes de productions delectricite 2, E.D.F. Bulletin de la direction des etudes et recherches serie B reseaux electriques, materials electriques, 1990.
- [6] D. P. Bertsekas, G. S. Lauer, Jr., N. R. Sandell, and T. A. Posburgh. Optimal short-term scheduling of large-scale power systems. *IEEE Transactions of Automatic Control*, AC-28(1), January 1983.
- [7] H. Brannland, D. Sjelvgren, and Bebenk J. A. Short term generation scheduling with security constraints. *IEEE Transactions on Power Systems*, 3(1), February 1988.

- [8] P. Burelli, G. D'Amoara, M. Innorta, and P. Marannino. Short-term hydro-thermal generation scheduling with inter-area power flow constraints. In *Proceedings of the Tenth Power Systems Computation Conference*, Graz, Austria, 19–24 August 1990.
- [9] J. Burton, J. Gleason, and J. Powell. Dynamic programming in the optimization of power generators. In The Fall Conference of the Operations Research Society of America, 1974.
- [10] Li Chao-an and D. L. Streifert. Incremental network flow programming for short term hydro thermal scheduling. In *Proceedings of the Tenth Power Systems Computation Conference*, Graz, Austria, 19–24 August 1990.
- [11] A. I. Cohen and V. R. Sherkat. Optimization based methods for operations scheduling. *Proceedings of the IEEE*, 75(12), December 1987.
- [12] J. G. Culy and E. G. Read. Short run gaming in the Victorian wholesale electricity market. Report to the ESIRU, NZ Institute of Economic Research (Inc.), Wellington, New Zealand, October 1994.
- [13] J. G. Culy, E. G. Read, and B. D. Wright. The evolution of New Zealand's electricity supply structure. Working paper, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1996.
- [14] H. G. Daellenbach, J. A. George, D. C. McNickle. *Introduction to Operations Research Techniques*. Second Edition. Allyn and Bacon Inc. 1983.
- [15] G. Debreu. *Theory of Value*. Yale University Press, New Haven, CT, 1959.
- [16] G. R. Drayton, B. W. Lamar, E. G. Read, and A. Turner. Optimisation of generation and spinning reserve in an electrical power system. In *Proceedings of the 28th Annual Conference on the Operational Research Society of New Zealand*, pages 142–149, August 1992.
- [17] G. R. Drayton-Bright. Generation and reserve scheduling program GARSP. Energy Modelling Research Group, Department of Management, University of Canterbury, 1994.

- [18] G. R. Drayton-Bright. Wholesale electricity market simulation: Case study. Energy Modelling Research Group, Department of Management, University of Canterbury, 1995.
- [19] G. R. Drayton-Bright, E. G. Read, and B. M. Graydon. Decentralised dispatch research: an iterative bidding model. Contract report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995.
- [20] G. R. Drayton-Bright, E. G. Read, and B. M. Graydon. Decentralised dispatch research: initial results. Contract report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995.
- [21] G. R. Drayton-Bright, E. G. Read, and B. M. Graydon. Decentralised dispatch research: modelling issues. Contract report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995.
- [22] G. R. Drayton-Bright, E. G. Read, and B. M. Graydon. Decentralised dispatch research: offers based on multiple scenarios in a day-ahead market. Contract report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995.
- [23] G. R. Drayton-Bright, E. G. Read, and B. M. Graydon. Decentralised dispatch research: sensitivity to system assumptions in a day-ahead market. Contract report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995.
- [24] G. R. Drayton-Bright, E. G. Read, and B. M. Graydon. Decentralised dispatch research: storage modelling issues. Contract report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995.
- [25] G. R. Drayton-Bright, E. G. Read, and A. L. Kerr. The cost of providing reserves. Contract report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1994.

- [26] G. R. Drayton, E. G. Read, B. W. Lamar, and G. J. Bell. River chain optimisation. Contract report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1994.
- [27] G. R. Drayton, E. G. Read. *Reserve Dispatch Offers: Theory and Examples*. Electricity Corporation of New Zealand (ECNZ), 1996.
- [28] Electricity Task Force. Structure regulation and ownership of the electricity market. Report to New Zealand Government, New Zealand Government, 1989.
- [29] L. Engles, R. E. Larson, J. Peschon, and K. N. Stanton. Dynamic programming applied to hydro and thermal generation scheduling. In *Application of Optimization Techniques in Power System Engineering*. IEEE Tutorial Course, 1976.
- [30] L. A. F. M. Ferreira, T. Anderson, C. F. Imarato, T. E. Miller, C. K. Pang, A. Svoboda, and A. F. Vojdani. Short-term resource scheduling in multi-area hydro-thermal power systems. *Electrical Power and Energy Systems*, 11(3), July 1989.
- [31] R. J. Gilbert and E. P Kahn. *International Comparisons of Electricity Regulation*. Cambridge University Press 1996.
- [32] R. Green. The British electricity spot market. In proceedings of the 17th Annual International Energy Conference. Stavenger, May 25–27, 1994.
- [33] R. Green. Competition and the structure of the UK's energy industries. In proceedings of the 17th Annual International Energy Conference. Stavenger, May 25–27, 1994.
- [34] G. Grimwood. Scheduling, Pricing and Dispatch Software: A Mathematical Description. Trans Power (NZ) Ltd. 12 September 1996.
- [35] A. Halim, A. Rashid, and K. M. Nor. An efficient method for optimal scheduling of mixed head hydro and thermal plants. *IEEE Transactions on Power Systems*, 6(2), May 1991.
- [36] Hirst, Eric, Kirby and Brendan. Restructuring—the devil is in the details. In *Electricity Journal*, 8(10), December 1995.

- [37] W. J. Hobbs, G. H. S. Warner, and G. B. Shelbe. An enhanced dynamic programming approach for unit commitment. *IEEE Transactions on Power Systems*, 3(3), August 1988.
- [38] W. W. Hogan. Project Independence Evaluation System: *Structure and Algorithms*. *Proceedings of the Symposia in Applied Mathematics*, 21, 1977.
- [39] R. B. Johnson, S. S. Oren, A. J. Svoboda. Equity and efficiency of unit commitment in competitive electricity markets. *Proceedings of the 1996 EMF Conference*.
- [40] R. J. Kaye, Felix F. Wu, P. Varaiya. Pricing for System Security. *IEEE Transactions on Power Systems*, 1995.
- [41] A. L. Kerr and E. G. Read. Optimal scheduling of thermal stations: a structured analytical method. Research report, Energy Modelling Research Group, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995.
- [42] A. L. Kerr and E. G. Read. Short-term hydro scheduling using integer programming: management and modelling issues. *Proceedings of the 32nd Annual Conference of the Operations Research Society of New Zealand*, 29-30 August 1996.
- [43] L. K. Kirchmayer. *Economic Operation of Power Systems*. John Wiley & Sons, New York, 1958.
- [44] G. X. Luo, H. Habibollahzadeh, and A. Semlyen. Short-term hydrothermal dispatch: detailed model and solutions. *IEEE Transactions on Power Systems*, 1990.
- [45] R. T. Miller and A. J. Turner. A spreadsheet model for Hydro-thermal-HVDC reserve scheduling. Technical report, Trans Power (NZ) Ltd., 1991.
- [46] M. R. Mohan, K. Kuppusamy, and M. Abdullah. Optimal short-term hydrothermal scheduling using decomposition approach and linear programming method. *Electrical Power and Energy Systems*, 14(1), February 1992.
- [47] J. A. Muckstadt and S. A. Koenig. An application of Lagrangian relaxation to scheduling in power generation systems. *Operations Research*, 25(3), May-June 1977.
- [48] F. H. Murphy. Design strategies for energy market models. *Energy Models and Studies*. North-Holland Publishing Company, 1983.

- [49] F. H. Murphy, J. J. Conti, S. H. Shaw and R. Sanders. Modelling and forecasting energy markets with the intermediate future forecasting system. *Operations Research*, 36(3), May-June 1988.
- [50] P. Oliveira, S. McKee, and C. Coles. Optimal scheduling of a hydrothermal power generation system. *European Journal of Operational Research*, 71, 1993.
- [51] A. Philpott, D. Ryan and Zakeri. Inexact cuts in stochastic Benders decomposition. *Proceedings of the 32nd Annual Conference of the Operations Research Society of New Zealand*, 29-30 August 1996.
- [52] M. K. Rakic, M. L. Markovic, and Z. M. Msrkovic. Short-term operation of combined power systems: a decomposition technique. *Proceedings of the Tenth Power Systems Computation Conference*, 19-24 August 1990.
- [53] E. G. Read. An alternative approach to short-term scheduling. Internal report, The - New Zealand Electricity Department, July 1979.
- [54] E. G. Read. The inclusion of pumped storage in a power system optimisation scheme. Internal report, New Zealand Electricity Department, March 1979.
- [55] E. G. Read. *Optimal Operation of Power Systems*. PhD thesis, Department of Economics and Operations Research, University of Canterbury, Christchurch, New Zealand, 1979.
- [56] E. G. Read, G. R. Drayton-Bright, B. J. Ring. An Integrated Energy/Reserve Market for New Zealand. In *Deregulation of Electric Utilities*. G. Zaccours and A. LaPointe (editors). Kluwer, Boston. (to appear)
- [57] E. G. Read. Transmission Pricing in New Zealand. *Utilities Policy*. 6(3) 1997. (to appear)
- [58] B. J. Ring. *Nodal Spot Pricing in a Competitive Electricity Market*. PhD thesis, Department of Management, University of Canterbury, Christchurch, New Zealand, 1996.
- [59] T. J. Scott. *Regulation of Gaming and Contracts in a Wholesale Electricity Market*. PhD thesis, Department of Management, University of Canterbury, Christchurch, New Zealand, 1997.

- [60] T. J. Scott and E. G. Read. Single period market response curves for a duopolistic electricity market. In *Proceedings of the 28th Annual Conference of the New Zealand Operational Research Society*, University of Canterbury, Christchurch, New Zealand, 24–25 August 1992.
- [61] T. J. Scott and E. G. Read. Optimising reservoir management in a deregulated electricity market. In *Proceedings of the 29th Annual Conference of the New Zealand Operational Research Society*, University of Auckland, New Zealand, 23–24 August 1993.
- [62] J. J. Shaw, R. F. Gendron, and D. P. Bertsekas. Optimal scheduling of large hydrothermal power systems. *IEEE Transactions on Power Apparatus and Systems*, PAS-104(2), February 1985.
- [63] V. L. Smith. *Research in Experimental Economics*. JAI Press Inc., 1979.
- [64] V. L. Smith. *Experimental Economics*. Edward Elgar Publishing Limited, 1990.
- [65] V. L. Smith. *Papers in Experimental Economics*. Cambridge University Press 1991.
- [66] M. J. H. Sterling. Power system control. Research report 6, IEEE Control Engineering Series, 1978.
- [67] S. K. Tong and S. M. Shahidehpour. Hydrothermal unit commitment with probabilistic constraints using segmentation method. *IEEE Transactions on Power Systems*, 5(1), February 1990.
- [68] H. Uzawa. Iterative Methods for Concave Programming. In *Studies of Linear and Non-linear Programming*. K. J. Arrow, L. Hurwicz and H. Uzawa, eds. Stanford University Press, 1958.
- [69] H. Waterer. Hydroelectric unit commitment, subject to uncertain demand. *Proceedings of the 32nd Annual Conference of the Operations Research Society of New Zealand*, 29–30 August 1996.
- [70] WEMDG. New Zealand wholesale electricity market. Report to New Zealand Government, New Zealand Government, 1994.
- [71] WEMS. Towards a competitive wholesale electricity market. Report to New Zealand Government, New Zealand Government, 1992.

- [72] O. E. Williamson. *Economic Organization—Firms, Markets and Policy Control*, Wheatsheaf Books, 1986.
- [73] Jin-Shyr Yang and Nanming Chen. Short term hydrothermal coordination using multi-pass dynamic programming. *IEEE Transactions on Power Systems*, 4(3), August 1989.

Appendix A

The Reference Model

This appendix describes an integrated hydro river chain (hydro group) energy and reserve optimisation model which includes a new linear approximation to the multi-unit hydro station flow function and TWD costs, called the Fan Approximation and a fairly detailed river model.

Section A.1 gives some background in the New Zealand context. Section A.2 applies the Fan Approximation to the problem of creating a linear model of a hydro station's flow function for the purposes of modelling reserves. Section A.3 describes a traditional model of a river chain for the optimisation of group energy output. Some extensions to this model, which deal with end effects, relevant to the application of the model in the market environment, are also presented. Finally the Fan Approximation is merged with the river model to give an integrated energy and reserve optimisation for a hydro system.

A.1 Background

The traditional hydro station LP formulation uses a method of linearising the efficiency profile of a hydro station taking special care to model the efficient points of the 'cusp

curves'. This approach is adequate to model the economic option associated with operating units at their most efficient loading point, MELP, rather than at capacity, but, when the concept of economic operation of units below MELP for reserve purposes is introduced a more detailed model is required. Thus, not only does the framework need to model the reserve response of the unit/station accurately, but it also needs to take account of the cost of operating units over their entire operating range. Further, in the New Zealand system, a significant number of hydro stations are capable of tail water depressed (TWD) operation, another aspect that needs to be accounted for.

A.2 The Fan Approximation

This section applies the Fan Approximation to the problem of creating a linear model of a hydro station's flow function. Section A.2.1 describes the flow function for a hydro unit. Section A.2.2 shows how the Fan Approximation can be applied to the station flow function. Section A.2.3 adds elements to the basic formulation that enable the model to calculate the amount of reserve available in a limited time frame(s). Finally, Section A.2.5 summarises the hydro station LP formulation.

A.2.1 The Unit Flow Function

This implementation of the Fan Approximation will be developed using an example hydro station. Assume that the example station has five identical 20 MW units. First, the flow function for a single unit needs to be examined. Assume that the flow on a single unit in this example can be represented by the non-linear function of the unit's energy production, $f(g)$, as in Equation 6-1; the curve is shown in Figure 6-1. (Use of an inverse quadratic function to approximate the flow function of a hydro station is a standard technique.)

$$f(g)_{g>0} = \frac{g}{\left(\frac{1}{6} + \frac{1}{9}g - \frac{1}{288}g^2\right)}$$

Equation 6-1

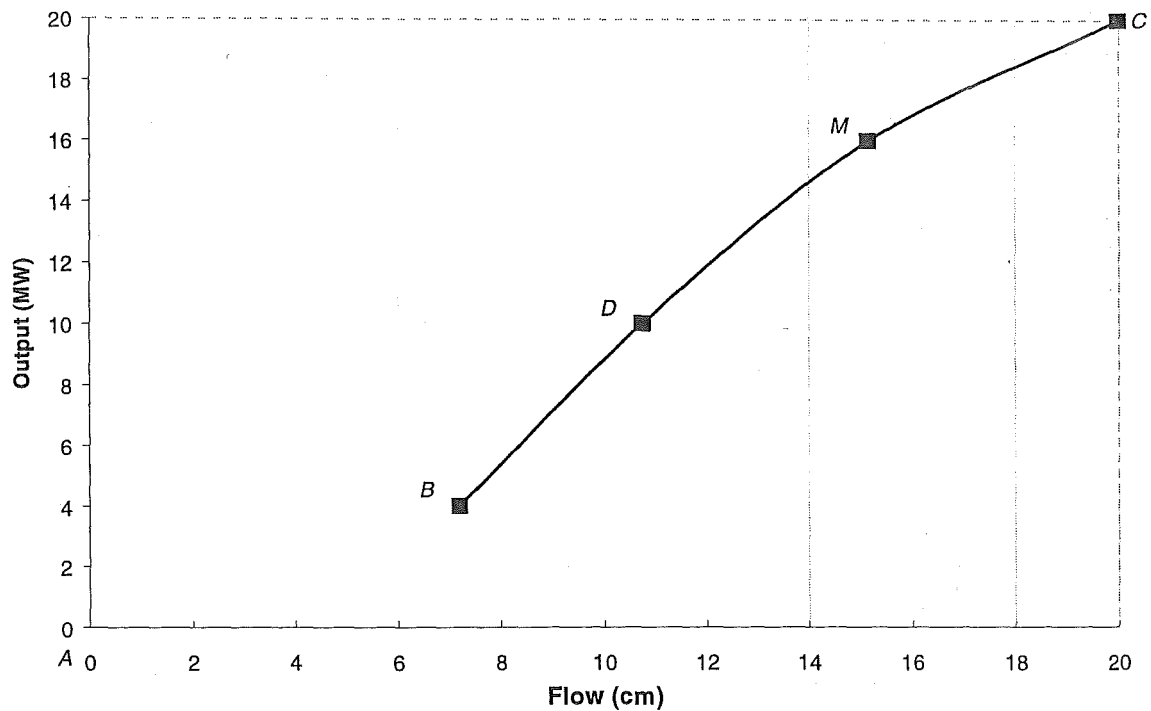


Figure 6-1: Example unit efficiency function for a hydro unit

The unit's Most Efficient Loading Point (MELP) can be calculated by differentiation of the quadratic flow function to give Equation 6-2.

$$MELP = \frac{-b}{2c} = \frac{-\frac{1}{9}}{2^{-\frac{1}{288}}} = 16$$

Equation 6-2

Thus, point *M* on Figure 6-1 is the unit's MELP. Point *D* lies at half load, while point *B* represents the unit's minimum loading point (MULP).

A.2.2 Defining the Station Level Approximation

As described in Appendix D, the output versus spare capacity diagram for a single unit can be scaled up to the station level. The feasible operating region can be divided radially into regions of various efficiency along the rays projecting from the origin, *A*, and through a number of key points:

- *C*, the units maximum output;
- *M*, the most efficiency loading point (MELP);

- B , the minimum unit loading point (MULP);
- D , an arbitrary intermediate point

This gives the three regions, or 'wedges' shown in Figure 6-2. The *AMC* region represents operation of units between their MELP and maximum output, which provides a certain amount of 'natural' reserves, i.e. it is spare capacity that is provided purely by operating units efficiently. The *ADM* region results from operation of units below MELP with a fairly low ratio of spare capacity to energy. The last region, *ABD*, represents operation of units inefficiently, i.e. with a high ratio of spare capacity to energy.

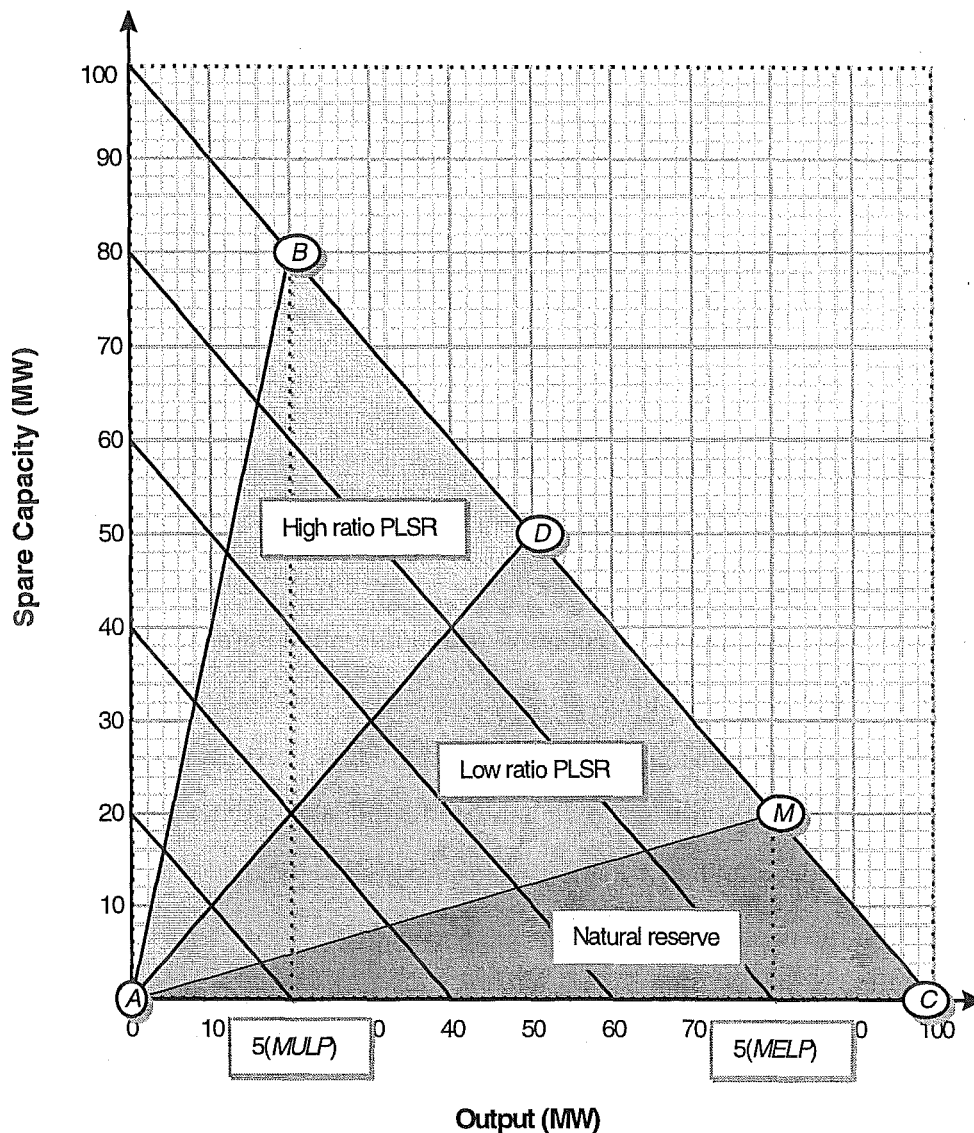


Figure 6-2: A single unit's feasible operating region divided into efficiency wedges

The falling diagonal lines, inside the shaded regions, correspond to the capacity of 1, 2, 3 and 4 committed units, and the final 45-degree line corresponds to the station's total capacity of 100 MW from all five units. As described by Appendix D, the rays leading from the origin through the points *C*, *M*, *D* and *B* are referred to as 'reference rays' with the first ray indexed as ray zero. Will denote the output along the *x*-axis *g*, the spare capacity up the *y*-axis, *z*, and the flow from any combination of *g* and *z*, *f*.

Now if (g^w, z^w) , represents a point on ray w , two key properties of the ray w can be calculated: the slope of the ray, a^w , which is the ratio of spare capacity to output, Equation 6-3; and the average flow along the ray, Equation 6-4.

$$a^w = z^w / g^w .$$

Equation 6-3

$$f^w = f(g^w) / g^w$$

Equation 6-4

For example, along ray 1, each unit of output has associated with it a constant (a^1) units of spare capacity, i.e. $z^1 = a^1 g$ and costs f^1 units of flow so $q^1 = f^1 g$. Any point between the reference rays can be expressed as a linear combination of the rays it lies between. This defines the Fan Approximation to the overall station flow function. The corresponding LP formulation of this approximation uses the variables:

- g for the generation along the x -axis of Figure 6-2, i.e. units operated at maximum output;
- and z^w for the amount of spare capacity in the w^{th} 'wedge', i.e. that lying between reference ray $w - 1$ and w

Counting the x -axis as the ray zero, there will be $W + 1$ variables required, where W is the index of the last ray. These variables are g, z^1, z^2, \dots, z^W . Calculating the flow coefficients for the variables is next.

For g the average flow on ray zero $c^0 = f^0$ is used. For z^1 the coefficient $c^1 = (f^1 - f^0) / (a^1 - a^0)$, is calculated. Generally, for z^w , Equation 6-5 is calculated. The total flow resulting from any combination of these variables is given by Equation 6-6.

$$c^w = \frac{(f^w - f^{w-1})}{(a^w - a^{w-1})}$$

Equation 6-5

$$q = c^0 g + \sum_{w=1}^W c^w z^w$$

Equation 6-6

It is logical that the sum of the output and spare capacity variables cannot exceed the station's total capability, \bar{o} , hence the constraint given by Equation 6-7 is defined.

$$g + \sum_{w=1}^W z^w \leq \bar{o}$$

Equation 6-7

For the five unit hydro station example with the feasible region divided as per Figure 6-2; four variables are required, g , z^1 , z^2 and z^3 . Note that more or less 'wedges' could be used depending on the variability of the average cost function and on the degree of accuracy required in the approximation.

For ray AC (ray 0, defining the bottom of wedge 1), the calculations in Equation 6-8 are made based on the unit maximum output point of 20 MW and the unit flow function. (Note that these coefficients have been calculated using the unit flow function, but it would be equally valid to use the station flow function.) Equation 6-9 shows the calculations for ray AM leading to the MELP of 16 MW (ray 1, defining the top of wedge 1). Equation 6-10 shows the calculations for ray AD (ray 2, defining the top of wedge 2). And finally Equation 6-11 shows the calculations for ray AB (ray 3, defining the top of wedge 3) leading to the MULP. Hence, the total station flow at any point will be given by Equation 6-12

$$\begin{aligned} a^0 &= 0 \text{ MW / MW} \\ f^0 &= f(20) / 20 = 1 \text{ cm / MW} \\ c^0 &= f^0 = 1 \text{ cm / MW} \end{aligned}$$

Equation 6-8

$$\begin{aligned}
 a^1 &= 4/16 = 0.25 \text{ MW / MW} \\
 f^1 &= f(16)/16 = 0.947368 \text{ cm / MW} \\
 c^1 &= \frac{(f^1 - f^0)}{(a^1 - a^0)} = -0.21053 \text{ cm / MW}
 \end{aligned}$$

Equation 6-9

$$\begin{aligned}
 a^2 &= 10/10 = 1 \text{ MW / MW} \\
 f^2 &= f(10)/10 = 1.074627 \text{ cm / MW} \\
 c^2 &= \frac{(f^2 - f^1)}{(a^2 - a^1)} = 0.169678 \text{ cm / MW}
 \end{aligned}$$

Equation 6-10

$$\begin{aligned}
 a^3 &= 16/4 = 4 \text{ MW / MW} \\
 f^3 &= f(4)/4 = 1.8 \text{ cm / MW} \\
 c^3 &= \frac{(f^3 - f^2)}{(a^3 - a^2)} = 0.241791 \text{ cm / MW}
 \end{aligned}$$

Equation 6-11

$$q = g - 0.21053z^1 + 0.169678z^2 + 0.241791z^3$$

Equation 6-12

Now, the first reserve variable, z^1 , represents the amount of spare capacity available due to operating units at their MELP rather than at maximum output. The LP will see z^1 as desirable because every MW of z^1 saves a small amount of release, for the same output level, so the formulation will have to place a limit on z^1 appropriately—constraints defining limits on the z^w are referred to as ‘reference ray’ constraints.

In the example, the point at which all units are operating at MELP is M , which is defined as $(g, z) = (80, 20)$. The limit on z^1 is defined by ray one, which has associated with it a slope of $a^1 = 0.25$. It follows that a constraint that says that the ratio of z^1 to g must be less than or equal to 0.25 should be introduced into the LP problem, i.e. $-0.25g + z^1 \leq 0$. To test this, let's examine the point M , where $(g, z) = (80, 20)$, so the constraint evaluates to $-0.25 \times 80 + 20 \leq 0$, or, $-20 + 20 = 0$ as expected.

The value of the flow approximation can also be calculated, which should be equal to the original function, at this point, because it lies on a ray, and the rays were used to define the approximation. The flow is given by $q = 80 - 0.21053 \times 20 = 75.7894$ cm, which should be equal to the true cost of operating five units at their MELP, which is given by $q = 5 \cdot 16 / \left(\frac{1}{6} + \frac{1}{9}(16) - \frac{1}{288}(16)^2 \right) = 75.7894$ cm as expected.

To further the example, consider point D . Using g and z^1 alone cannot define D , so some z^2 must be used. Thus D is made up of a combination of g , z^1 and z^2 . The output g is 50 MW and the first ray constraint must be binding, so z^1 is given by $z^1 = a^1 g = 0.25 \times 50 = 12.5$, and the remainder of the reserve, 37.5 MW, must be z^2 and can be calculated as $z^2 = (a^2 - a^1)g = (1 - 0.25)50 = 37.5$. Now, the flow at D is given by $q = 50 - 0.21053 \times 12.5 + 0.169678 \times 37.5 = 53.7313$ cm, which is correct for five units running at 10 MW each, according to Figure 6-1.

Now, if the incentive to operate at low loadings is high enough (i.e. the station can provide reserve in this region and the reserve price is high) the LP will want to use as much z^2 as possible, so z^2 must be constrained appropriately. This constraint has a similar form to the first ray constraint above, $-(a^2 - a^1)g + z^2 \leq 0$, or, $-0.75g + z^2 \leq 0$. At Point D this constraint should be binding, so let's test that: $-0.75 \times 50 + 37.5 \leq 0$, or, $-37.5 + 37.5 = 0$ as expected.

This exercise leads to the general form of reference ray constraint w :, shown in Equation 6-13; but note that $a^0 = 0$, which gives the slightly simplified constraint for ray 1, shown earlier.

$$-(a^w - a^{w-1})g + z^w \leq 0$$

Equation 6-13

A.2.3 Modelling the Cost of Motoring Plant

Appendix D shows how the PLSR framework can be extended to include TWD capability. Figure 6-3 shows how this relates to the feasible operating range of a hydro sta-

tion. The new region $AEFB$ represents operation of units in TWD mode, assuming that TWD is the least desirable operating mode—if it were not, the region could be shown lying between PLSR wedges with lower cost and those with higher cost.

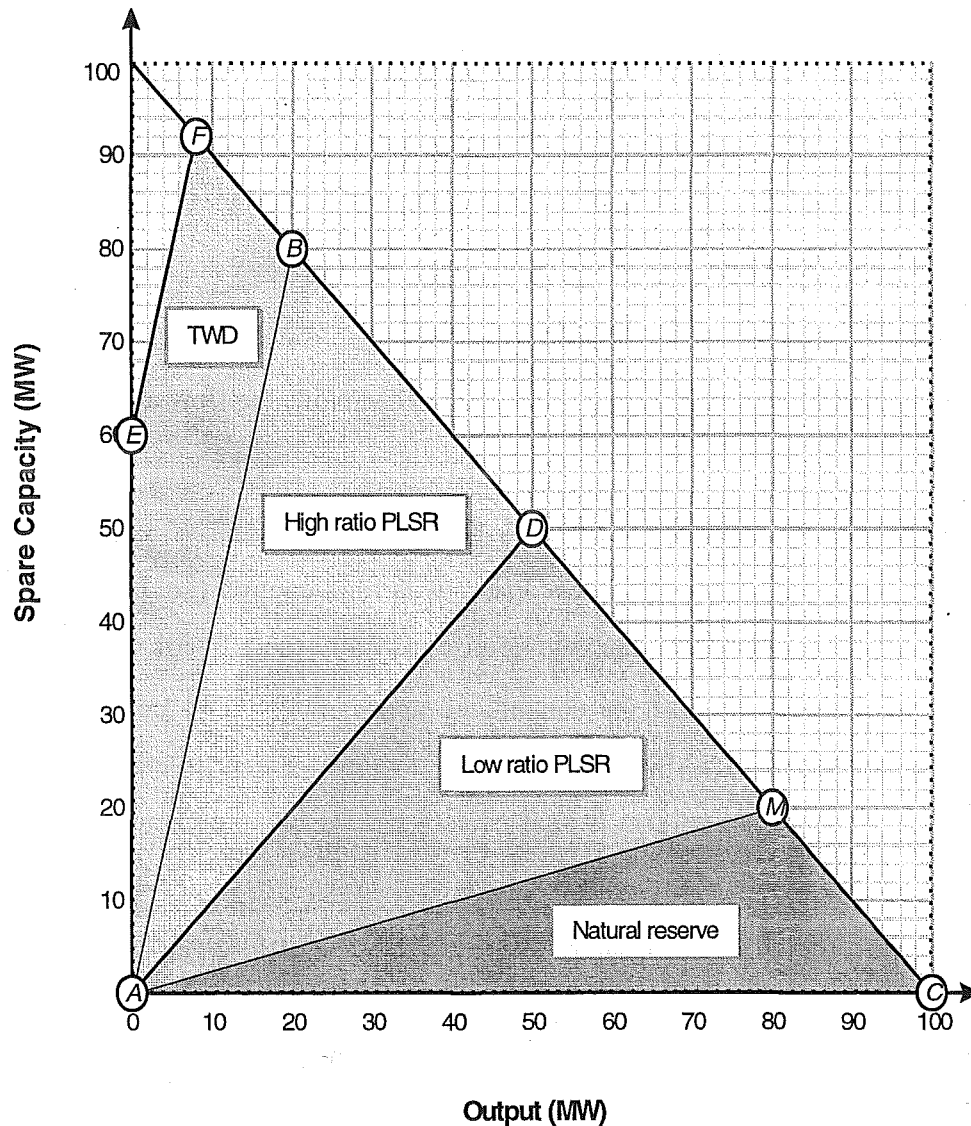


Figure 6-3: The Fan Approximation for PLSR and TWD

Wedges of TWD can be added to the LP formulation by adding a set of variables representing the amount of committed capacity in the TWD wedge. (Note that, in a linear framework, the fact that TWD can only be provided in integer units is ignored, and it is assumed that some mechanism is used to either provide appropriate bounds on TWD or

post optimally integerise the solution.) These variables must be included in the station capacity constraint, along with generation and the PLSR wedges, but are also subject to simple upper bounds.

Finally, the cost of TWD must be accounted for by either adding a cost term to the global objective function or by adding a small fraction of the total TWD to the right hand side of the station energy demand constraint (if one exists).

A.2.4 Calculating Delivered Reserve

Appendix D developed an LP formulation of the time-limited reserve capability of a hydro station with PLSR and TWD capability. It defines 'wedges' in a similar fashion to that described here, except that the variables used represent the amount of reserve delivered in a given time frame (for example 6 or 60-seconds). Then a Capacity Utilisation Factor (*CUF*) is applied to each wedge and block to convert *x*-second reserve into units of capacity, on the left-hand side of the station capacity constraint, which is of the form shown in Equation 6-14.

$$g + \sum_{w=1}^W CUF^{w,x} r^w \leq \bar{o}$$

Equation 6-14

In Equation 6-14 $CUF^{w,x}$ is the capacity utilisation factor for PLSR or TWD wedge *w* for the station's *x*-second reserve capability function. In contrast, the formulation developed here, for approximating a hydro station flow function, uses the variables z^w and z^b to represent the amount of spare capacity and total capacity committed to TWD respectively. However, if the appropriate *x*-second *CUF* values for each wedge and block were known, the amount of *x*-second reserve available could be calculated using Equation 6-15.

$$r^x = \sum_{w=1}^W 1/_{CUF^{w,x}} r^w \quad x \in \{6, 60\}$$

Equation 6-15

Hence, to allow the hydro station LP formulation to calculate the amount of x -second reserve from z^w the variable r^x is simply added to the above constraint. An estimated price, or appropriate bounds on r^x , can then be applied to derive the required reserve response from the model, allowing the underlying Fan Approximation framework to handle the economics of doing so.

A.2.5 Summary

The Fan Approximation to the flow function of a station uses the following notation:

- a^w for the slope of the ray defining the upper boundary of spare capacity wedge w ,
- c^w for the coefficient of flow in spare capacity wedge w ,
- \bar{o} for the station's maximum output,
- \bar{z}^b for the size of TWD block b ,
- ε^b for the fraction of energy consumed in operating TWD block b (optional),
- e for the energy required from the hydro station (optional),
- r^x for the amount of reserve available in x seconds,
- $CUF^{w,x}$ for the x -second 'Capacity Utilisation Factor' for PLSR wedge w ,
- $CUF^{b,x}$ for the x -second 'Capacity Utilisation Factor' for TWD wedge b .
- g for the MW station output,
- z^w for the MW spare capacity in PLSR wedge w ,
- z^b for the MW capacity committed to TWD block b ,
- ψ for the preset (constant) value of water in storage at the station

The hydro station LP formulation would include the following:

The objective function components (cost minimisation assumed):

$$\text{Minimise } \dots + \psi \left(c^0 g + \sum_{w=1}^W c^w z^w \right) + \dots$$

$\dots, g, z^w, \dots \geq 0$

Equation 6-16

The station capacity constraint:

$$g + \sum_{w=1}^W z^w + \sum_{b=1}^B z^b \leq \bar{o}$$

Equation 6-17

The reference ray constraints:

$$-(a^w - a^{w-1})g + z^w \leq 0 \quad w = 1, \dots, W$$

Equation 6-18

TWD block upper bounds:

$$z^b \leq \bar{z} \quad b = 1, \dots, B$$

Equation 6-19

Reserve delivery definition constraints:

$$r^x = \sum_{w=1}^W \gamma_{CUF^{w,x}} r^w + \sum_{b=1}^B \gamma_{CUF^{b,x}} r^b \quad x \in \{6, 60\}$$

Equation 6-20

Energy demand constraint (optional):

$$g - \sum_{b=1}^B \epsilon^b z^b \geq e$$

Equation 6-21

These equations may be supplemented by additional bounds on the station's output.

A.3 Storage and River Chains

This basic hydro river chain model, described in Chapter 2, is incomplete with respect to:

- how releases made during the current planning horizon that will arrive outside of the horizon are valued (see Figure 6-4);
- how storage targets should be set for head ponds;
- how flows that enter the model from the previous day are represented

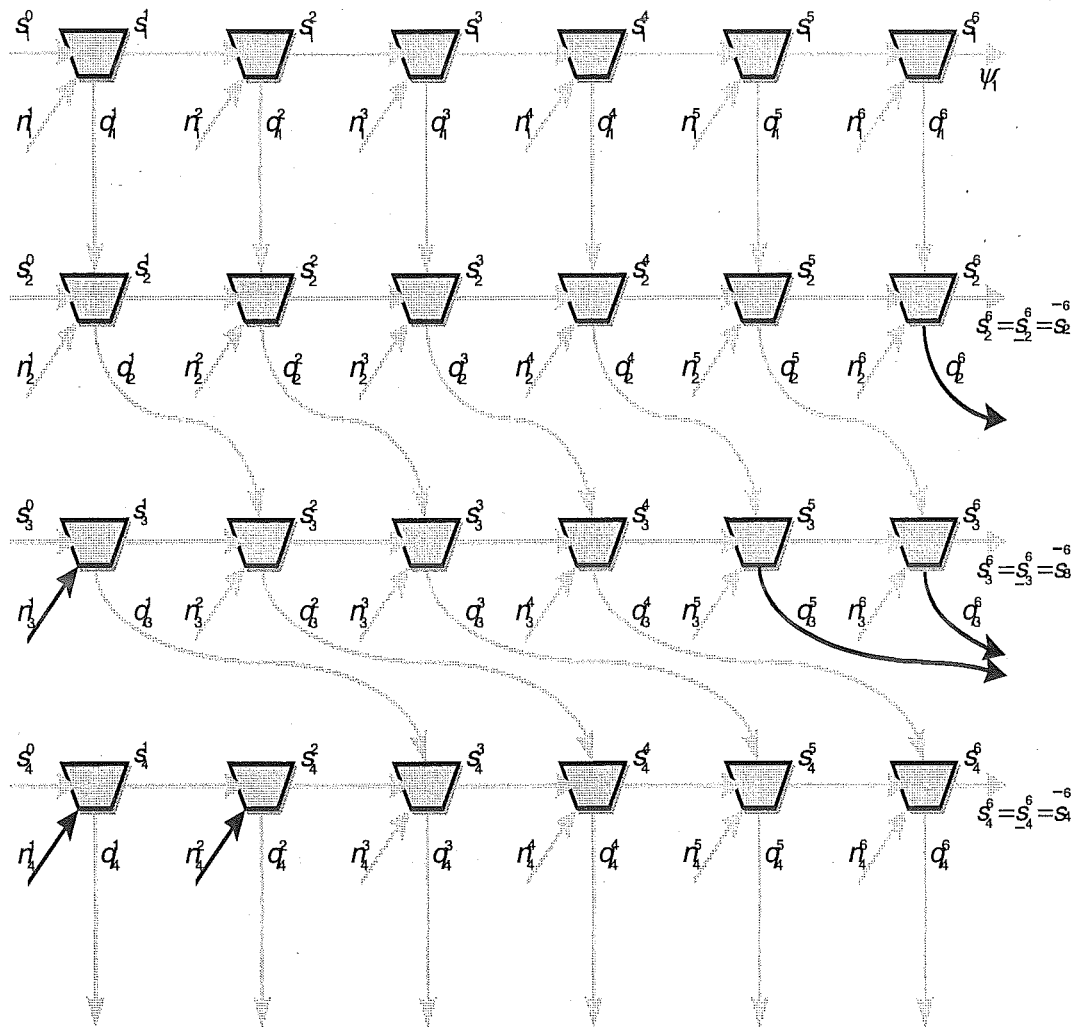


Figure 6-4: Hydro river chain network model

The river chain model is designed to represent the real system over a *limited* horizon, so it is unaware of economic and physical constraints on operation outside of that horizon. The basic formulation presented earlier attempted to offset this by valuing water, which it does two ways:

- 1) Directly via water values at the major reservoirs. (The use of water values as a measure of cost is valid because water values are effectively the 'fuel costs' at hydro stations. They give the expected value of the water in terms of thermal

costs forgone, either now, or in the future, because the water will be used to generate instead of thermal plant.)

2) Indirectly via the end-of-horizon storage targets on the remaining reservoirs.

Storage targets directly determine how much water will be available for release, while water values determine whether release is economic. These water values are very important parameters because they ensure that adequate provision is made for future requirements. The major complication is that simple storage targets are not sufficient to model end-of-horizon effects when delay times mean that there may be significant flows-in-transit, i.e. flows that would not arrive at their destination within the current planning horizon. There is little point ensuring that a target is met at the end of the day if flows-in-transit mean that the river will dry up or flood immediately thereafter, it is not surprising then that optimisation models tend to produce precisely these kind of effects unless formulated not to.

Typical problems that one might observe in the solutions of a 'naive' formulation, such as the basic hydro model presented earlier, are uneconomically high levels of release in final periods, and, unbalanced end of horizon storage in intermediate reservoirs. A simple approach to minimising the impact of these 'end effects' on current decision-making is to extend the 24-hour horizon to 48 hours. This may be an adequate provision in some cases, but one must still consider how flows-in-transit are to be handled at the conclusion of that extended horizon. Additionally the number of variables and constraints will double, increasing the model's solution time by at least this much—an important consideration if the model is to be used multiple times in one planning period.

However, unless a reservoir is expected to attain its bounds in any of the post horizon periods, it is certain that the water value will be the same in each of those periods. In such a situation, there is no need to model the post horizon periods separately unless there are other constraints needing to be modelled—which seems unlikely for reservoirs large enough to have a water value assigned. Consequently, the preset water value may be interpreted as applying to all water arriving at or after the end of the horizon, provided no storage or other constraints are expected to be binding in the near future. Thus,

for major storage reservoirs, it may be sufficient simply to aggregate all flows-in-transit into the final period storage value.

This approach would not be appropriate for head ponds. By definition, they are expected to attain their bounds at least once within the planning horizon, and are required to meet a preset end of horizon storage target. Thus, each flow arc that will arrive at a head pond after the current planning horizon may need to be modelled separately to ensure that the model does not, in optimising the current horizon's releases, sacrifice post horizon feasibility. To do so accurately, either the post horizon periods must be modelled completely or strict bounds on these flows must be set based on some exogenously determined solution.

The array of possibilities seems daunting but, when one considers the framework in which this model is to be used, the appropriate approach is more obvious. Several possible situations can be envisaged, the most likely of which are:

- 1) The model is used to determine the best combination of storage and release for a 'typical' day under specific long-term conditions. (This is called this the Daily Release Cycle Planning Model.)
- 2) The model is used to determine storage and release given specific initial conditions and required end conditions and/or a set of generation targets or system prices. (This is called the Block Implementation Planning Model.)

One can easily envisage how these models would be used in a market environment in which a river chain is managed independently. The first model would be used prior to an actual day to define the optimal 'cycle' for the river chain's head ponds given expected generation patterns and constraint conditions for a 'typical' day. These cycles would be defined by their beginning and ending storage levels and flows-in-transit. The second model would be used to plan the river chain's actual dispatch close to real time, acting in response to actual generation targets produced by the market. Its objective would be to allow, as much as is practicable, for the river chain to be left in 'equilibrium' at the end of the horizon. Thus, it would use the first model's optimal cycle to set appropriate

constraints on end storage and flows-in-transit. The following sections describe these models in more detail.

A.3.1 Daily Release Cycle Planning Model

Apart from storage bounds for head ponds, this model would be given as input:

- initial storage levels for major storage reservoirs;
- and preset water values for major storage reservoirs, that have been determined by a long term, most likely annual, model

The river chain optimisation model would be asked to determine the best ending *and* beginning storage for the chain's head ponds. Under these conditions the model:

- requires that the initial and final storage levels for head ponds are equal but leaves the model to determine the best levels, i.e. the storage is 're-cycled' from the end of the horizon to the start;
- and allows delayed flows to 'wrap around' in a cycle, i.e. delayed flows from the end of the horizon appear in the appropriate period earlier in the day

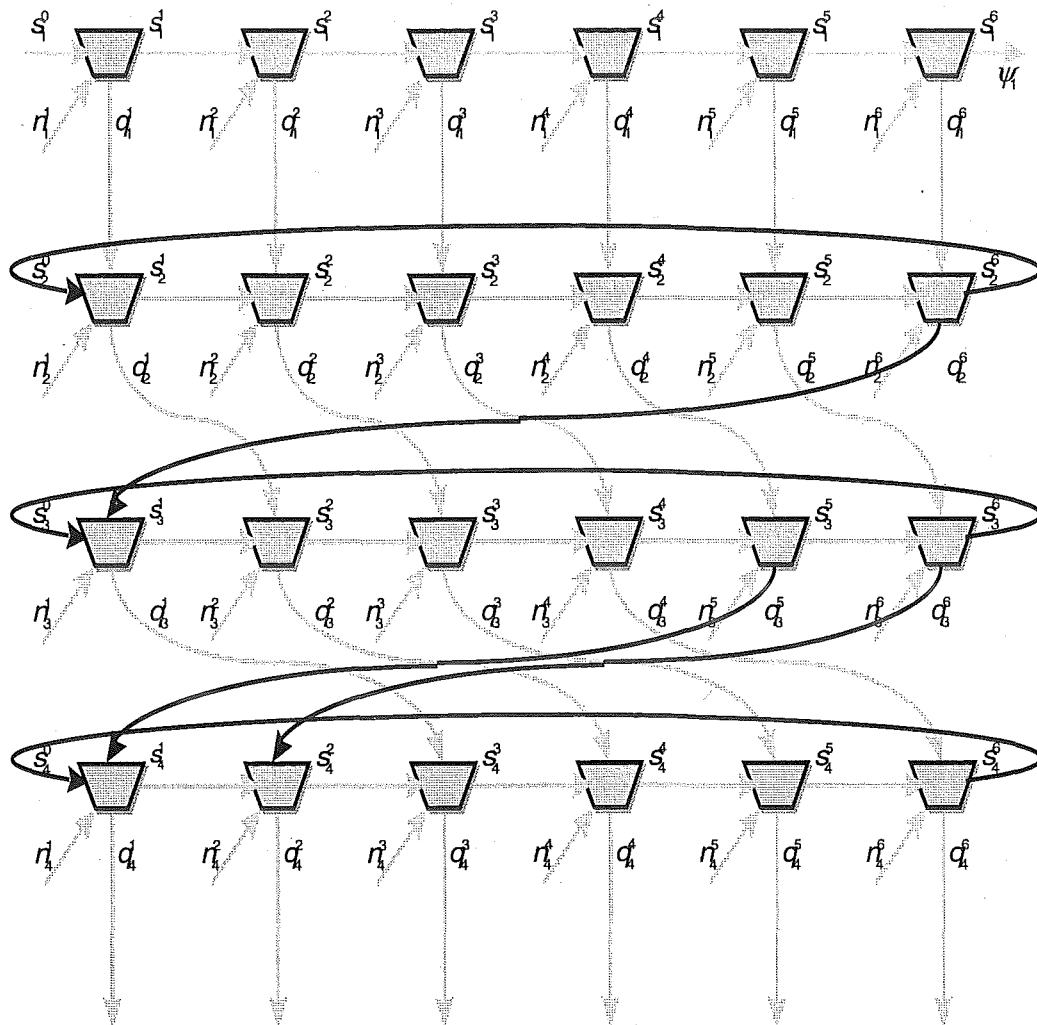


Figure 6-5: Network flow diagram for the Daily Release Cycle Planning Model

The network flow diagram of Figure 6-5 shows the network structure and storage bounds for this model. This style of model is suitable for determining the optimum daily cycle for a river system. It implicitly assumes that the day before and the day after are identical to the planning day. Implementation of this approach in the LP formulation follows the rules set out in the following paragraphs.

An important feature of this model, is the implication its solutions have for management of head ponds; storage targets control their releases. By 'recycling' storage and flows-in-transit in the manner described above, the optimisation model produces the

optimal daily management policy, i.e. optimal storage targets for these reservoirs. This makes this form of model valuable in planning not just short-term but medium term operation. It is also useful for scenario analysis and answering management 'what-if' type questions.

Operationally, one of these models may be used for determining the optimal system response for a weekday and another for weekends when load patterns are quite different. The intermediate days require an approach that allows flows in transit from the previous day and into the next day to be different. This is actually equivalent to the second model, discussed in the next section.

A.3.2 Block Implementation Planning Model

In response to system price or energy target signals, a river chain may not want to keep reservoirs in equilibrium according to the solution to the Daily Release Cycle Planning Model. Rather, they will want to respond to market conditions, capitalising on higher than expected prices, etc. or simply finding the optimal transition from one day to the next. Under these circumstances flows-in-transit at the beginning and end of the day may not be equal, as they must be in the Daily Release Cycle Planning Model.

When using this model as a planning tool, it is fair to assume that the day before the planning day was an average day. Thus, the flow levels from the equilibrium model may be used to 'adjust' the tributary inputs into the first few periods of the Block Implementation Planning Model. At the other end of the horizon, the flows-in-transit can be aggregated with the end of period storage and, assuming the next day is average, adjust the storage target upwards appropriately.

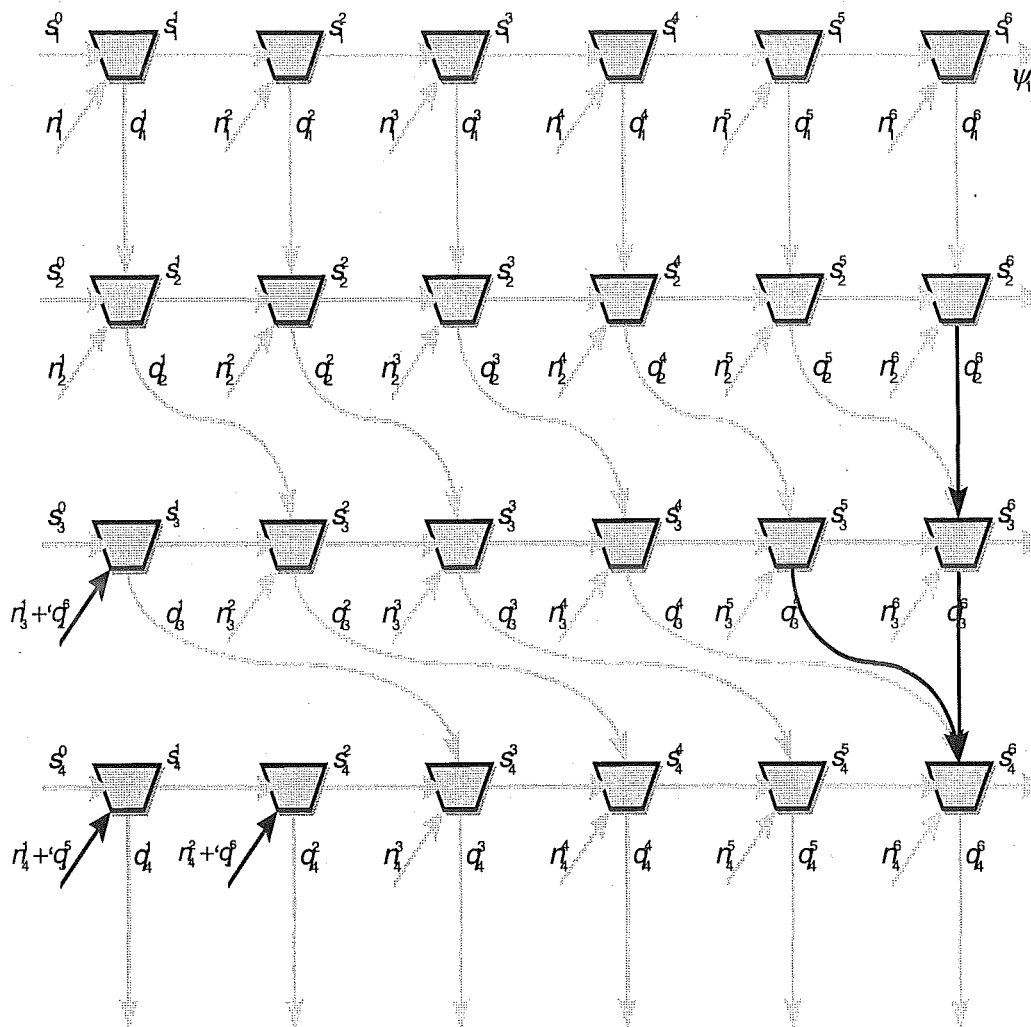


Figure 6-6: Network flow diagram for the Block Implementation Planning Model

Figure 6-6 shows the network layout for this model, with changed arcs shown in black. Note that by aggregating all flows-in-transit with the final period storage, this is implicitly assuming a constant water value over all future periods in which these flows arrive. This may or may not be a valid assumption. If not, 'sets' of flows with equal implied water value can be separated and apply separate targets to them.

Without recycling of flows, this model may eventually run head ponds down; thus, this model is most appropriate for planning a real-time daily dispatch in response to system prices or targets that may not allow equilibrium to hold. Targets, from an equilib-

rium flow model, should always be used to set end-of-horizon storage targets so that the system can move safely from one day to the next.

A.3.3 Smoothing Storage Trajectories and Release Patterns

Any LP formulation may have a range of alternative optimal solutions. In this case there could be a series of solutions with different storage trajectories and release patterns but the same overall objective function value. This type of behaviour is potentially destructive to the process of forming simple supply curves for the energy market. Rather than attempting to overcome this by taking more samples or repeatedly solving the model with slightly perturbed input or starting conditions, it seems logical to eliminate some of the alternatives by adding enhancing the model's objective function. This approach could be used to take into account the following:

- some solutions may have much smoother storage trajectories than others;
- and some solutions may operate station more consistently than others (smooth trajectories are desirable since they imply lower maintenance costs through less unit switching, erosion etc)

On these criteria the model may be refined to largely eliminate the occurrence of undesirable alternative optima. Specifically, a small penalty factor, κ , may be added to the objective function, which 'smooths' the storage/release profiles by either:

- penalising changes in station release from one period to the next;
- or penalising changes in storage levels from one period to the next;
- or penalising changes in the proportion of the energy target which a station produces from one period to the next

The first and second approach might unnecessarily constrain the solution by (incorrectly) forcing the release profile or storage trajectory to be constant. The third approach is less restrictive because it allows each station movements up and down that mirror the changes in the river chain's overall energy target.

To implement this approach, a new set of variables is introduced:

- x_i^t for the increase in station energy output relative to the river chain's target;

- y_i^t for the decrease in station energy output relative to the river chain's target

These variables would appear in the cost minimisation objective function with the term $\kappa \sum_{t=1}^T \sum_{i=1}^I (x_i^t + y_i^t)$. The size of the smoothing factor, κ , is important. Too large and it may change the optimal solution in terms of the original objective. Too low and it will not be effective in eliminating undesirable alternative optima. One must also realise that the above constraints are intertemporal, and so are likely to increase the solution time somewhat. This trait must be traded off against improvements in solution quality.

Finally, when the model is free to determine the optimal beginning and ending storage levels for head ponds, it may find optimal solutions with 'free' storage trajectories, i.e. ones that do not hit any bounds (with positive multipliers). In this case, it would be desirable if the model chose the trajectory with the highest end storage.

To implement this, a small value of θ can be applied to the end of horizon storage level at head ponds, to encourage any 'free' trajectories to the upper extent of their ranges. The objective function term in the cost minimisation objective function would be of the form $-\theta \sum_i s_i^T$.

Appendix B

Overview of WEMSIM

This appendix outlines the Wholesale Electricity Market Simulation Model (WEMSIM), the modelling system used to produce the experimental results for this thesis. It is composed of three main parts:

- 1) A spreadsheet front-end to the model which stores input data, solutions, analyses the solutions to produce market distortion estimates etc.
- 2) A C++ module implemented as a dynamic link library that can be linked to the spreadsheet front-end and is responsible for reading data from and writing data to the spreadsheet
- 3) A C++ module which builds the required LP problems and invokes CPLEX to solve them, passing the results back to the data engine for reporting

WEMSIM was developed on IBM OS/2 Warp Version 3.0 using Watcom C/C++ 10.0 and Lotus 1-2-3 2.1 for OS/2 and was later ported to Microsoft Windows NT 3.51 using Microsoft Visual C++ 4.0 and Microsoft Excel 7.0 for Windows 95. The LP engine uses the CPLEX Version 4.0 callable library. On the OS/2 platform, the Lotus External Program API was used to pass data between the spreadsheet and the C++ program. On

Windows NT, the program used OLE automation. In total, the application contains about 15,000 lines of C++ code.

Wholesale Electricity Market Simulation (WEMSIM)											
Market type: Integrated energy/reserve.											
Comparison				Energy Cost		Penalties		Profit			
Profit	Revenue	Penalty	Cost	Cent	Decent	Energy	Reserve	Cent	Decent		
-0.30%	-0.08%	0.2660%	0.0448%	\$ 245,627.	\$ 1,253,147.	\$ 1,253,708.	\$ 2,760.	22.	\$ (245,627.)	\$ 711,750.	\$ 707,57
Wko (okay)											
		0.45%	0.0000%	\$ 2,316.	\$ -219,269.	\$ 219,269.	\$ 979.	0.			
-0.54%	2.16%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 2,807.	\$ 2,79	
-2.55%	0.19%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 8,989.	\$ 8,76	
0.05%	-0.08%		0.0000%	\$ 2,316.	\$ 219,269.	\$ 219,269.	\$ -	\$ -	\$ (2,316.)	\$ (179,573.)	\$ (179,86
-0.19%	-0.10%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 44,720.	\$ 44,63	
-0.19%	-0.04%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 30,373.	\$ 30,31	
-0.25%	-0.25%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 51,644.	\$ 51,51	
-0.29%	-0.19%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 88,021.	\$ 87,76	
-0.18%	-0.18%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 23,622.	\$ 23,58	
-0.23%	-0.10%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 74,710.	\$ 74,63	
-0.17%	-0.16%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 46,305.	\$ 46,22	
Wkm (okay)											
		6.59%	6.5330%	\$ -	\$ 41,033.	\$ 43,713.	\$ 0.	22.			
7.72%	4.50%		6.5330%	\$ -	\$ 41,033.	\$ 43,713.	\$ -	\$ -	\$ (27,517.)	\$ (29,84	
5.00%	5.06%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 24,167.	\$ 25,37	
4.64%	4.70%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 14,873.	\$ 15,56	
-0.85%	-1.33%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 3,640.	\$ 3,60	
Wlk (okay)											
		-0.98%	-0.9767%	\$ 84,816.	\$ 563,513.	\$ 558,010.	\$ -	\$ -			
0.02%	-0.27%		0.0000%	\$ -	\$ 182,935.	\$ 182,935.	\$ -	\$ -	\$ (171,475.)	\$ (171,50	
-0.27%	-0.27%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 81,212.	\$ 81,04	
0.06%	-0.16%		0.0000%	\$ 83,780.	\$ 362,881.	\$ 362,881.	\$ -	\$ -	\$ (83,780.)	\$ (258,234.)	\$ (258,39
-0.12%	-0.12%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 79,438.	\$ 79,34	
-0.12%	-0.12%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 79,438.	\$ 79,34	
-0.14%	-0.14%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 180,409.	\$ 180,15	
-0.07%	-0.07%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 73,151.	\$ 73,10	
-0.27%	-0.27%		0.0000%	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 48,977.	\$ 48,84	
-20.44%	-22.51%		-22.6434%	\$ 454.	\$ 15,763.	\$ 12,194.	\$ -	\$ -	\$ (454.)	\$ 1,029.	\$ 81
-100.00%	-100.00%		#####	\$ 382.	\$ 1,934.	\$ -	\$ -	\$ -	\$ (382.)	\$ 51.	\$ -
		0.00%	0.0000%	\$ 155,136.	\$ 113,275.	\$ 113,275.	\$ -	\$ -	\$ (155,136.)	\$ 21,760.	\$ 21,39
Man (okay)											
-1.70%	-0.27%		0.0000%	\$ 155,136.	\$ 113,275.	\$ 113,275.	\$ -	\$ -	\$ (155,136.)	\$ 21,760.	\$ 21,39

Figure 6-7: The WEMSIM spreadsheet on Microsoft Windows NT 4.0

Figure 6-7 is a captured image of WEMSIM running on Windows NT 4.0. The OS/2 version was used for most of the day-ahead and periodic-update market experiments with the Windows NT version being used for later experiments. The hardware used for early experiments was an IBM compatible Pentium 90MHz with 256kB L2 cache and 32MB of RAM. Later experiments were performed on a Pentium 166 MHz with 512kB write-through L2 cache and 32MB of EDO RAM.

WEMSIM builds several different types of LP problem depending on the phase of the market dispatch and the type of market being modelled. The integrated energy and reserve central coordination LP consists of 4,500 rows and 3,500 columns and uses 25–90 seconds of CPU time to solve. The smaller market clearing LP problems takes about seven seconds.

The steps required to perform one experiment of the energy market are:

- 1) Solve the centrally coordinated LP problem and store the result by selecting
WEMSIM/Run/Centrally Coordinated then WEMSIM/Keep/Centrally Co-ordinated".
- 2) Solve the centrally coordinated LP problem for five demand scenarios and record each generator's response and the prices by selecting
WEMSIM/Run/Scenarios. Each scenario involves a full centrally coordinated LP but the optimal basis is saved each time so that only the first scenario need be solved from scratch.
- 3) Prepare the offers. This function is done automatically by formulae in the spreadsheet.
- 4) Solve the market dispatch or real-time simulation by selecting
WEMSIM/Run/Market Dispatch.

For the periodic-update market, this process is repeated for each period of the day. In total, a run of the periodic-update market over a number of days can take several hours CPU time.

Apart from the solution of LP problems the WEMSIM menu allows the user to store and delete scenarios, display graphs of the results and review LP and log files from the DLL.

Appendix C

Energy Market Results

This appendix presents the experimental results in some detail. The aim of this analysis is to answer the research questions related to the efficiency of the simplified energy market. Sections C.1 and C.2 present the results of an initial investigation of fixed versus sculpted offers in a day-ahead market design. Section C.3 attempts to improve on these results by using more than one demand scenario to prepare the offers. Section C.4 takes the most promising offers from the day-ahead market and tests their performance when generators are allowed to update their offer every four hours. Section C.5 measures the sensitivity of the observed results to permutations of the input data.

The overall methodology is to compare a simulated market dispatch to what could be achieved with a centrally coordinated model. The energy market experiments use a 'base day' with two demand peaks. The base demand is deliberately high; to reflect likely demand patterns when the market will actually be implemented. Thermal stations in the North Island are base loaded up to Huntly, with New Plymouth being 'marginal'. Both North and South Island hydro river chains are fairly heavily utilised.

In reality, generators will not know the centrally coordinated solution, and will not necessarily be motivated to reproduce it if they did. Here this second issue is ignored, although it is critical in practice. However, it is not unreasonable to suppose that generators will estimate market prices and dispatched quantities for the day ahead. So, in these experiments, the centrally coordinated model with its estimation of demand and of other generators' physical situations is run to determine prices and group's responses to those prices. This is equivalent to each generator having their own simulation, from which to draw expected market prices, and on which to base their offers. These offers are then submitted to a market clearing process which accepts the cheapest energy/reserve offers until the projected demand is met.

Finally, the centrally coordinated model is run again, with added constraints to reflect commitments made in the market. Generators attempt to meet their commitments, suffering a penalty (transaction cost) on a \$/MWh basis for failing to do so. This approach implicitly assumes that generators trade optimally to find the most desirable dispatch given the targets and penalties they face. The measures of market distortion calculated were detailed earlier, and include:

- 1) Direct cost distortion (percent increase in direct costs over the centrally coordinated dispatch).
- 2) Transaction cost inclusive distortion (direct costs plus transaction costs).
- 3) Total targets defaulted on (MWh).
- 4) Total spill or increase in spills (MWh).

C.1 Sculpted Offers

Producing an offer tailored to each period seems intuitive. However, with only one raw observation to work from (as would result from a single simulation of the market), deciding on a form for the supply curve requires more assumptions, and a number of processes might be considered. For example, it would be possible to do a parametric analysis, with the centrally coordinated model, to determine the amounts and prices of output

for each generator in each period. The parametric analysis would alter either the overall demand or market price for one period at a time and observe the output response by each generator.

Unfortunately this is likely to be far too optimistic, as it assumes the market prices or demands will stay the same in the other periods, whereas in reality, a higher demand is likely over several periods or even the whole day. For hydro generators, with limited water available, this could result in offers that would be infeasible if accepted.

The easiest approach to forming a sculpted offer is to take the price from the simulation in a period and construct a 'band' by specifying a range around the output by the generator in that period. Capacity below this band is offered for free, and any above, up to the maximum, is offered at a much higher price. In these experiments, the range offered is set the same in each period.

The 'attraction' of sculpted offers is to allow hydro generators to try to follow their preferred reservoir storage trajectories. The offer should achieve this because the 'free' capacity offered below the central band will most likely be taken up, while there is a very low probability that anything above that band will be taken up. Naturally, if no leeway at all is allowed, this process will exactly reproduce the ideal trajectories, and in the deterministic case, this will be identical to the centrally coordinated dispatch. On the other hand, by effectively denying the market any choice with regard to hydro scheduling, this strategy seems likely to impose significant costs under uncertainty.

One way of forming an offer from an observed price and quantity pair is to assume, on average, the generator would be comfortable producing q MW at p \$ per MW in each period that the same price and quantity apply. Thus, the observed output level could be taken and converted to a range of levels using some method, with q MW being the midpoint. Above this range one assumes that some very high price, p^{max} in Figure 6-8, would have to be reached before the generator is prepared to offer more. (This does not necessarily mean that the generator can operate in that range and if the offer is taken up, it may be forced to buy in power from the market at a very high price.) Below this range,

a price of \$0.00 is assumed so that flow constraints, minimum operating levels, etc. are satisfied.

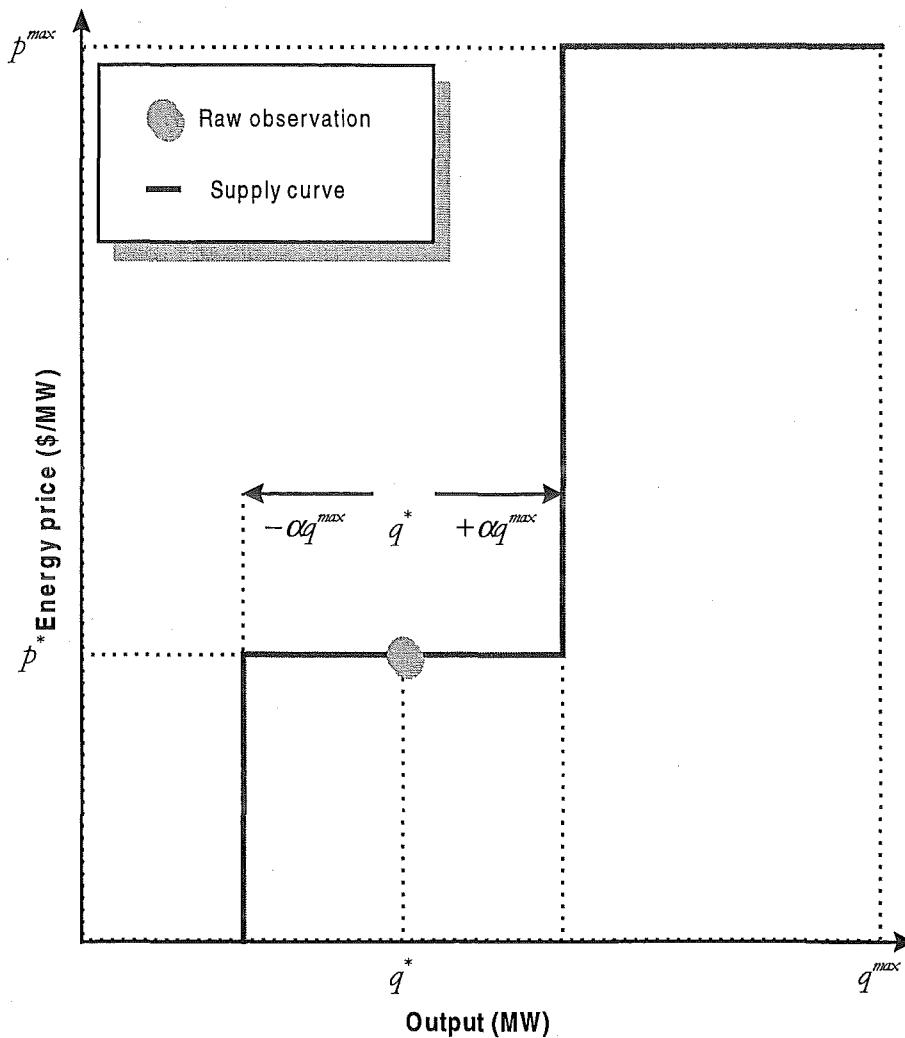


Figure 6-8: A sculpted offer supply curve

Creating a single period supply curve is a relatively simple task, requiring the creation of three sub-bands for each offer. The middle band is centred on the observed quantity (q) and has a width set to a specified fraction 2α of the maximum capacity, with the price being the same as the original observed price (p). The other bands allocate the remaining generation capacity above and below that middle band, priced at \$0.00 and

the p^{max} respectively. An example of how the supply curve is created from an observed price and quantity pair is shown in Figure 6-8. In mathematical terms, the band definitions are as in Table 6-1.

<i>Band Section</i>	<i>p</i>	<i>q</i>
<i>Band 1</i>	0.0	$q^* - \alpha q^{max}$
<i>Band 2</i>	p^*	$2\alpha q^{max}$
<i>Band 3</i>	p^{max}	$g^{max} - (q^* + \alpha q^{max})$

Table 6-1: Sculpted offers band formulae

C.1.1 Simple Three-band Sculpted Offers

These experiments use the sculpted offer creation calculations described above, and vary the parameter α , which determines the width of the central band, over the range 0.002–1.0, and assumes the generators are able to accurately forecast day ahead demand, i.e. they use a single accurate scenario to form their offers. The results show that:

- with extremely narrow bands around the ideal trajectory, the dispatch follows the ideal almost exactly, with percentage direct cost between 0.02–2.8% , and transaction cost inclusive distortions between 0.04–3.5% for widths of 0.002–0.1;
- but, as the width of the central band is increased, targets begin to diverge, many becoming infeasible and large transaction costs result (percentage direct cost distortions were between 3.9–17.6%, and distortions including transaction costs were between 8.2–72.9% for band widths of 0.2–1.0)

The performance of these simple three-band sculpted offers depends heavily on the accuracy of day-ahead demand forecasts. Uncertainty factors would force generators to widen bands and face increased penalties, resulting in an overall loss in productive efficiency. To test this effect, the next set of experiments introduces random +/-5.0% errors into the demand forecasts.

Width	Penalty						
	\$148.00	\$74.00	\$37.00	\$18.00	\$9.00	\$4.50	\$2.25
0.002	2.50	1.80	1.80	1.80	0.90	0.30	0.09
0.020	2.00	1.60	1.60	1.60	0.90	0.30	0.08
0.100	2.30	2.20	1.90	1.10	0.70	0.20	0.09
0.200	4.60	3.90	2.70	1.20	0.60	0.20	0.10
0.400	11.50	7.70	4.50	1.90	0.80	0.40	0.11
1.000	16.10	13.20	10.00	4.80	2.60	0.70	0.16

Table 6-2: Three-band sculpted offers: direct cost distortion (%)

Width	Penalty						
	\$148.00	\$74.00	\$37.00	\$18.00	\$9.00	\$4.50	\$2.25
0.002	2.60	2.50	2.20	2.00	1.70	1.10	0.60
0.020	2.00	1.90	1.80	1.70	1.50	1.00	0.60
0.100	2.80	2.50	2.30	1.80	1.30	0.80	0.50
0.200	6.60	5.60	4.50	3.00	1.90	1.10	0.60
0.400	27.40	18.90	12.30	7.40	4.30	2.40	1.30
1.000	58.70	37.10	24.20	15.00	9.20	5.20	2.80

Table 6-3: Three-band sculpted offers: transaction cost inclusive distortion (%)

Width	Penalty						
	\$148.00	\$74.00	\$37.00	\$18.00	\$9.00	\$4.50	\$2.25
0.002	4.0	53.0	53.0	66.0	501.0	929.0	1358.0
0.020	1.0	22.0	23.0	37.0	393.0	834.0	1225.0
0.100	21.0	23.0	51.0	230.0	414.0	786.0	1012.0
0.200	77.0	129.0	266.0	576.0	824.0	1145.0	1371.0
0.400	608.0	865.0	1189.0	1714.0	2227.0	2574.0	3011.0
1.000	1633.0	1830.0	2173.0	3217.0	4171.0	5775.0	6652.0

Table 6-4: Three-band sculpted offers: target defaults (MWh)

Referring to the results of Table 6-2 to Table 6-4, in contrast to the deterministic case, which produces superior results as the width of the central band is decreased, a narrow central band can be expected to produce worse results under uncertainty. This is because, with a narrow central band, the market has very limited choice for hydro dispatch and must work around this with thermal plant. Also, if the solutions are forced to jump outside the central bands, the market sets targets in an arbitrary way because all the other non-allocated bands are at some very high level. This happens for some experiments with the width of the central band very low and all generators under estimating demand,

and so offering less in the lower band. Consequently, arbitrary targets are set for some generators and they were unable to meet them, resulting in high costs.

Overall, with uncertainty in the offer formation phase, the experimental results show:

- bands of width 0.02 perform the best, with narrower central bands performing worse than in the certainty case;
- and bands of width of 0.1 provide the most stable results in terms of overall system direct costs over a range of uncertainty scenarios with distortions in the region of 2–3%

As the width of bands increases, defaults on target increase, up to a width of 0.4, when they level off. A by-product of high penalties under uncertainty is spill, which increases significantly with widths above 0.2. When penalty rates are varied, the distortions remain fairly stable with less than 2% distortion achievable for widths between 0.2–1.0, with penalty rates from \$148.00 to \$2.25 respectively.

Overall, simple three-band sculpted offers performed reasonably well in these initial experiments, with about 2.0% direct cost distortion plus an additional 0.0–1.0% in transaction costs.

C.1.2 Multi-band Sculpted Offers

This section examines sculpted offers with five bands. This type of offer is quite sophisticated, involving a different multi-band offer for each period. There are many ways to generate such an offer but for simplicity, the single observation from the market simulation is used. This is similar to the way in which the three-band sculpted offer is formed.

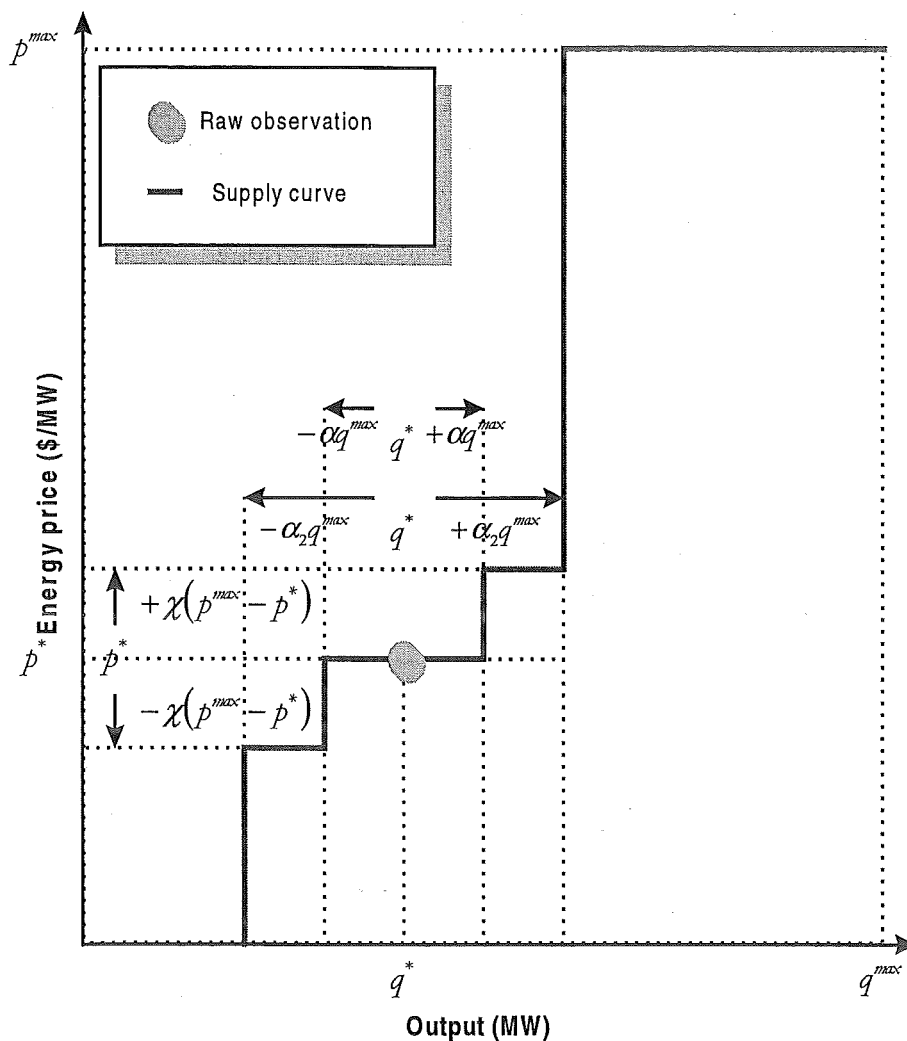


Figure 6-9: Five-band sculpted offer

As with the three-band sculpted offers, the lowest band is assumed to be a quantity offered for free. This ensures that this quantity is taken up, to cover river flows. The top band is offered at a high price, so it is unlikely to be taken up. The central band consists of some percentage of the total possible offer, centred on the observed output level from the market simulation—the price of this band is also taken from that simulation.

These three bands were all that was used to form the three-band sculpted offer. But in this design two additional bands are added around the centre. These two bands, one above, and one below the central band are also calculated as a percentage of the maxi-

mum possible offer, based on the parameter α_2 . They are priced at some fraction χ of the distance from zero to the observed price and the observed price to the top band price, respectively, see Figure 6-9.

Obviously, with a number of parameters used to create the offer, a very large number of experiments could be carried out. Initially, one parameter was explored at a time with the following experiments:

- 1) A set of experiments with $\chi = 0.5$ and $\alpha_2 = 0.01$. (These parameters were tested with the same range of widths of the central band, uncertain demand scenarios and penalty values as the experiments for the three-band offer previously.)
- 2) A set of experiments with a central band of width 0.04 of the maximum offer, the intermediate widths set to 0.04 each, and χ allowed to vary from 0.0–1.0.

It is possible to add additional bands, beyond the five used here, in a similar manner. In the limit this would give a four-piece linear offer of the form shown Figure 6-10. Altering the value of χ is equivalent to changing the slopes of the two central linear sections. Offers with many bands could approximate this offer, however, experiments using offers of this kind are not investigated further, because it is clear from experiments presented later that offers formed using more than one forecast scenario yield superior results to offers using single scenarios.

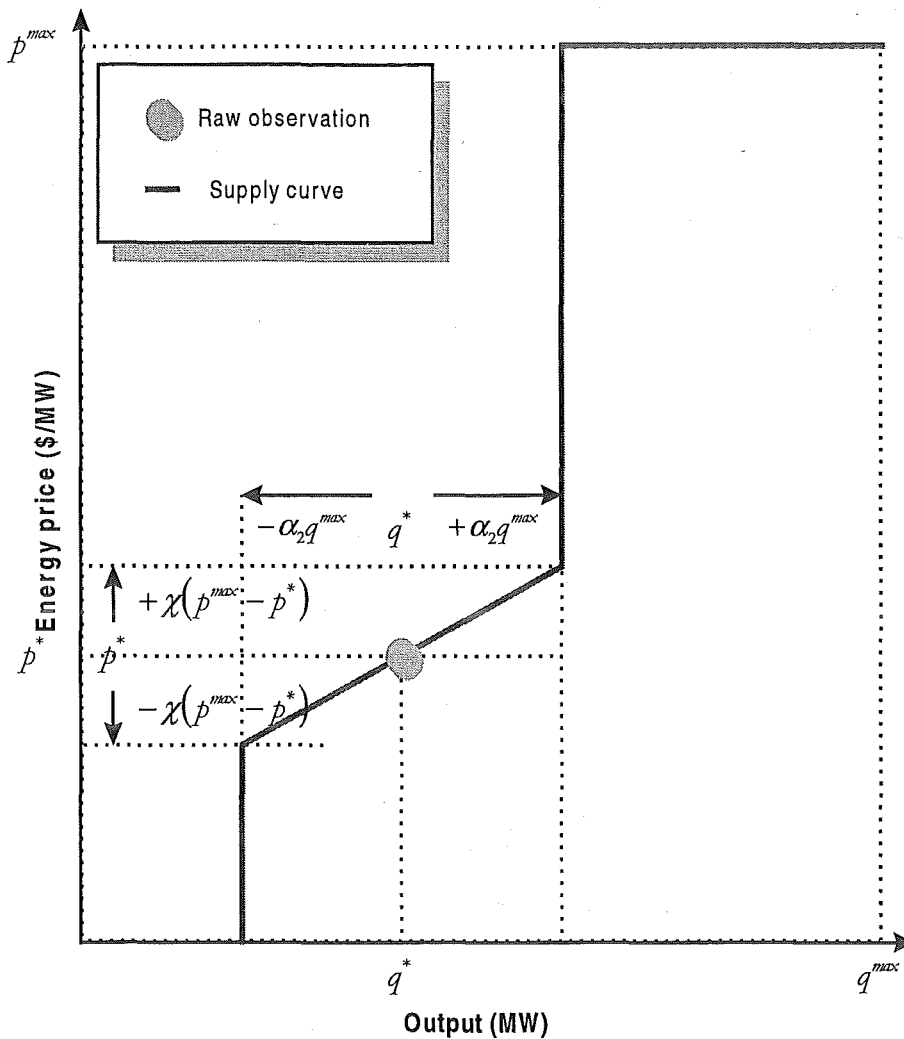


Figure 6-10: Piece-wise linear sculpted offer

Referring to Table 6-5 to Table 6-8, these results show a slight improvement over the simple three-band sculpted offers. These results are for offers with the two intermediate bands at a price halfway between the observed and zero and the observed and the top band, respectively. Both of these bands had a width of 0.01 of the total generation possible. The width of the central band is allowed to vary between 0.02–1.0 of the possible offer, as for the three-band sculpted offers.

Width	Penalty						
	\$148.00	\$74.00	\$37.00	\$18.00	\$9.00	\$4.50	\$2.25
0.002	0.49	0.03	0.03	0.00	0.00	0.01	0.00
0.020	0.29	-0.02	-0.01	-0.02	-0.01	-0.01	-0.01
0.100	-0.01	-0.05	0.24	-0.06	0.02	-0.01	-0.01
0.200	-1.04	-1.00	-0.35	-0.08	-0.04	-0.04	-0.02

Table 6-5: Multi-band sculpted offers: direct cost distortion (%)

Width	Penalty						
	\$148.00	\$74.00	\$37.00	\$18.00	\$9.00	\$4.50	\$2.25
0.002	0.57	0.48	0.26	0.14	0.06	0.03	0.02
0.020	0.29	0.25	0.12	0.05	0.02	0.01	0.00
0.10	0.25	0.11	0.08	0.05	0.03	0.00	0.00
0.20	-0.74	-0.88	-0.63	-0.40	-0.20	-0.12	-0.07

Table 6-6: Multi-band sculpted offers: transaction cost inclusive distortion (%)

Table 6-7 and Table 6-8, for target defaults and spill, generally show less defaults on targets but more spill for the five-band offers for the 0.02 and 0.1 band widths.

Width	Penalty						
	\$148.00	\$74.00	\$37.00	\$18.00	\$9.00	\$4.50	\$2.25
0.002	3.0	35.0	35.0	43.0	35.0	30.0	39.0
0.020	0.0	21.0	19.0	22.0	15.0	1.0	16.0
0.100	10.0	12.0	-25.0	33.0	9.0	16.0	26.0
0.200	12.0	10.0	-42.0	-101.0	-101.0	-105.0	-125.0

Table 6-7: Multi-bands sculpted offers: target defaults (MWh)

Width	Penalty						
	\$148.00	\$74.00	\$37.00	\$18.00	\$9.00	\$4.50	\$2.25
0.002	-26.0	-9.0	-9.0	-20.0	-23.0	-2.0	0.0
0.020	-8.0	5.0	5.0	5.0	-3.0	-1.0	0.0
0.100	-87.0	-105.0	48.0	-41.0	-53.0	-27.0	-7.0
0.200	-544.0	-538.0	-225.0	5.0	-30.0	16.0	22.0

Table 6-8: Multi-band sculpted offers: increase in spills (MWh)

The second set of results in Table 6-9 to Table 6-12 show reduced costs over the three-band sculpted offers for higher penalty rates. For lower penalty rates, performance is similar to the three-band sculpted offers. The increase in cost relative to the centralised solution remains quite stable for the high and medium penalty rates but falls for the

lowest penalty rate of \$4.50. The increase in costs is also quite uniform across the different values of χ .

In retrospect it seems intuitive since all the intermediate bands across generators are at the same price hence, they all raise or lower the price together and, except for the thermal offer, the coordinator faces identical decisions for all values of χ except zero and one.

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
1.00	1.46	1.45	1.40	0.29
0.80	1.55	1.52	1.44	0.29
0.60	1.52	1.49	1.44	0.29
0.40	1.54	1.51	1.42	0.30
0.20	1.48	1.47	1.43	0.29
0.10	1.43	1.43	1.37	0.31
0.00	1.65	1.64	1.40	0.31

Table 6-9: Multi-band sculpted offers: direct cost distortion (%)

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
1.00	1.60	1.50	1.50	0.90
0.80	1.60	1.60	1.50	0.90
0.60	1.50	1.50	1.50	0.90
0.40	1.70	1.60	1.50	0.90
0.20	1.50	1.50	1.50	0.90
0.10	1.50	1.40	1.40	0.90
0.00	1.70	1.70	1.60	0.90

Table 6-10: Multi-band sculpted offers: transaction cost inclusive distortion (%)

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
1.00	4.0	5.0	18.0	722.0
0.80	2.0	5.0	23.0	753.0
0.60	0.0	3.0	17.0	746.0
0.40	5.0	8.0	30.0	741.0
0.20	2.0	2.0	13.0	733.0
0.10	2.0	2.0	17.0	722.0
0.00	2.0	2.0	58.0	776.0

Table 6-11: Multi-band sculpted offers: target defaults (MW)

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
1.00	211.0	210.0	201.0	125.0
0.80	259.0	249.0	237.0	119.0
0.60	254.0	248.0	246.0	126.0
0.40	235.0	226.0	217.0	120.0
0.20	228.0	227.0	223.0	121.0
0.10	302.0	300.0	289.0	156.0
0.00	359.0	357.0	286.0	150.0

Multi-band sculpted offers: spills (MW)

It is difficult to conclude based on these results that multi-band sculpted offers show performance superior to three-band sculpted offers. A 2% increase in total costs is the best result previously obtained with a three-band sculpted offer with \$148.00 penalties. The second experiment certainly shows superior results to this but it must be kept in mind that a χ value of 1 actually equates to a three-band offer, with a central band 4% wide.

Therefore in this experiment, it appears that a three-band offer is at least as good as the five-band ones, however, the method of offer formation used in this section is mainly for convenience of demonstration and comparison. It is easy to imagine a more sophisticated heuristic that could perform better.

C.2 Fixed Offers

The simplest offer proposed is a single supply curve that describes a generator's availability for all periods of the day. Such a curve could probably be expected to have more bands than an offer which applies to one period only and should provide the market with more apparent flexibility than that provided by the sculpted offers examined above.

Two main steps can be followed to create the fixed supply curve for each generator:

- 1) Create a monotonically increasing supply curve based on all the observed price and quantity pairs taken from a market simulation, handling inconsistent or non-monotone observations in some way.

- 2) Insert sub-bands. (The term sub-band is used to distinguish bands created in order to limit risk exposure, from the original bands appearing in the raw curves.)

Approximately one sub-band is added for each band in the raw curve, including the implicit bands at \$0.00 and the maximum price. For example, a generator that has two different price bands in the raw curve will have a supply curve created consisting of seven sub-bands as necessary to limit generators' risk exposure.

The experiments discussed in this section use supply curves of this form and created using the procedure described as follows.

C.2.1 Building a Fixed Offer

The price and quantity pairs from market simulation can be interpreted as points on the supply curve of that generator during that period. To generate the offers used in the experiments one attempts to interpret the information from these points to form complete supply curves, employing certain assumptions, as detailed below. Although it is conceptually possible to develop a formulation by which the offers themselves could be optimised, the discussion only considers heuristic offer formation processes, because they are easier to implement, and are better able to account for more subjective considerations, such as risk exposure.

From the viewpoint of simplicity of the market, it might be desirable to have offers from each generator that apply for all the periods in the day ahead. Such an offer could have only a few price bands or many—although 12 seems a logical limit for a 12 period horizon.

The most obvious way to produce such a curve, using the market simulation data available, is to plot all the generator's responses from their model for the day on a single graph and attempt to draw a step-wise supply curve through them. As noted earlier, if the points form a monotone increasing series this can be done. But in the presence of active inter-temporal constraints, the curve may not be uniquely defined.

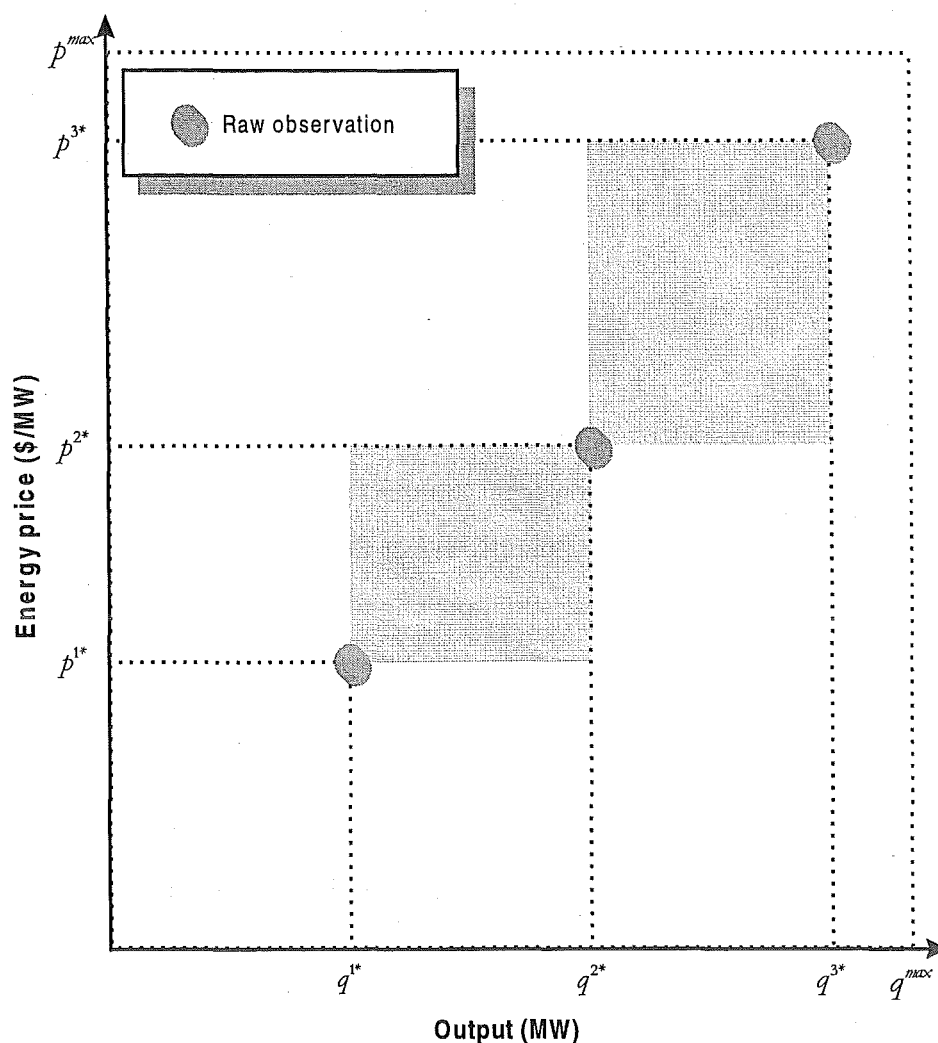


Figure 6-11: Typical response to system energy price

Consider the situation depicted in Figure 6-11, a supply curve could be drawn anywhere within the blue rectangle and be consistent with the data; defining this are:

- the left hand side and top of this rectangle as the upper curve, representing a generator which will not generate additional power until the top price is achieved;
- and the bottom and right hand side the lower curve, representing a generator which would be willing to generate additional power at the lower price

It may not be the case that a supply curve constructed along either of these paths is feasible if applied to every period of the day, even if defined by the upper curve, which does not release water in the doubtful region. Each point on the graph represents what can be achieved in one period, given the generation in all the others. The presence of energy constraints mean that a hydro generator might be able to supply that amount of power at that price in any one period but have insufficient water to sustain this generation for all periods, which is what a supply curve implies. The converse can also happen potentially resulting in increased spill in the day or future days.

In all reasonable circumstances, it will be feasible for the hydro generator to operate at somewhere near the mean generation from the market simulation for the whole day, since generation at a constant rate is generally easier than following a daily cycle—ignoring the possibility that the physical situation might change by, for example, unexpected tributary inflows. Hence, the shape of the supply curve can be modified in the direction of this average generation—in fact the mid-point of the range of possible generation, rather than the average, is used for simplicity, in order to produce more cautious offers, i.e. offers which are more likely to be feasible if taken up. This process of ‘centralisation’ also mimics the process by which offers could be expected to anchor around a contract level, which is likely to be moderate.

If the points do not form a monotone increasing series, then no single supply curve can be formed which is consistent with all the points. When this happens, it is an indication that the supply curve for the generator did not stay the same throughout the day. In these cases separate supply curves for peak and off peak, or even separate supply curves for each period, may be needed to produce a result similar to the centralised dispatch. If a single supply curve for the whole day is required, however, the situation is probably best handled by choosing the most cautious offer, in the sense of taking the curve that is closest to the mean generation.

The prices can also be adjusted to produce offers which leave the generator less open to infeasible targets being set by the market clearing process. Although all the points in one step might have the same price in the ideal dispatch, the generator could increase

the prices at the upper end of the range of that band and decrease the prices at the lower end. This means the market would be more likely to accept the lower range of the band and less likely to accept the upper range. The result is a more diversified, curve which gives the market freedom to choose any point in the range, but guides the central market clearing process towards setting generator targets in the central part of the range, i.e. targets more likely to be feasible.

C.2.2 Calculating a Monotone Supply Curve

Consider the set of observations depicted in Figure 6-12, derived from a run of WEMSIM. The first step is to create a monotonically increasing supply curve from the observations. Obviously one can define the supply curve between the minimum and maximum quantity for each price, as shown in Figure 6-13 but there are several regions (shaded) for which it is not clear what the form of the supply curve should be.

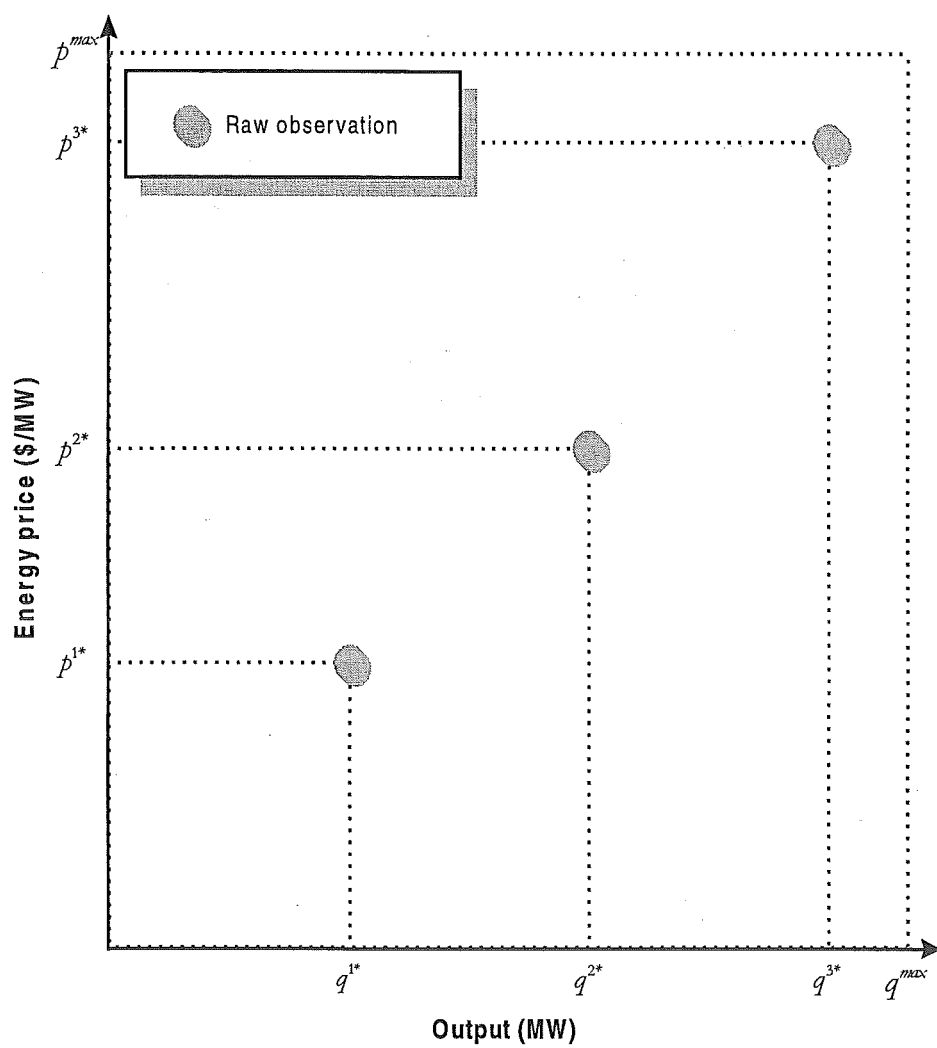


Figure 6-12: A generator's price and quantity observations

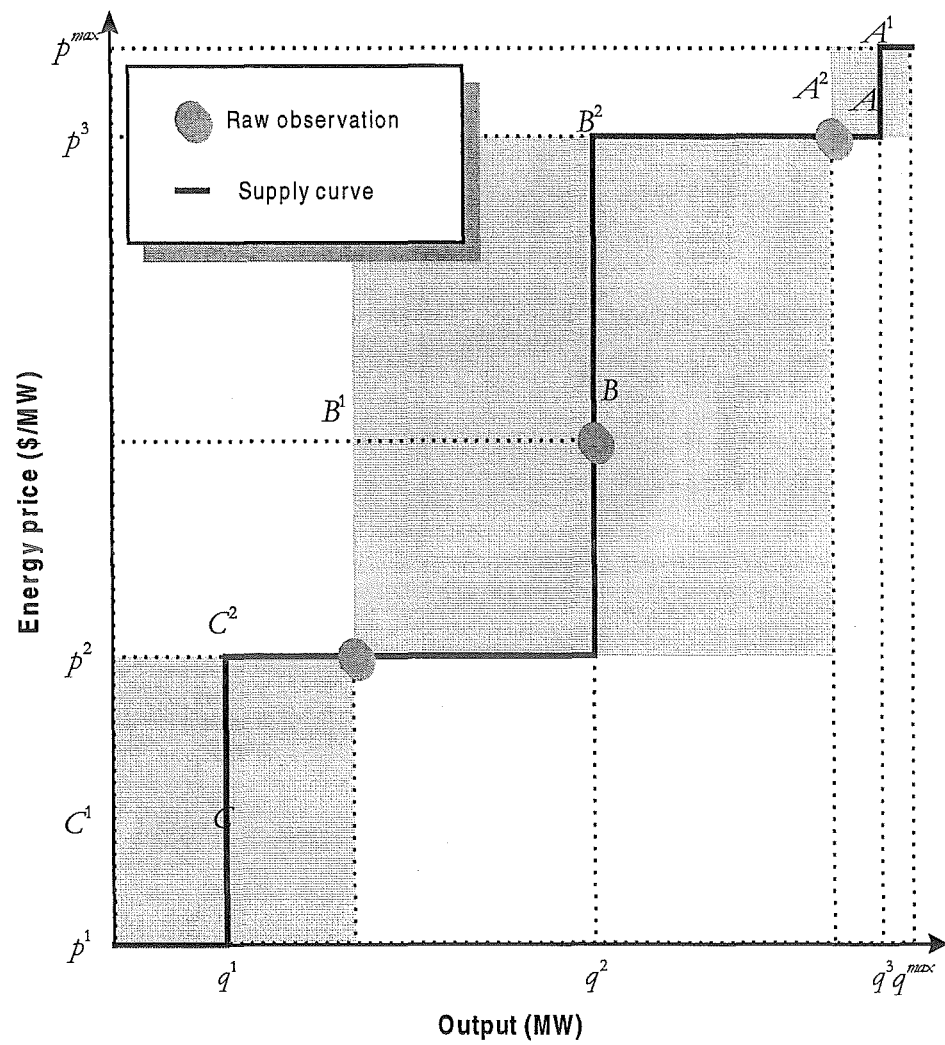


Figure 6-13: Creating the monotone supply curve

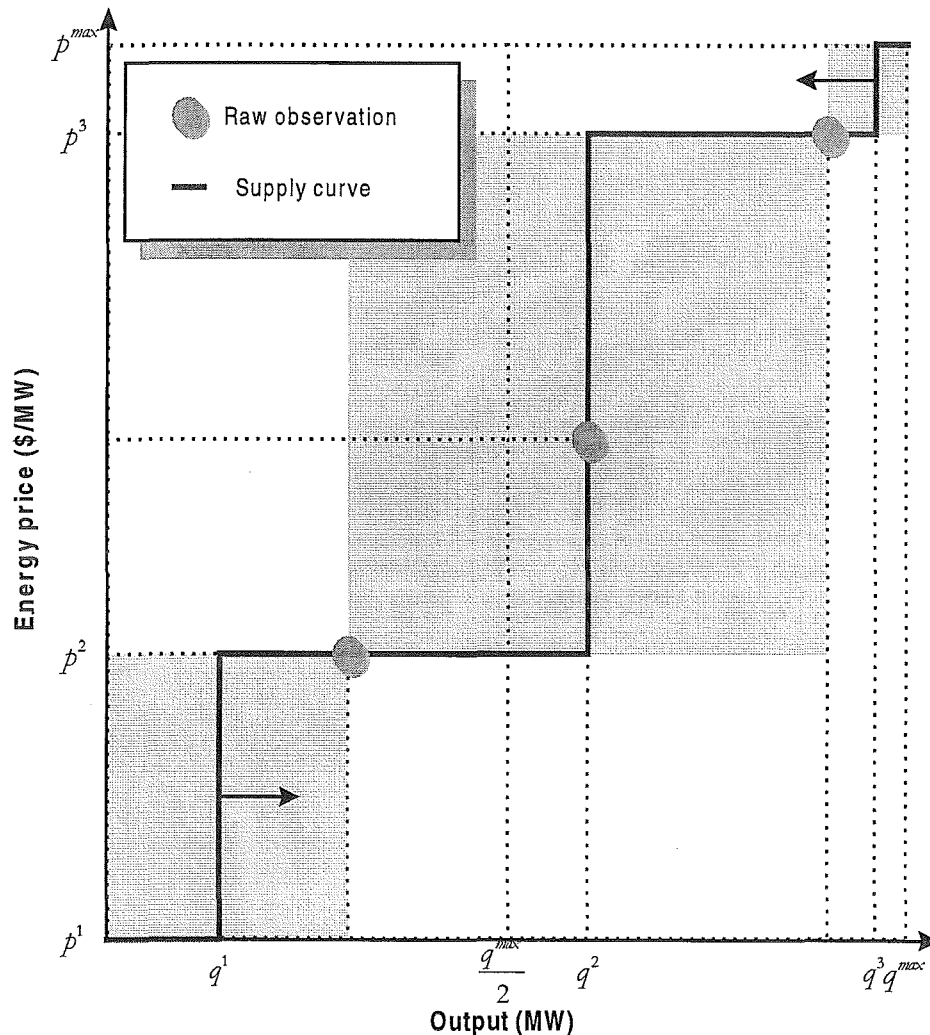


Figure 6-14: Creating the cautious supply curve

Consider the shaded region A shown in Figure 6-14. The supply curve must be defined so that it connects the observation adjoining A to the top right corner of A—the vertex defined by (p^{max}, q^{max}) .

Assuming that one would actually be prepared to offer more than q^1 at a price p^{max} , the most cautious approach would result in the supply curve being defined along edges A^1 and A^2 . In this upper part of the generation range, ‘caution’ is defined to include wishing to avoid having targets assigned which are too high to be met and also reluctance to offer the market power at a lower price than necessary.

The same cautious approach would result in the supply curve for region B being defined along edges B^1 and B^2 . On the other hand, consider shaded region C in which the supply curve must be defined to connect the observation at top right to the origin. Caution could be interpreted here, too, in terms of the generator withholding supplies from the market. But it can also be argued that, in order to ensure that minimum flow constraints and the like are met, a cautious approach would result in the supply curve being defined along edges C^1 and C^2 —contractual commitments imply a similar policy.

The monotone supply curve is defined as in Figure 6-14. This process of centralising can be described in general terms as:

- for any undefined output range greater than the mean level, add the output range to the upper band concerned;
- for any undefined output range less than the mean level, add the output range to the lower band concerned. For simplicity, in the actual experiments, as shown in the diagrams, half of the possible offer is used rather than the mean generation

Note that the same procedure applies when the mean output lies within an undefined range, except that both the upper and lower bands will have that part of the range that lies above and below the mean, respectively, added to them.

C.2.3 Handling Inconsistent Observations

If the storage levels of a hydro generator's reservoirs reach their bounds during the day, one may obtain a set of observed price and quantity pairs which does not allow determination of a single monotone supply curve.

Consider the set of observations shown in Figure 6-15. One can create the monotone supply curve using the same method as is used to create the curve depicted in Figure 6-14, but there are two observations (in bold) which do not enable the definition of the supply curve in the shaded region D . To define the supply curve in region D one can employ the same centralising approach as used earlier, i.e. the supply curve is defined as the left and upper edges of region D , edges D^1 and D^2 .

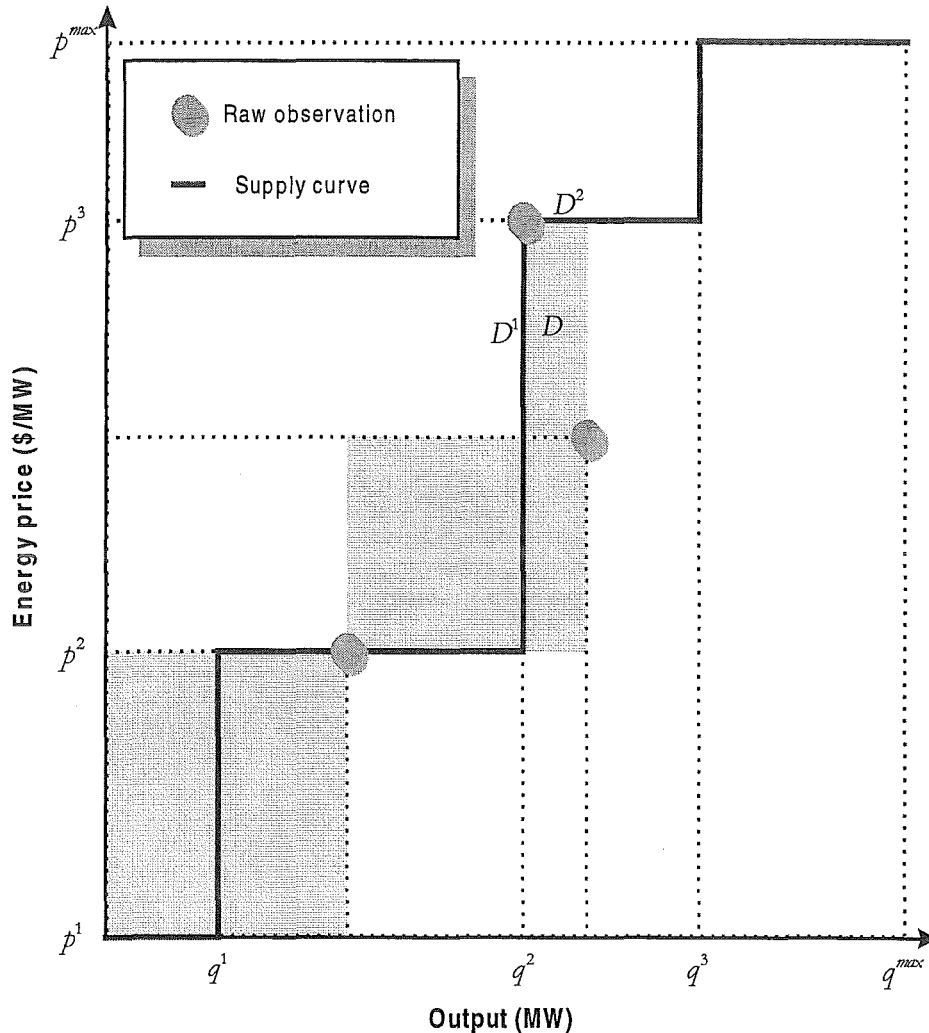


Figure 6-15: Inconsistent observations

A simple variation of the process described above is used in the first set of experiments here. It allows the specification of three bands, with one of them having a price of \$0.00. The centralising policy described above is only partially applied, with the supply curves constructed by applying a 'caution factor' to the basic upper supply curve resulting from the plot. The caution factor pulls all the quantities on the supply curve towards the mean generation for the day. For example, a caution factor set to 10% means the quantities moved 10% of the distance towards the mean generation level.

The simplest form of supply curve for a generator would be a three-band curve derived directly from the set of price and quantity pairs. This is rather unrealistic, though, as it implies there is no uncertainty or risk involved and that each generator could expect the market to respect its energy constraints in taking up offers centred on the mean of each band, rather than opting for the extremes. Initial experiments suggested that these assumptions are unrealistic and costly, thus generators must somehow 'tighten up' their offer around the preferred generation levels.

Specifying additional sub-bands is one way of achieving this. Sub-bands guide the market solution away from the extreme lower and upper ends of the bands of the raw curve. Generators might want to do this because of energy and flow constraints.

Although the point at the extreme upper end of the range of generation in a band may be achievable, at the price given by a generator in any one period, it may not be sustainable over the whole horizon. Yet the latter is what a supply curve passing through such a point implies and it is possible that the market might take up such an offer over a large part of the day, not just a few periods. Indeed, if there are many offers at similar prices, one may be taken up completely, for many periods and the other not taken up at all, simply because there is a minuscule difference in price, reflecting slightly different forecasts by the generators concerned. In such situations it might be impossible to meet the targets set. To avoid this, generators might make the upper range of generation in a band less attractive to the market by increasing its price. Similarly they might decrease the price of the lower range in the band, to make it more likely that at least some of the band will be taken up.

Consequently, the principle reason for adding extra sub-bands is to spread the implicit risk that arises if only a few large bands are specified. Additionally, it may be desirable to generate a richer supply curve for that part of the generator's operating range lying outside the range of the observations taken from the market simulation.

One way of interpreting the monotone bands is to assume that, on average, the generator would be happy to generate between q^- MW and q^+ MW at p \$ per MW. However, to make the curve more realistic, for the reasons discussed above, let each band be

comprised of three sub-bands: a lower, a middle, and an upper sub-band, each of which has a different price. Figure 6-16 depicts three bands corresponding to prices p^1 , p^2 , and p^3 .

For illustrative purposes this discussion will focus on the modifications to be made to band two which, as it currently stands, offers generation in the range q^{2-} to q^{2+} at a price p^2 .

The prices for each segment are shown on the p -axis. The price for the lower sub-band is halfway between the price for band two, p^2 and the price corresponding to the next lowest band, p^1 . Note that the upper sub-band derived from band one will have the same price as the lower sub-band from band two and hence they will form a single band.

In terms of the number of bands in the final supply curve, the general rule is there are two sub-bands for each band in the monotone supply curve, with each upper sub-band being concatenated with the lower sub-band above it, at a price half-way between those of the two raw bands concerned. A still richer curve could be obtained by setting distinct prices for each sub-band. Similarly, the price for the upper sub-band is $(p^2 + p^3)/2$. To determine the offered quantity for each sub-band two parameters are defined:

- 1) β_{LO} , the proportion of the band quantity that is apportioned to the lower sub-band.
- 2) β_{HI} , the proportion of the band quantity that is apportioned to the lower and middle sub-bands.

Using these parameters one can easily manipulate the quantities offered in each sub-band, so long as both β and β_{HI} are less than or equal to one and $\beta_{LO} \leq \beta_{HI}$. For example, to obtain the cautious supply curve in Figure 6-16 $\beta_{LO} = 0.0$ is set and $\beta_{HI} = 1.0$.

In mathematical terms, the sub-band definitions for band two shown in Figure 6-16 are as in Table 6-12.

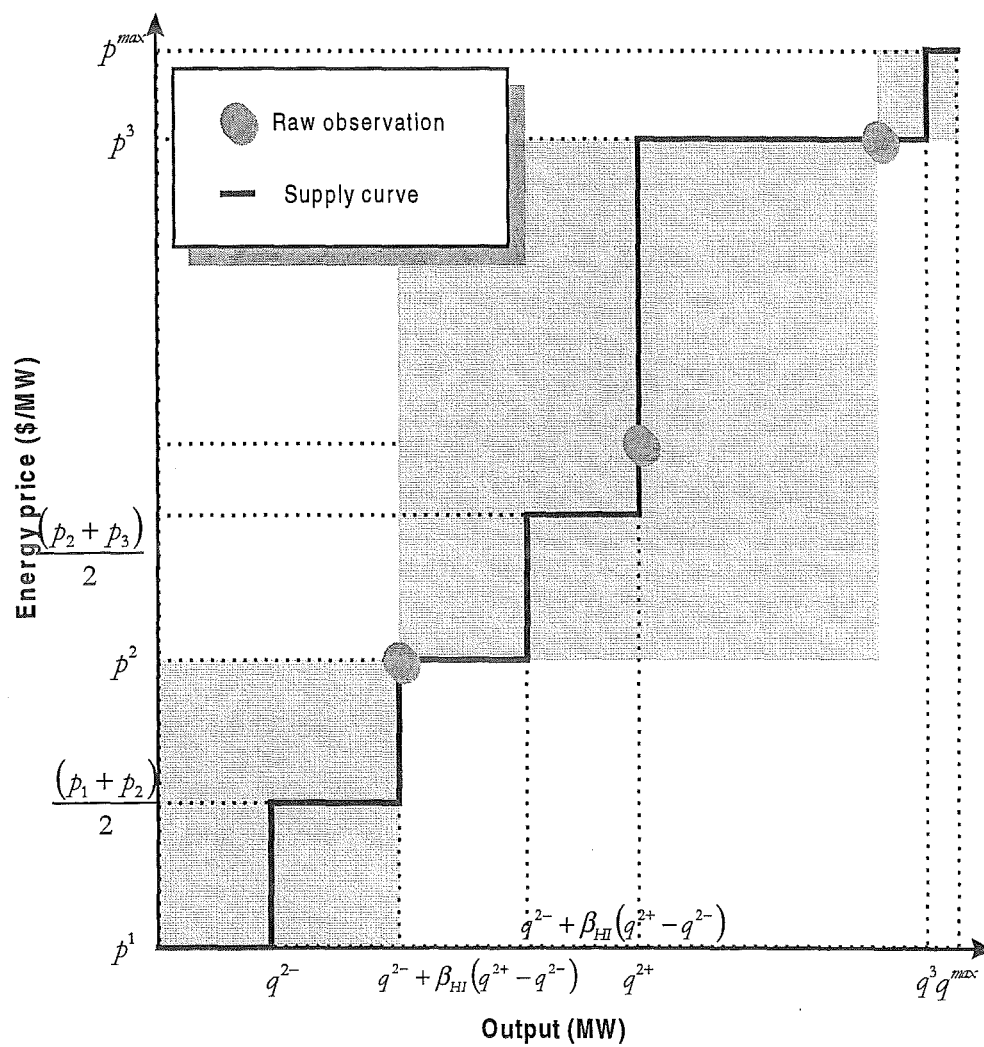


Figure 6-16: Band splitting

Band 2 Section	p	q
Sub-band 1	$(p_1 + p_2)/2$	$\beta_{LO}(g_2^+ - g_2^-)$
Sub-band 2	p_2	$(\beta_{HI} - \beta_{LO})(g_2^+ - g_2^-)$
Sub-band 3	$(p_2 + p_3)/2$	$(1 - \beta_{LO})(g_2^+ - g_2^-)$

Table 6-12: Sub-band formulae

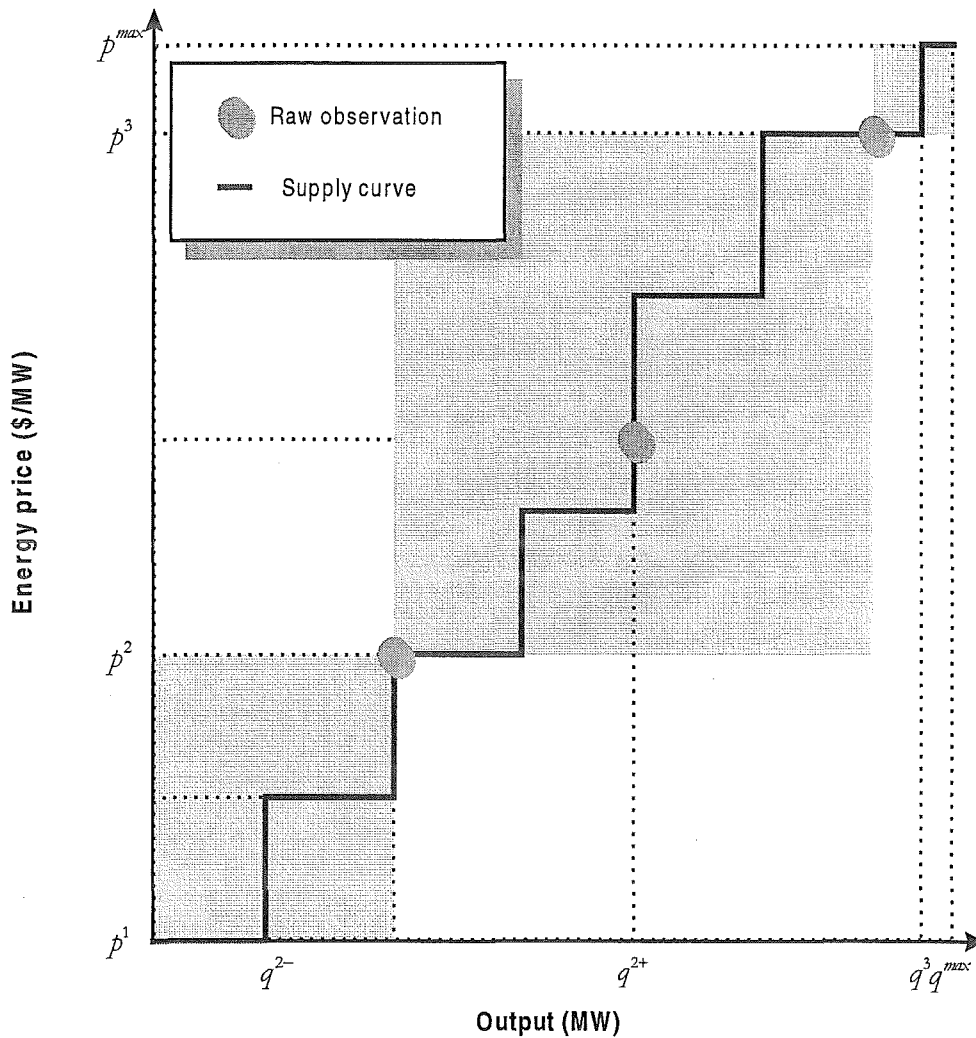


Figure 6-17: Final supply curve

C.2.4 Three-band Fixed Offers

The aim of this set of experiments is to estimate the cost of a day-ahead market based on very simple fixed offers. A set of three cases with varying demand profiles is trialed. The hydro groups are assumed to correctly forecast demand and prices, and form their offers using the simple three-band offer creation process described earlier while the thermals are modelled as an eight piece supply curve.

Results from these experiments indicate that fixed offers of this type perform quite poorly. In all cases the targets set by the market clearing process diverge from the optimal dispatch to a high extent. The market clearing process typically took up the entire offer in a band from a generator for the whole day. Yet these are the maximum amounts, minus the caution factor, that a generator would offer at this price for one period, not an amount at which they could generate all day consequently, the targets are often unattainably high.

In part, this is an artefact of the way the offers are constructed and the market clearing process. Because many of the bands are at the same price, the market clearing is indifferent between the offers and might arbitrarily take all of one and none of another.

In reality, so many offers at exactly the same price would be unlikely, however, the real situation could still result in equally arbitrary results. Most of the offers will be very close in price, as the generators are attempting to estimate the market clearing prices. In this case a very small difference in price would be the difference between one generator's offer being completely taken up and another's completely rejected and vice versa. Although this may not have a very great direct economic input, it probably would result in unacceptable release scheduling on rivers additionally, even if the de-centralised system does turn out to have similar costs to the centralised one, if the targets are consistently unattainable, or far too low, the credibility of the system will be undermined. After all, there seems little point in having 'targets' if they are rendered meaningless by the real-time dispatch.

In the real-time dispatch process the generators whose targets had been set too low were able to make up the short fall in generation of those whose targets were too high. This process is assumed to execute perfectly in the model—although at a significant cost. Whether a real real-time dispatcher and the generators would do so well in reality is debatable, however, the high penalty applied more than covers the cost of any imperfection in this process. On the other hand, uncertainty in the generators' offer formation is not modelled, rather, it is assumed they knew perfectly what is going to occur on the

day. Misestimating market conditions would, most likely, make the offers and targets diverge even further from the ideal dispatch.

One way to improve the performance of the market dispatch in comparison to the ‘ideal’ dispatch is to limit the extend to which the market clearing process can set extreme targets—by increasing the number of bands in the offer. The next section examines the performance of multi-band fixed offers.

C.2.5 Multi-band Fixed Offers

These 5-band fixed offers are formed according to the setting of β_{LO} and β_{HI} . Any offer that has β and β_{HI} close together results in the upper and lower sub-bands being assigned a large proportion of the original raw band resulting in offers that differ greatly from the original observations.

Thus these experiments exclude those combinations where $\beta_{HI} - \beta_{LO} \leq 0.1$, ensuring that the quantity defined for the middle sub-band is at least 20% of that defined for the raw band. Experiments cover combinations of β_{LO} and β_{HI} in increments of 0.2. As β_{LO} increases the generator’s offer becomes more ‘generous’ by offering a substantial proportion of each raw band at a lower price. Hence these experiments assume values of $\beta_{LO} > 0.5$ are unlikely. Additionally, one assumes generators are “risk neutral” in their offer creation and thus choose values of β_{LO} and β_{HI} that are distanced equally from the centre.

β_{LO} β_{HI}	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.15	2.02	2.71	2.90	2.36	2.04	1.84	2.08	2.37	1.99	1.34
0.1		1.95	2.99	2.93	1.84	1.66	2.13	2.36	1.89	1.74	2.07
0.2			2.64	3.44	2.21	2.20	1.88	2.29	1.81	1.38	1.90
0.3				2.31	2.18	1.64	2.10	2.41	1.49	1.25	1.61
0.4					2.27	2.11	2.01	1.97	1.74	1.44	1.56
0.5						2.39	2.33	2.25	1.92	1.58	1.61
0.6							2.83	2.55	2.39	1.92	1.83
0.7								3.04	2.74	2.57	2.37
0.8									3.57	3.17	3.11
0.9										4.39	4.17
1.0											5.13

Table 6-13: Multi-band fixed offers: direct cost distortion (%)

β β_{HI}	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	9.59	7.80	2.71	2.90	2.36	2.04	1.84	2.08	2.41	2.35	1.87
0.1		2.01	2.99	2.93	1.84	1.66	2.13	2.36	1.93	2.10	2.74
0.2			2.64	3.44	2.21	2.20	1.88	2.29	1.86	1.74	2.57
0.3				2.31	2.18	1.64	2.10	2.41	1.53	1.61	1.61
0.4					2.27	2.11	2.01	1.97	1.78	1.80	1.56
0.5						2.39	2.33	2.25	1.96	1.94	1.61
0.6							2.97	2.55	2.44	2.28	2.50
0.7								3.04	2.79	2.93	3.05
0.8									3.62	3.53	3.26
0.9										4.75	4.53
1.0											6.84

Table 6-14: Multi-band fixed offers: transaction cost inclusive distortion (%)

To take some account of uncertainty, these experiments are repeated for generators over-estimating demand by five percent and under estimating demand by five percent. The results when averaged for values of β and β_{HI} in the likely parameter range described above are shown in Table 6-12 and Table 6-14:

- The minimum cost distortions, including and excluding transaction costs of 1.96% and 4.64%, occur when $\beta_{LO} = 0.1$ with $\beta_{HI} = 0.9$ and $\beta_{LO} = 0.1$ with $\beta_{HI} = 0.4$ respectively, and,

- the minimum distortion, including and excluding penalties in the likely parameter range of 3.06% and 11.48% occur when $\beta_{LO} = 0.1$ with $\beta_{HI} = 0.3$ and $\beta_{LO} = 0$ with $\beta = 0.9$ respectively.

These results indicate an acceptable degree of stability over the range of likely parameter ranges, although the distortions are quite high overall.

Starting with a high penalty rate and working down, offers close to the original raw curve become more attractive. This is because the generators are penalised less for deviating from targets and prefer to follow their ideal trajectories. Lowering this rate reduces distortion but produces dispatches in which the instructions of the market are ignored. If one were to set a two percent distortion as a benchmark the penalty rate would need to be set below \$10.00 in these experiments reflecting the poor quality of the targets produced by these offers.

C.3 Offers Prepared from Multiple Scenarios

One way of improving the information content of an offer is to include more demand scenarios in the offer formation. This should produce better quality fixed and sculpted offers. This chapter examines fixed and sculpted offers formed using three scenario points per period.

C.3.1 Sculpted Offers with Multiple Scenarios

There are many methods that may be used to produce multi-band sculpted offers. The most intuitive approach is to use more forecasting data when forming an offer. Generators who base their offers on several demand scenarios, rather than on a single forecast, can make better provision for uncertainty. The offers tested here use three demand scenarios to form five-band sculpted offers. To the three demand scenarios used previously (-5.0%, 0.0, +5.0%) are added two more extreme cases (-8.0%, +8.0%).

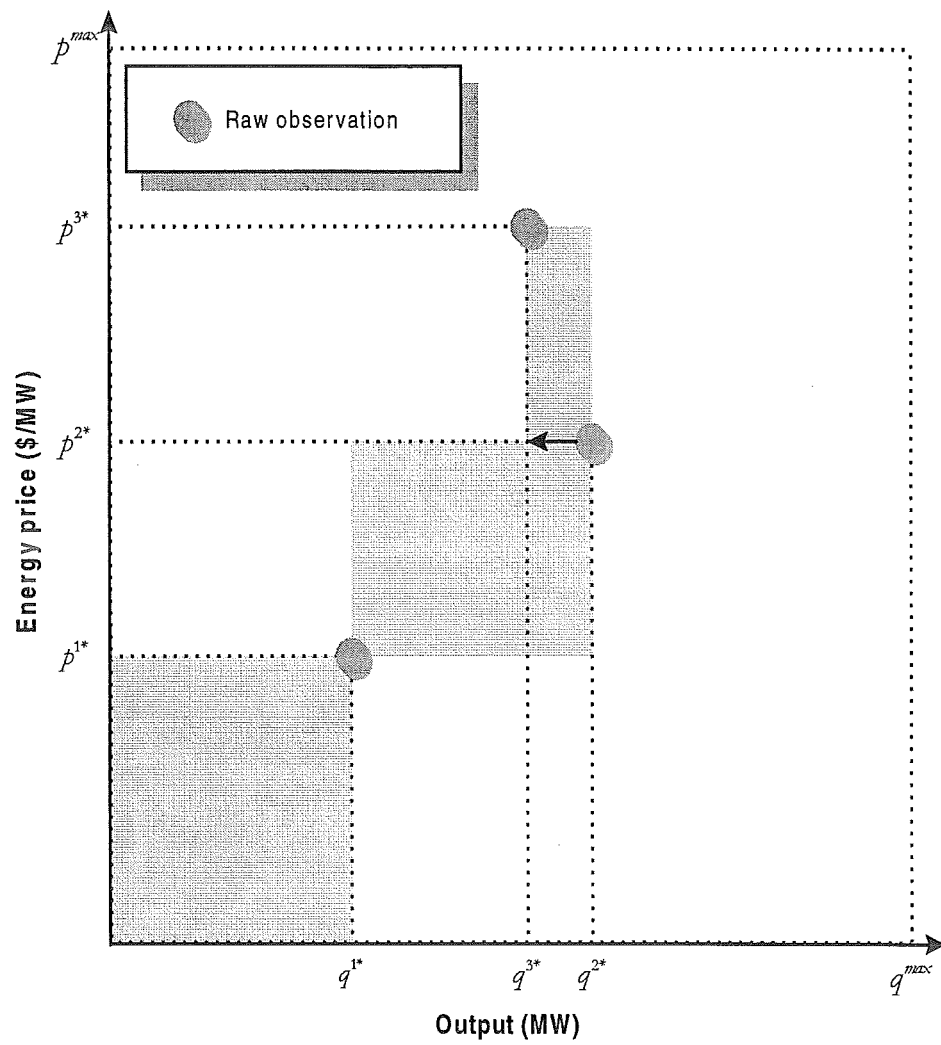


Figure 6-18: Resolving non-monotonicities

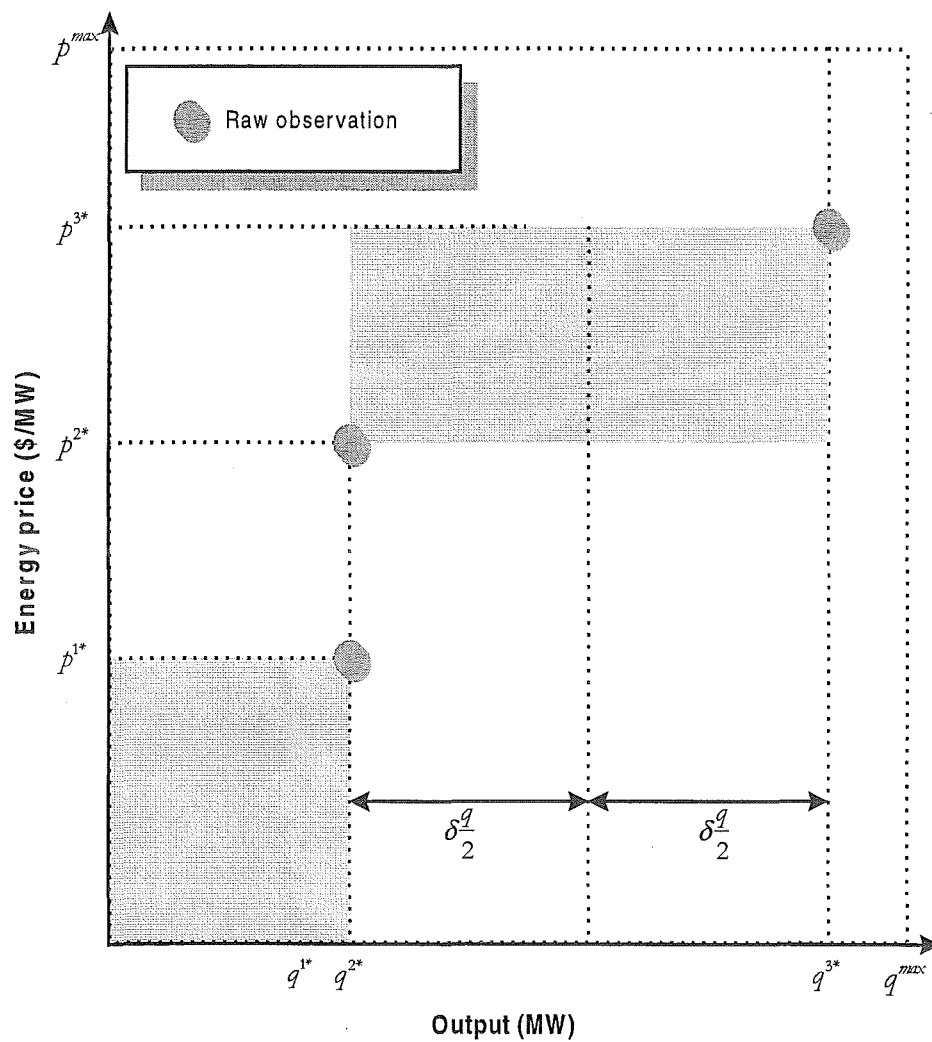


Figure 6-19: Forming bands

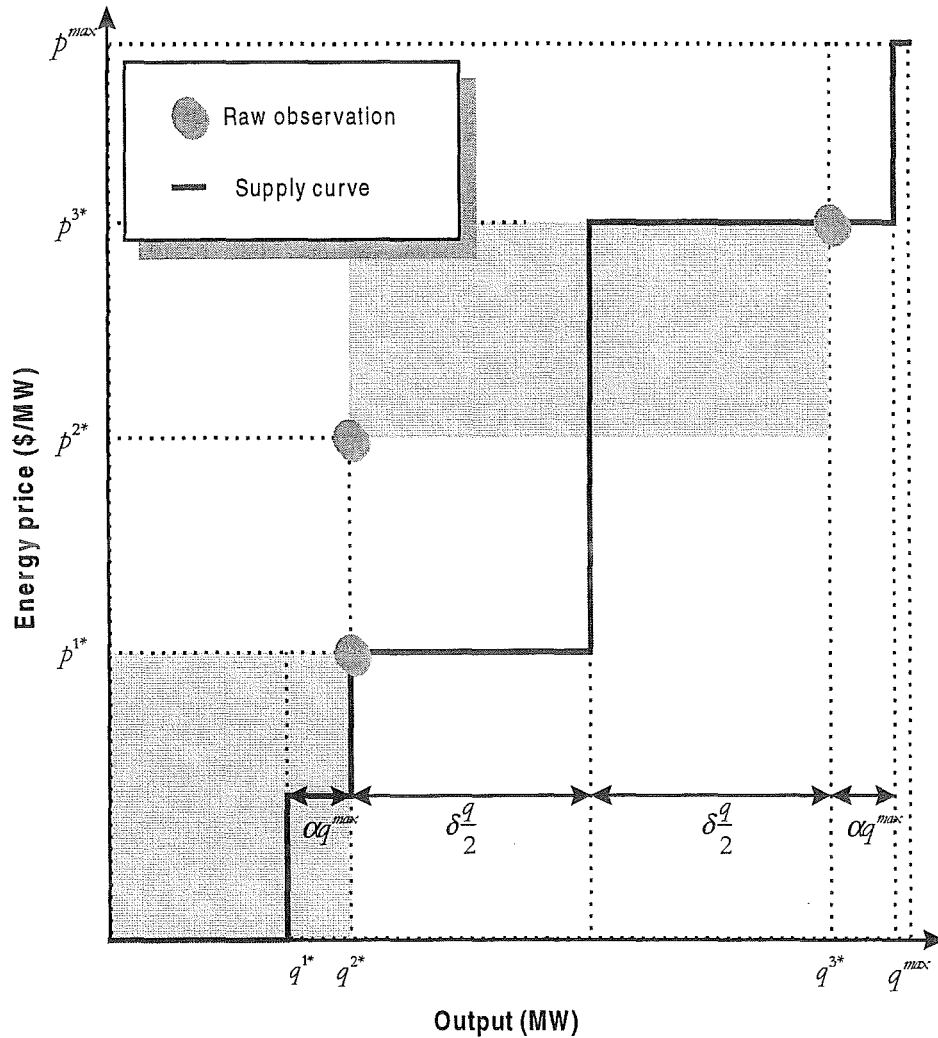


Figure 6-20: Extrapolating the outside bands

Each generator is assumed to base its offer upon three of these scenarios. For example, a company forecasting a low demand day uses the -8.0%, -5.0% and +0% demand scenarios to form its offer. In each period, the three points from the market simulation are formed into an offer by the following process:

- 1) Any non-monotonicities are resolved by reducing the quantity offered at the lower price to the quantity offered at the higher price, see Figure 6-18.

- 2) The bands offered between the observed points are at the same price as the closest point, i.e. they extend half the distance to the next point's quantity, see Figure 6-19.
- 3) The highest and lowest priced scenarios have their bands extended outwards by a proportion α of the total possible offer. Generation below these central bands is priced at \$0.00 per MW, and generation above at \$112.00 per MW, see Figure 6-20.

The method is used to form offers for a variety of forecasting scenarios:

- 1) All hydro groups forecasting accurately, i.e. using (-5.0%, +0.0%, +5.0%) to form their offers.
- 2) All hydro groups forecasting a low demand day, i.e. using (-8.0%, -5.0%, +0.0%) to form their offers.
- 3) All hydro groups forecasting a high demand day, i.e. using (+0.0%, +5.0%, +8.0%) to form their offers.
- 4) The Taupo and Waitaki groups forecasting a high day, the rest a low day.
- 5) The Taupo and Waitaki groups forecasting a low day, the rest a high day.

Experimental results are shown using these scenarios over a variety of penalty rates and a variety of values of α , the outer band extrapolation parameter. The results obtained using this kind of offer are the best so far observed. The direct and transaction cost inclusive distortions are shown in Table 6-15 and Table 6-16.

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
0.002	0.6	0.6	0.6	0.05
0.020	0.7	0.7	0.7	0.08
0.100	1.1	0.9	0.7	0.11
0.200	2.4	1.4	0.7	0.15

Table 6-15: Multi-band sculpted offers with 3 scenarios: direct cost distortion (%)

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
0.002	0.6	0.6	0.6	0.3
0.020	0.7	0.7	0.7	0.4
0.100	1.7	1.2	1.0	0.5
0.200	3.8	2.5	1.7	0.7

Table 6-16: Multi-band sculpted offers with 3 scenarios: transaction cost inclusive distortion (%)

The figures in Table 6-15 and Table 6-16 are significantly better than the best results for the three-band sculpted offers based on one demand scenario, which had a minimum increase of 2% for both direct and total costs. Note that, as in previous experiments, the results for narrower extrapolation widths are better and stay quite stable, until the penalty rate falls below \$18.00 per MW.

The total amount defaulted on is also very low for high and medium penalty rates when extrapolation outside the market simulation data is minimal. For low penalties, the amount of defaulting is higher because it is often cheaper to bear the penalty costs than to run their systems hard trying to meet targets. The drop off shows this, in direct costs, to be a less than 0.1% increase over the centralised solution for \$4.50 per MW penalties reported in the first table of this section.

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
0.002	0.0	0.0	5.0	339.0
0.020	2.0	2.0	4.0	345.0
0.100	22.0	36.0	91.0	462.0
0.200	54.0	169.0	316.0	671.0

Table 6-17: Multi-band sculpted offers with 3 scenarios: target defaults (MWh)

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
0.002	8.0	8.0	7.0	5.0
0.020	62.0	62.0	60.0	20.0
0.100	429.0	363.0	297.0	58.0
0.200	1268.0	757.0	486.0	143.0

Table 6-18: Multi-band sculpted offers with 3 scenarios: spill (MWh)

As discussed earlier, it is the non-monotonicities in response that produces a 'base' distortion that cannot be overcome. To further reduce the effect of the non-

monotonicities responses may be averaged for equi-priced periods—an approach examined in the following section.

C.3.2 Averaging Responses over Equi-priced Periods

Since hydro generators' offers are based largely upon the expected market price in each period, it seems reasonable for offers in periods that have the same market price to be similar. But, observation suggests that there is significant variation in the amount offered, presumably because there are so many alternative optima, contributing greatly to the incidence of observed non-monotonicities. This suggests a form of offer that has some form of smoothing or averaging of offers from similar periods. One approach would be a 'top down' method with the offer being formed on the basis of expected market prices and the proportion of the load carried by each generating company. These proportions would be averaged across periods with the same market price. The averages would be adjusted for the expected load in each period to give the offers.

It is also possible to imagine forming offers via a complex 'bottom up' approach starting with individual reservoir storage trajectories. Note that water values at a reservoir remain constant until it hits one of its bounds—hydro groups should be indifferent between releasing water from the reservoir in one period or in another, provided the reservoir has not hit a bound in any intervening period. To reflect this, an average release could be offered from each reservoir across all periods in an 'epoch' between occurrences of bounds being hit. These offers may be aggregated and adjusted for the expected load to form offers that can be submitted to the market.

This latter strategy for forming offers is not tested because of its complexity, however, experiments are performed using the top down approach across the same penalty rates and extrapolation parameters as the previous set of experiments.

Generator responses, from the market simulation, are divided by the total load in that period to give the proportion of load carried by that generator in that period, under that scenario. These proportions are then averaged in one of two ways:

- 1) The proportions of load carried are averaged across all periods with the same price. This is equivalent to the assumption that a generator will be indifferent as to which of these periods it generates more, or less, so long as the average remains the same. This would be true if no reservoirs reach their bounds in the intervening periods
- 2) The proportions of the load are averaged across contiguous periods with the same price. This is a slightly more relaxed assumption which allows that periods which have equal prices might none the less represent different physical situations, e.g. different parts of reservoir storage trajectories.

Multiplying the average proportion of the load carried by the expected load to give a quantity forms sculpted offers. This point is then combined with two similar points from other scenarios in the manner described earlier.

It might be expected that such offers would be more stable and also make it easier for the market to match the load pattern. Although it is probably the proportion of net hydro load which should be used to scale these hydro offers.

<i>Width</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$ 4.50</i>
<i>0.002</i>	1.5	1.3	1.0	0.23
<i>0.020</i>	1.6	1.3	0.9	0.22
<i>0.100</i>	1.8	1.5	1.0	0.19
<i>0.200</i>	3.2	1.8	1.1	0.23

Table 6-19: Multi-band sculpted offers with 3 scenarios with responses averaged across equi-priced periods: direct cost distortion (%)

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
0.002	1.8	1.5	1.3	0.6
0.020	1.8	1.5	1.3	0.6
0.100	2.6	1.9	1.6	0.7
0.200	5.4	3.4	2.3	0.9

Table 6-20: Multi-band sculpted offers with 3 scenarios with responses averaged across equi-priced periods: transaction cost inclusive distortion (%)

Perhaps surprisingly, the distortions are higher than for the standard multi-scenario offers which had no averaging across periods, even though these earlier offers required more artificial monotonisation of the offer curve. Indeed, the results are marginally worse than the best five-band sculpted offers based on a single demand scenario. The target defaults for this kind of offer is higher than the case with no averaging across periods. Similarly the amount of spill seen in these results is substantially higher.

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
0.002	9.0	24.0	109.0	488.0
0.020	10.0	31.0	124.0	483.0
0.100	31.0	72.0	180.0	612.0
0.200	85.0	242.0	366.0	839.0

Table 6-21: Multi-band sculpted offers with 3 scenarios with responses averaged across equi-priced periods: target defaults (MWh)

Width	Penalty			
	\$148.00	\$37.00	\$18.00	\$ 4.50
0.002	95.0	66.0	56.0	31.0
0.020	230.0	179.0	129.0	50.0
0.100	714.0	552.0	366.0	66.0
0.200	1638.0	876.0	568.0	132.0

Table 6-22: Multi-band sculpted offers with 3 scenarios with responses averaged across equi-priced periods: spills (MWh)

The results for offers formed using averaging across contiguous equi-priced periods are presented in Table 6-23 to Table 6-26. The cost distortions for these offers are better than those averaged across all equi-priced periods but not quite so good as with no averaging at all. One might conclude that generators are not indifferent about swapping gen-

eration between all equally priced periods, but only those which occur under similar physical situations.

<i>Width</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$ 4.50</i>
<i>0.002</i>	0.8	0.8	0.7	0.12
<i>0.020</i>	0.8	0.8	0.6	0.14
<i>0.100</i>	1.3	0.9	0.8	0.13
<i>0.200</i>	2.8	1.8	0.9	0.19

Table 6-23: Multi-band sculpted offers with 3 scenarios with responses averaged across equi-priced contiguous periods: direct cost distortion (%)

<i>Width</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$ 4.50</i>
<i>0.002</i>	0.9	0.8	0.8	0.4
<i>0.020</i>	0.9	0.8	0.7	0.4
<i>0.100</i>	1.8	1.3	1.1	0.5
<i>0.200</i>	5.3	3.1	2.1	0.8

Table 6-24: Multi-band sculpted offers with 3 scenarios with responses averaged across equi-priced contiguous periods: transaction cost inclusive distortion (%)

<i>Width</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$ 4.50</i>
<i>0.002</i>	4.0	4.0	34.0	318.0
<i>0.020</i>	4.0	5.0	38.0	294.0
<i>0.100</i>	17.0	69.0	97.0	467.0
<i>0.200</i>	96.0	199.0	390.0	757.0

Table 6-25: Multi-band sculpted offers with 3 scenarios with responses averaged across equi-priced contiguous periods: target defaults (MWh)

<i>Width</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$ 4.50</i>
<i>0.002</i>	38.0	38.0	31.0	21.0
<i>0.020</i>	114.0	114.0	99.0	30.0
<i>0.100</i>	602.0	398.0	377.0	77.0
<i>0.200</i>	1407.0	959.0	516.0	168.0

Table 6-26: Multi-band sculpted offers with 3 scenarios with responses averaged across equi-priced contiguous periods: spills (MWh)

The target defaults and spill using this type of offer also show improvement over offers averaged across all equi-priced periods.

Overall, offers formed using several scenarios (or market forecasts) are superior, in terms of overall system cost distortion, to offers formed on the information provided by a single forecast of the energy market.

Basing offers on several scenarios means generator may make allowance for the uncertainty in their forecast, and lower overall system costs result. Cost distortions as low as 0.6% are observed in these experiments, compared with 1.5% for the best offers based on a single forecast. This result is obtained earlier, for a five-band sculpted offer with each central band being 4% of the total generation.

Given that multiple scenarios improves the effectiveness of sculpted offers one might assume that the same is true for fixed offers, examined in the next section.

C.3.3 Multi-band Fixed Offers with Multiple Scenarios

Offers based on several scenarios have shown good performance up to this point, with direct cost distortions reduced from around 2%, achieved with a single scenario offer, to around 0.6% for offers based on three scenarios, however, fixed offers of this type have yet to be tested. This section tests fixed offers, constructed from several forecast scenarios.

These offers are formed in a similar way to the fixed offers based on single scenarios. The difference when using multiple scenarios is that the price and quantity pairs for all three scenarios are plotted together on the same graph and formed into a raw monotone curve.

In order to guide the market's solution toward choosing, on average, quantities from the middle of the original curve, additional intermediate price bands are added in the same manner as the previous fixed offers. Each offer is based upon three scenarios from the same set of five as is used for the multi-scenario sculpted offer experiments, i.e. the set (-8.0%, -5.0%, +0.0%, +5.0%, +8.0%) demand scenarios. For example, a company forecasting a low demand day uses the set (-8.0%, -5.0%, +0.0%) to form their offer.

A 'conservative' approach is used to form the monotone curve, resolving non-monotonicities by choosing the point with the lowest generation and highest price. This

means the lower demand points are effectively ignored when forming the offer. The offers are run for the same five uncertainty scenarios used for the sculpted offers, consisting of scenarios where forecasts could be different from each other, and from what actually occurs. Four parameter settings and three penalty rates are used giving a total of 60 experiments in this set.

Width	Penalty	
	\$148.00	\$18.00
(0.0,0.2)	6.0	2.6
(0.0,0.6)	4.2	2.4
(0.0,1.0)	5.9	2.6
(0.4,0.6)	3.6	2.2

Table 6-27: Multi-band fixed offers with 3 scenarios: direct cost distortion (%)

Width	Penalty	
	\$148.00	\$18.00
(0.0,0.2)	6.8	4.5
(0.0,0.6)	5.5	3.7
(0.0,1.0)	6.5	4.8
(0.4,0.6)	5.0	3.3

Table 6-28: Table 6-29: Multi-band fixed offers with 3 scenarios: transaction cost inclusive distortion (%)

Width	Penalty	
	\$148.00	\$18.00
(0.0,0.2)	31.0	615.0
(0.0,0.6)	49.0	400.0
(0.0,1.0)	25.0	691.0
(0.4,0.6)	53.0	340.0

Table 6-30: Table 6-31: Multi-band fixed offers with 3 scenarios: target defaults (MW)

Width	Penalty	
	\$148.00	\$18.00
(0.0,0.2)	1487.0	224.0
(0.0,0.6)	779.0	195.0
(0.0,1.0)	956.0	54.0
(0.4,0.6)	576.0	132.0

Table 6-32: Table 6-33: Multi-band fixed offers with 3 scenarios: spills (MW)

The results for this method of offer formation are poor. The direct cost increases over the centralised solution are substantially higher than the best results for fixed offers

based on single scenarios. However, the total costs are comparable with the results for fixed offers based on single scenarios, with the results for \$148.00 per MW penalties appearing slightly better and the results for \$18.00 per MW penalties slightly worse. The figures, reported in Table 6-27 to Table 6-32, show the average across the five uncertainty scenarios.

It would appear that the prospects for fixed offers formed using several scenarios are not particularly bright. The primary problem appears to be that fixed offer formation often results in the highest demand scenario dominating the offer.

The highest demand scenario usually calls for higher prices, and so it usually forms the bulk of the raw monotone curve that the offer is based on. Points lower in price are often discarded as the curve is made monotone. The result of this is an offer based mostly on the high demand scenario. This gives a conservative offer, in the sense that the generator offers less generation, at a higher price, than would be ideal.

The results of these conservative offers are higher direct costs, as the market is forced to select higher priced bands than is really necessary. However, the amount of defaulting on targets is reduced as the conservative offers result in targets generators can more easily reach, even if they misestimate demand.

Consequently, the poor performance of this offer formulation method is due to the dominance of the higher demand scenario in the offer. Unfortunately, it is not easy to see how this situation is to be avoided, unless the high demand scenario observations are ignored. But this would be almost the same as returning to the original formulation, using a single scenario. It might be concluded that there is no obvious way that the extra information provided by multiple forecasted scenarios can be incorporated effectively into a fixed offer.

C.4 The Periodic-update Market

The WEMSIM model was extended to simulate a market in which generators update their offers periodically. Initial experiments produced some indications of the physical

feasibility and efficiency of such a system, and allowed comparisons to be made with the day-ahead market.

The initial experiments performed were a parallel set to those in the previous section for sculpted offers. They used the same base demand day, and similar demand scenarios. However these demand scenarios were modified to reflect the generators' greater certainty about the likely demand in the first few periods. In previous experiments the whole day's demand was shifted up or down to give the demand scenarios, reflecting the fact that the generators' forecasts and offer formation would probably take place at least 12 hours ahead of the beginning of the day. However, if the generators update their offers, the offer formation process may only precede the dispatch by a few hours. Hence it is reasonable to assume that over such a time frame, generators' forecasts would be more accurate, and so the demand scenarios gradually diverge from the current observed level over the first few periods of the 24 hour planning horizon.

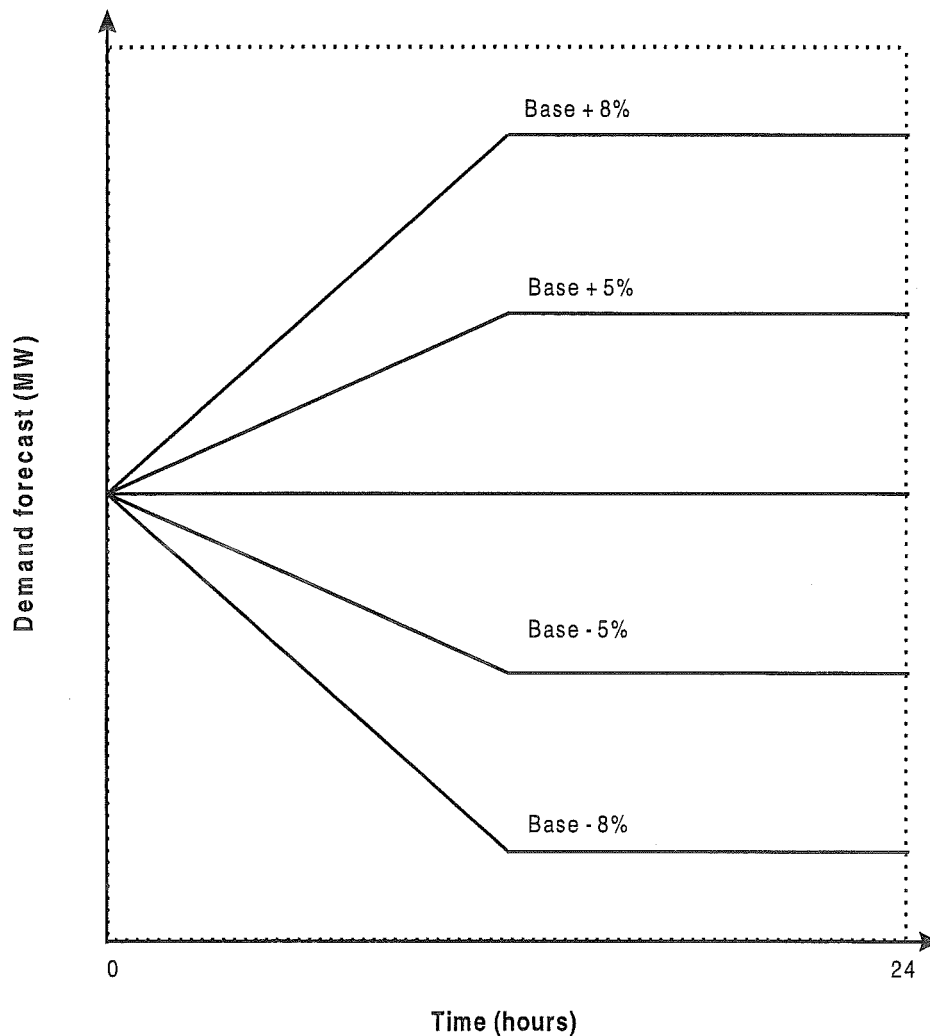


Figure 6-21: The forecast demand fans out into the day ahead over half of the horizon

Of these scenarios, each generator uses the three that they think are most likely to happen. For example, if a generator predicts the next 24 hours will be low in demand, then they will use the +0%, -5% and -8% scenarios to form their offer. The type of offers used in these experiments had up to five bands, and were different for each period of the day. For the rolling horizon model, these forecasts were updated each iteration, starting from the new observed demand.

Five experiments were carried out, using different combinations of generator expectations about the demand level. These were:

- 1) All hydro groups forecast demand accurately, i.e. use (+5%, 0%, -5%) scenarios.
- 2) All hydro groups forecast a high demand day, i.e. use (0%, +5%, +8%) scenarios.
- 3) All hydro groups forecast a low demand day, i.e. use (0%, -5%, -8%) scenarios.
- 4) Waikato and Waitaki expect a high day, the rest a low day.
- 5) Waikato and Waitaki expect a low day, the rest a high day.

The demand that actually occurs is always the base +0% day, and in these initial experiments it is assumed that the market forecasts accurately when clearing the market.

The penalties applied for missing the output targets in these experiments differ from those used previously. They are still applied as a rate per MW by which the target is missed, but, because only the current period's targets are fixed, and the future periods' targets are only indicative, the full penalty applies only to the current period's defaults. The future periods' target defaults, which are as yet only a projection, are penalised at a lower rate. These penalties for future periods represent the generator's desire to be in a position to meet the projected targets in the future, rather than any regulatory constraint or financial penalty. In these experiments the current period penalty on defaulting on output targets was \$148 per MW, and the penalty for defaulting in future periods was \$148/(number of periods ahead). For example the penalty modelled for the fourth period from the present is $\$148/4 = \37 MW. However, the horizon will roll forward, and so any defaults which remain in these future periods once they actually occur will be penalised at the full \$148/MWh.

These initial experiments with a regime that allows generators to update their offers show improved results compared with the model of a regime that cleared the market a day ahead. Table 6-34 shows the percentage increase in direct costs compared with the

ideal centralised solution. These costs consist of the thermal fuel and water from major reservoirs used over the day.

<i>Forecast</i>	<i>Periodic-update Market</i>	<i>Day-ahead Market</i>
<i>All accurate</i>	0.14	0.24
<i>All high</i>	0.25	0.24
<i>All low</i>	0.02	0.04
<i>Waikato, Waitaki high, rest low</i>	0.12	0.21
<i>Waikato, Waitaki low, rest high</i>	0.19	0.16
<i>Average</i>	0.14	0.18

Table 6-34: Multi-band sculpted offers in the periodic-update market: direct cost distortion (%)

On average, the rolling horizon model results in slightly lower cost distortions than the day-ahead model. The offers for the rolling horizon model consistently produce targets closer to the ideal dispatch than the fixed offers. This is presumably because the way in which uncertainty is progressively reduced in the rolling horizon model means that the scenarios used to form the offers are closer together, and closer to the ideal dispatch. Non-monotonicities between the scenarios, which must be resolved to create a supply curve offer, are also reduced when the scenarios are closer together.

No output target defaults occurred in any of the five experiments, as was the case with the corresponding day-ahead market.

Table 6-35 shows the increase in spill, in MW, compared with the ideal centralised solutions. These numbers are low, but not quite so low as the slight decrease in spill observed in the day-ahead market experiments. The spill occurred on the Waikato chain, past the Waipapa station, and the decrease in spill reported for the day-ahead market reflects the Waikato chain producing less in these experiments, due to the non-monotonicities in its offer. Apparently the cost of this increased spill was lower than the cost of the increased thermal and/or inefficient hydro generation in the day-ahead model.

<i>Forecast</i>	<i>Periodic-update Market (MW)</i>	<i>Day-ahead Market (MW)</i>
<i>All accurate</i>	5.0	-11.0
<i>All high</i>	3.0	-11.0
<i>All low</i>	1.0	1.0
<i>Waikato, Waitaki high, rest low</i>	-6.0	5.0
<i>Waikato, Waitaki low, rest high</i>	5.0	-1.0

Table 6-35: Multi-band sculpted offers in the periodic-update market: increase in spills (MWh)

From these initial results, it would appear that a market that allows generators to update their offers as the day progresses is superior to the day-ahead market in terms of the potential loss of efficiency. The rolling horizon showed an average increase in costs of 0.14% over the ideal centralised solution, compared with an increase of 0.18% for the day-ahead market. There were no defaults on the output targets set by the market in either set of experiments, indicating the physical feasibility of both regimes. The day-ahead market resulted in slightly less spill than either the rolling horizon market or the ideal centralised solution.

The reason for the reduced cost distortion in the rolling horizon market appears to be the lower uncertainty faced by the generators. This allows hydro generators to more closely target their preferred generation (and hence storage) trajectories. It also reduces the incidence of non-monotonicities in the offers in such a market, compared with the day-ahead market. In the simple market considered here, non-monotonicities in generators' offers must be resolved to form a supply curve, and in the model this is achieved conservatively, by reducing the amount of generation from the lower priced observation. Hence there is a bias towards less capacity being offered by the generators that have these non-monotonicities, and the market has to make up the additional capacity from other generators, at increased cost.

C.4.1 Periodic-update versus Day-ahead Market

To gain insights into the possible advantages of a rolling horizon market regime, a solution from the rolling horizon modelling system was compared with the equivalent solution from the day-ahead modelling system. The day chosen was one in which the Wai-

kato and Waitaki generators overestimated the demand by 5.0%, and the Waikaremoana, Manapouri, and Clutha generators underestimated demand by 5.0%. The thermal generators offered their marginal fuel costs, and the market correctly forecast the demand.

In the rolling horizon model the final quantities a generator offers may differ considerably from previous offers which the generator has made in the preceding 24 hours. Table 6-36 shows how the offers made by the Waikato group evolve through the day. The quantities shown are for the central band of the offer. Usually small quantities will be offered on either side of this band, at higher and lower prices. Across the top of the table are the actual periods, and each column contains the offers made in that period for the coming 24 hour horizon. Hence the offers in the lower part of the table are actually for the next day. To see how the offer for a particular period alters during the day, look across the rows of the table. For example consider the offer for period 7. In period 1, the amount offered is 801MW. In period 2, the amount offered for period 7 has risen to 803 MW, and as the day progresses, this falls to the final offer of 779 MW.

	Actual Period											
	1	2	3	4	5	6	7	8	9	10	11	12
1	464											
2	410	497										
3	450	543	470									
4	831	792	805	826								
5	108 9	107 0	108 9	107 1	107 6							
6	807	805	776	794	789	791						
7	801	803	811	790	790	787	779					
8	874	869	870	858	857	857	863	860				
9	108 2	109 1	106 9	110 3	112 3	110 6	110 6	110 6	106 2			
10	841	841	841	860	860	852	842	831	832	831		
11	823	759	787	808	813	809	809	800	826	819	819	
12	790	677	782	772	773	784	774	762	742	717	695	667
1		517	533	539	535	545	539	540	529	519	526	525
2			397	413	420	414	418	415	448	460	466	485
3				466	465	463	462	464	461	503	514	532
4					819	803	802	801	806	806	806	800
5						108 9	109 1	1118	1118	108 8	109 8	109 8
6							784	803	816	756	770	810
7								791	770	741	757	802
8									877	879	879	876
9										114 4	113 3	113 3
10											812	847
11												687

Table 6-36: Central band of the Waikato offer

Other generators show similar variation in their offers. The graphs below show the amounts of generation in the scenarios used by each generator, and the resulting offers. The horizontal axis shows each two-hour period. The stacked bars on the graphs show the quantities of generation in the group's offer for that period. In the case of the rolling horizon model, the quantities shown represent the final offer for that period. The lines on the figure detail the three scenarios used by the generator to form their offer.

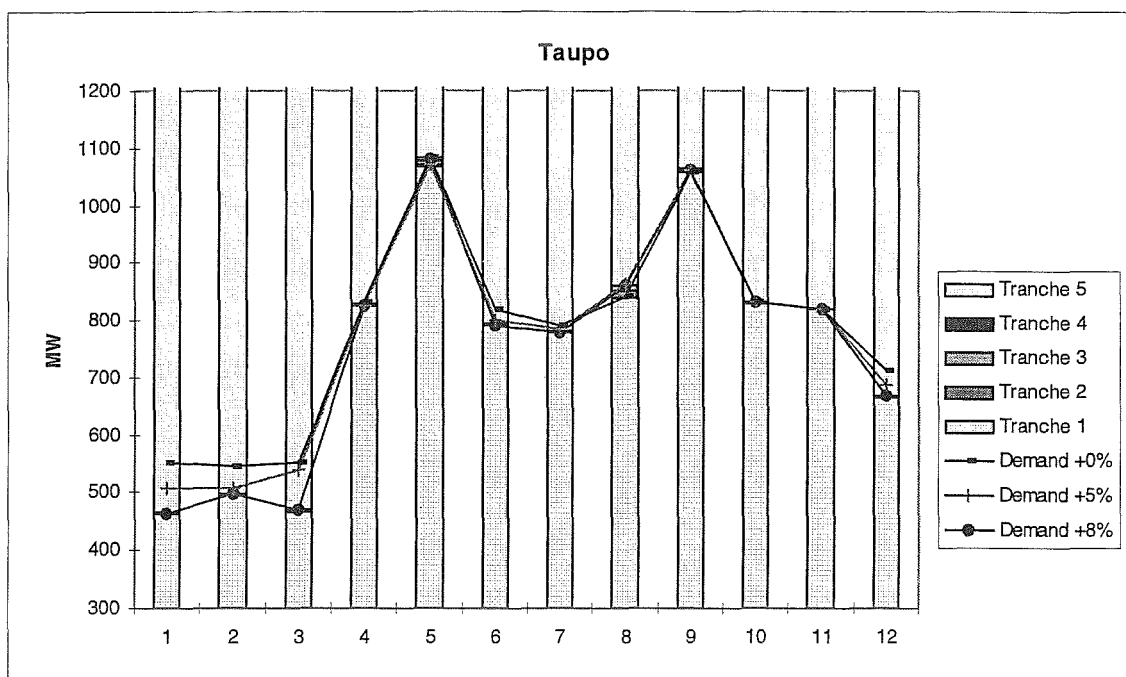


Figure 6-22: Periodic-update market offer profile for Waikato

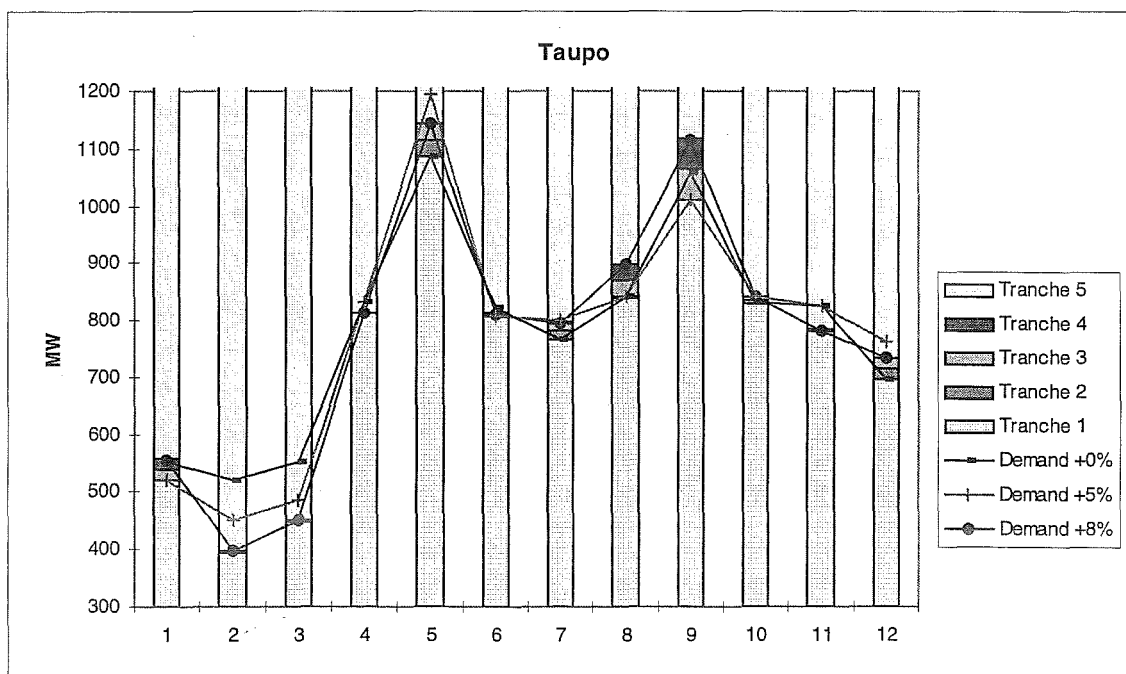


Figure 6-23: Day-ahead market offer profile for Waikato

The Taupo group forecast a high day, so they used the +0%, +5%, and +8% demand scenarios. These scenarios exhibited considerable non-monotonicities in the initial peri-

ods of both the rolling horizon and day-ahead experiments. A group's offer must be monotone, that is, a higher level of generation must have a correspondingly higher price. However, because of intertemporal effects, this is often not the case with the scenarios generated for hydro systems. For example, in a higher demand scenario (which may have a higher price), a generator may wish to generate less during off peak periods in order to release more during the peak periods. Consequently during the off peak period a generator might wish to submit a non-monotone offer, with a lower level of generation at a higher price. Because the offer is required to be a supply curve the generator must modify it. The offer formation process used in these experiments makes offers monotone by truncating the amount of generation offered at the lower price back to the level offered at the higher price.

This effect is seen in the initial periods of both the rolling horizon and the day-ahead profiles. The lines representing the +0% and +5% scenarios are above the line representing the +8% scenario. Consequently the offer bands which correspond to the lower demand scenarios are reduced to the lower generation level of the +8% scenario, as can be seen by the position of the bars representing the offers. The result is that in the initial periods, the Waikato group offers less than the ideal centralised solution, in both the day-ahead and rolling horizon models. This in turn affects the thermal generators as they produce more to make up the deficit, as shown in Figure 6-24 and Figure 6-25.

One major difference between the day-ahead and rolling horizon model results is the wider offer bands that the former submits. Presumably this is because of the reduced level of uncertainty in the rolling horizon model, leading to scenarios which are closer together, and hence narrower offer bands. The overall conclusion is that the rolling horizon model produces a closer match between the actual and ideal generation than the day-ahead model.

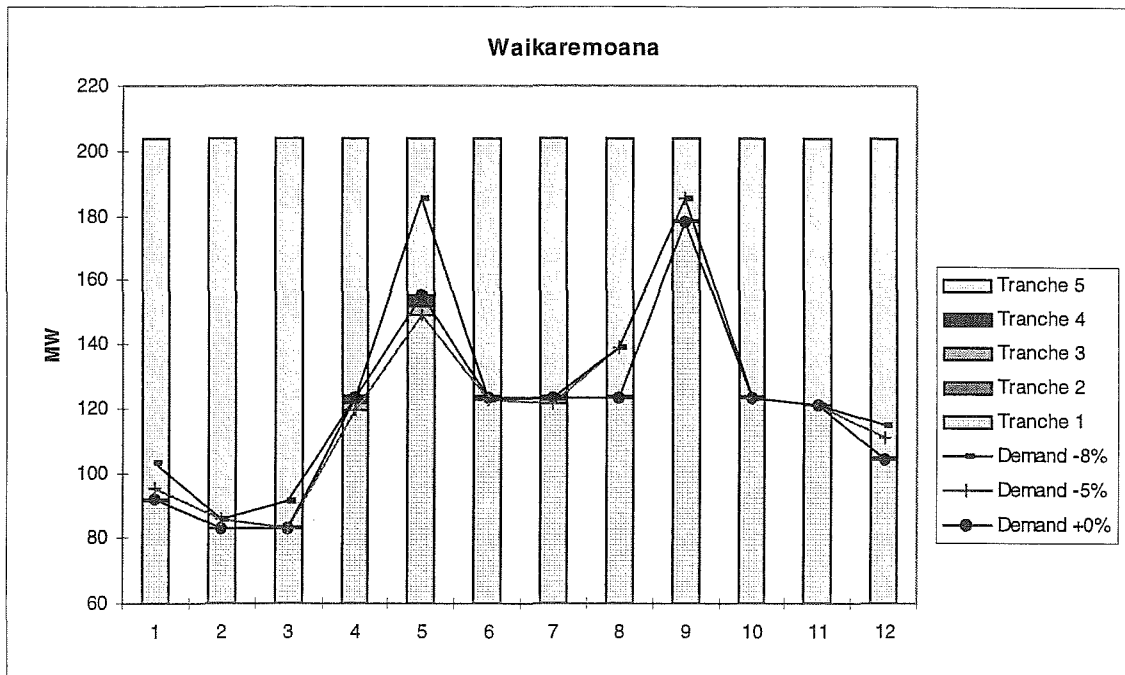


Figure 6-24: Periodic-update market offer profile for Waikaremoana

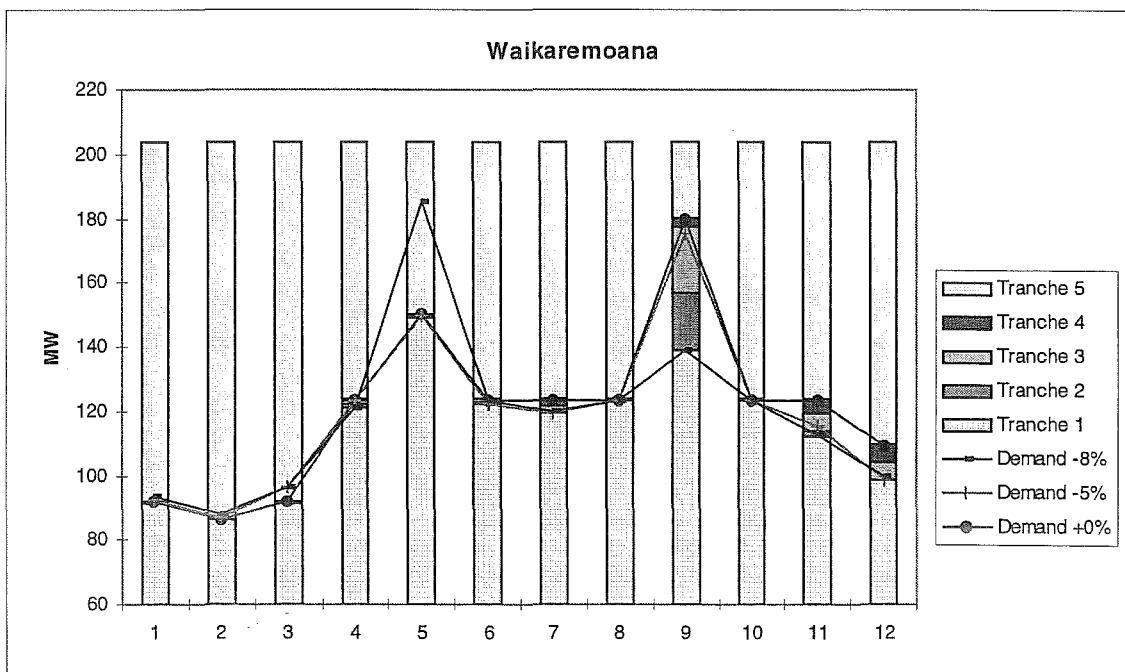


Figure 6-25: Day-ahead market offer profile for Waikaremoana

The Waikaremoana group predicted a low day, and so used the +0%, -5%, and -8% scenarios to form their offers. The high first peak of the -8% scenario in both models is due

to swapping generation between the peak periods that have the same price. The rolling horizon model also has a high second peak in the -8% scenario because the generator's viewpoint changes through the day. In period 5, the -8% scenario has a high generation for period 5 (the first peak), and a lower generation for period 9 (the second peak). However, by the time the model rolls forward to period 9, it has swapped the generation between the daily peaks, and is now generating more in the second peak (period 9), and less in the first peak of the next day (period 5 tomorrow). The thermal generators offer in their marginal costs, and so their offer is the same for both models, and remains the same throughout the day.

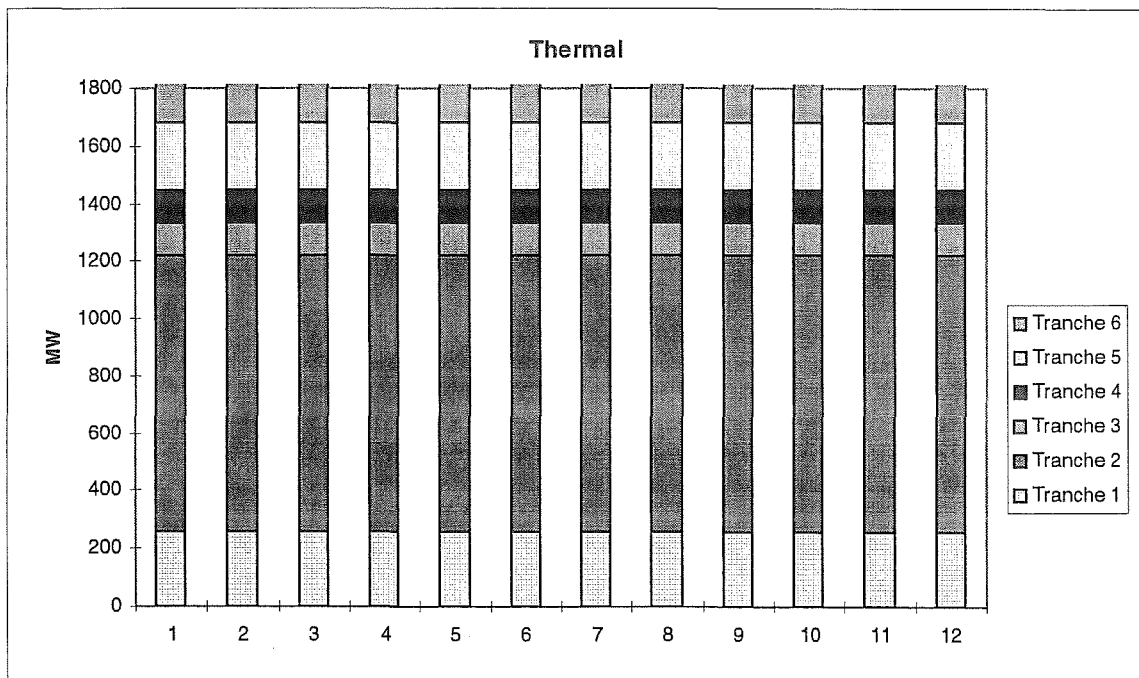


Figure 6-26: Offer profile for thermals

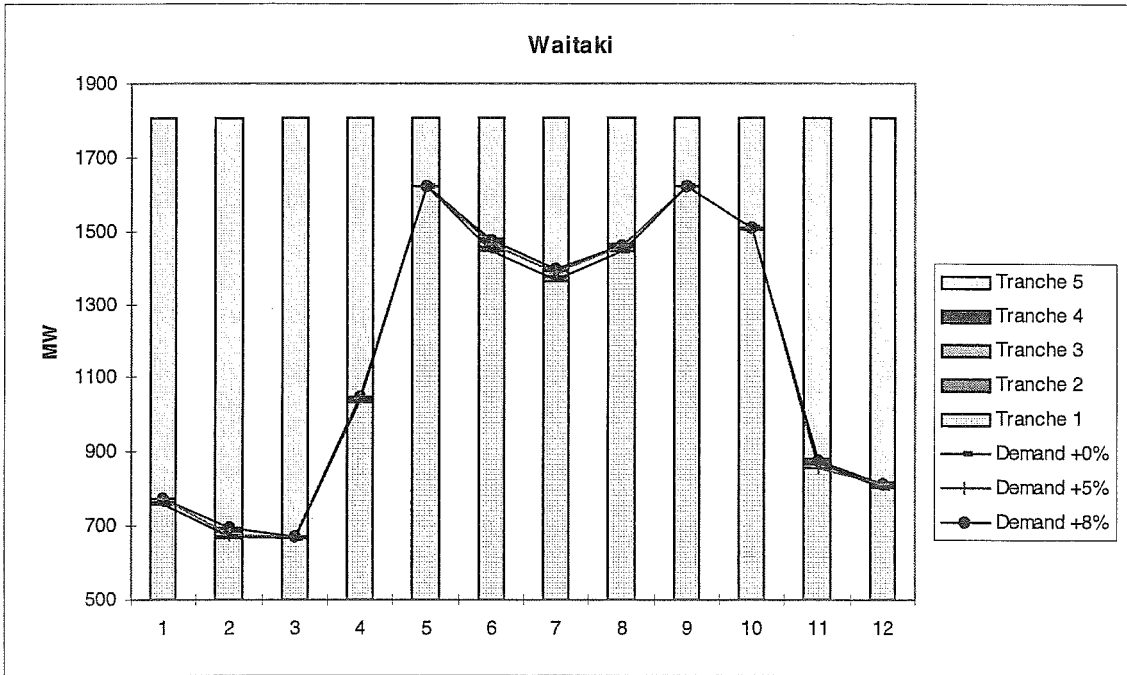


Figure 6-27: Periodic-update market offer profile for Waitaki

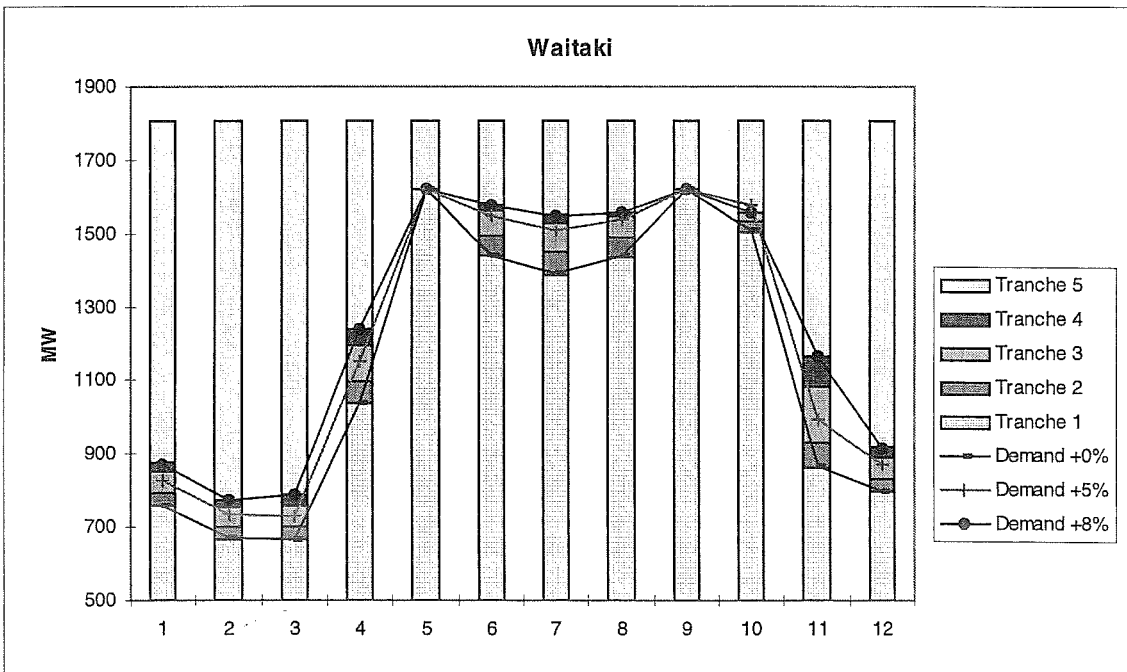


Figure 6-28: Day-ahead market offer profile for Waitaki

The fixed offers from the Waitaki system have broader bands, reflecting the greater uncertainty and the flexibility of the Waitaki system with its large reservoirs.

Manapouri

The chart displays the power output in MW for 12 periods. The Y-axis ranges from 0 to 600 MW. The legend identifies five tranches (Tranche 1 to Tranche 5) and three demand scenarios (Demand -8%, Demand -5%, Demand +0%). The output is consistently near 600 MW across all periods, with minor variations in the pattern of the stacked tranches.

Period	Tranche 1 (MW)	Tranche 2 (MW)	Tranche 3 (MW)	Tranche 4 (MW)	Tranche 5 (MW)
1	0	0	0	0	590
2	0	0	0	0	590
3	0	0	0	0	590
4	0	0	0	0	590
5	0	0	0	0	590
6	0	0	0	0	590
7	0	0	0	0	590
8	0	0	0	0	590
9	0	0	0	0	590
10	0	0	0	0	590
11	0	0	0	0	590
12	0	0	0	0	590

Manapouri

Y-axis: MW (0 to 600)

X-axis: 1 to 12

Legend:

- Tranche 5 (White)
- Tranche 4 (Light Gray)
- Tranche 3 (Medium Gray)
- Tranche 2 (Dark Gray)
- Tranche 1 (White)
- Demand -8% (Line with square markers)
- Demand -5% (Line with triangle markers)
- Demand +0% (Line with circle markers)

Period	Tranche 1 (MW)	Tranche 2 (MW)	Tranche 3 (MW)	Tranche 4 (MW)	Tranche 5 (MW)	Demand -8% (MW)	Demand -5% (MW)	Demand +0% (MW)
1	585	0	0	0	0	0	0	0
2	585	0	0	0	0	0	0	0
3	585	0	0	0	0	0	0	0
4	585	0	0	0	0	0	0	0
5	585	0	0	0	0	0	0	0
6	585	0	0	0	0	0	0	0
7	585	0	0	0	0	0	0	0
8	585	0	0	0	0	0	0	0
9	585	0	0	0	0	0	0	0
10	585	0	0	0	0	0	0	0
11	585	0	0	0	0	0	0	0
12	585	0	0	0	0	0	0	0

Figure 6-30: Day-ahead market offer profile for Manapouri

The generation profiles for the Clyde group also exhibit non-monotonicities in the scenarios, and truncations of the offers. However, because the group expected a low day, using the +0%, -5%, and -8% scenarios, the truncations often meant a reduction to the generation level of the 0% scenario. This resulted in targets that were close to the 0% level and hence to the ideal solution.

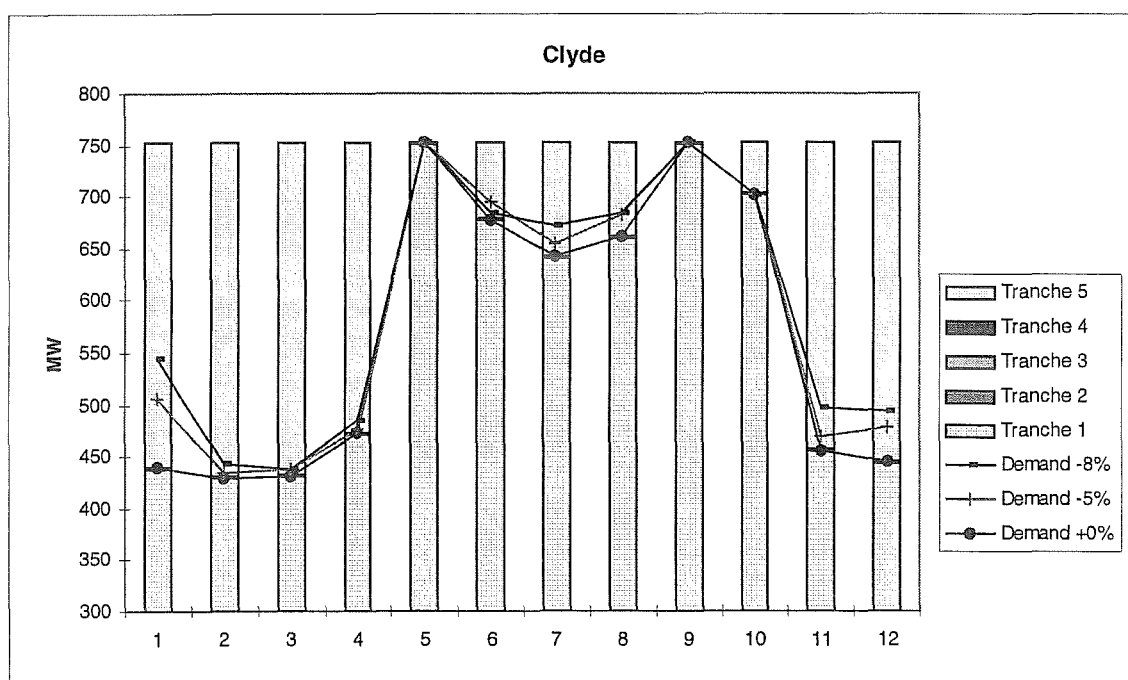


Figure 6-31: Periodic-update market offer profile for Clyde

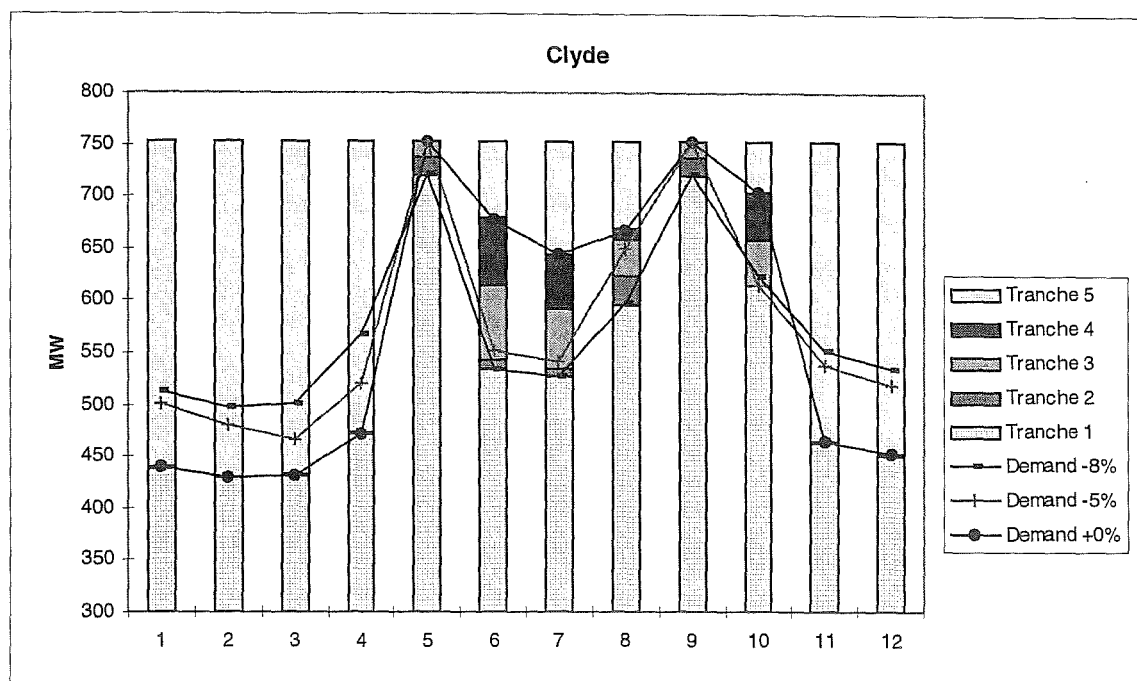


Figure 6-32: Day-ahead market offer profile for Clyde

With the rolling horizon model it is also possible that the group generation responses from the scenarios could actually be non-monotone over the entire day, not just for off-peak periods. The sum of all the energy offered by a generator could thus be less when they were expecting a higher demand day. This undesirable result is caused by the combination of the rolling horizon and the fan shaped demand scenarios. Consider the base and -5 % scenarios shown earlier. Suppose a hydro generator has a maximum amount of water which can be released over the day, and that this constraint is binding. Then the amount released in the current period will be different for the two scenarios shown above. The higher demand scenario will expect to need more water later in the day, so it will actually schedule less release in the current period. However, the horizon is then rolled forward one period and the scenarios are re-evaluated. Again the higher demand scenario will release less, in order to be able to meet the expected need later in the horizon. The same will be true of every period as the horizon rolls forward through the day, repeatedly revealing that current demand is lower than had been projected, but still projecting higher loads in future periods. So the higher demand scenario will result in lower releases in every period.

This effect transfers across to the offers generators submit, and so to the targets set. Against this, generators who are in a position to do so will probably be willing to use more water in total during the day if they expect higher demand, and consequently higher prices. In the rolling horizon experiment discussed here, the former effect dominates in the Taupo and Clyde river-chains, and the latter in the Waitaki river-chain.

The graphs below are similar to those in the previous section, but they compare the amounts of generation offered with the output targets set by the market, and with what actually occurs in the real time dispatch, under both kinds of regime, across the day. As before, the stacked bars on the graphs show the quantities of generation in the group's offer for that period, with the quantities shown representing the final offer for that period in the case of the rolling horizon model. The lines detail the output targets set by the market, the final real time dispatch, and the ideal centralised solution.

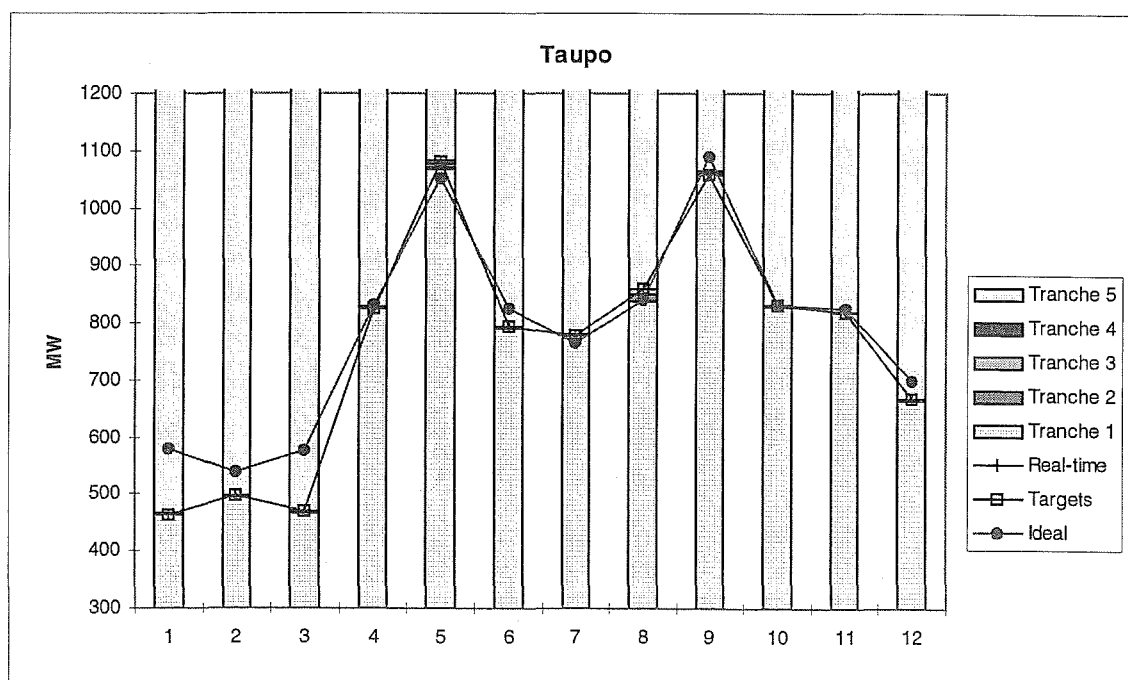


Figure 6-33: Periodic-update market energy output profile for Waikato

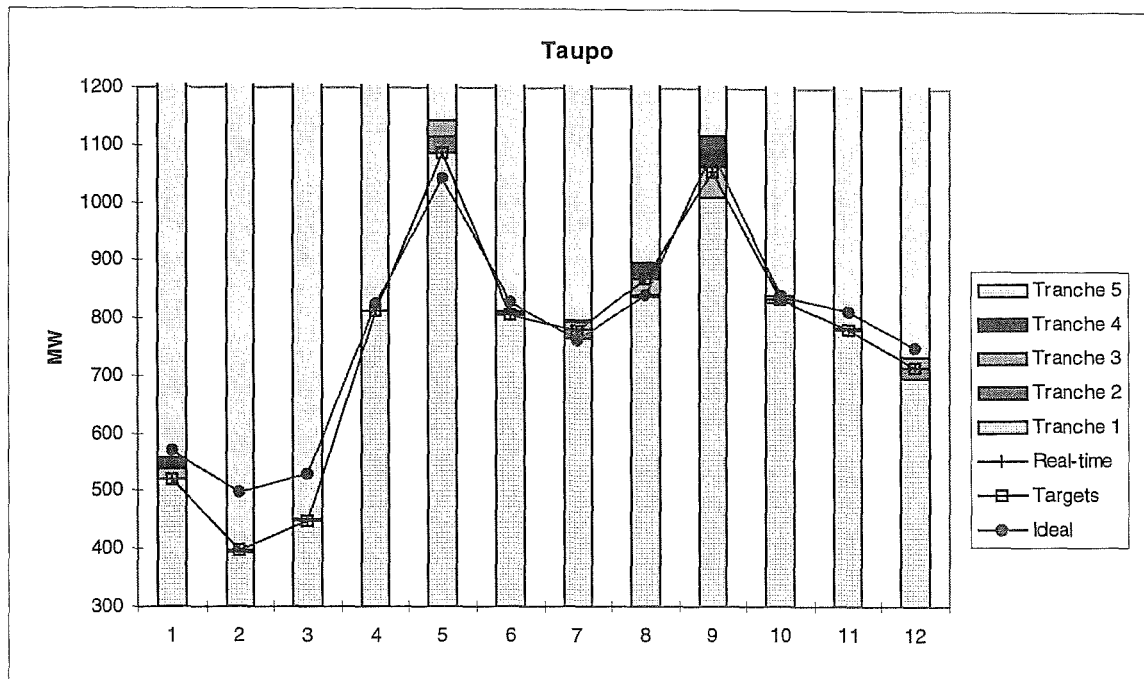


Figure 6-34: Day-ahead market energy output profile for Waikato

The graphs show the real time dispatch matching the targets perfectly, with no defaults. This was the case for all generators in all of these initial rolling horizon experiments, as well as the equivalent day-ahead experiments, indicating that the targets set are physically feasible. The targets and real time dispatch are generally reasonably close to the ideal centralised dispatch, except for the first three periods, which have targets considerably lower than the ideal solution. Recall that the offers in these periods had considerable non-monotonicities between the scenarios, and so the amount of generation was reduced when the offer was being formed. The result is lower than ideal generation from the Taupo group in these periods. To compensate, the market was forced to set higher targets for the thermal generators, as shown in Figure 6-33 and Figure 6-34, causing a considerable loss in efficiency relative to the idealised centralised solution. There is also some swapping of generation between the peak periods, compared with the centralised dispatch, but this probably is not costing the system very much.

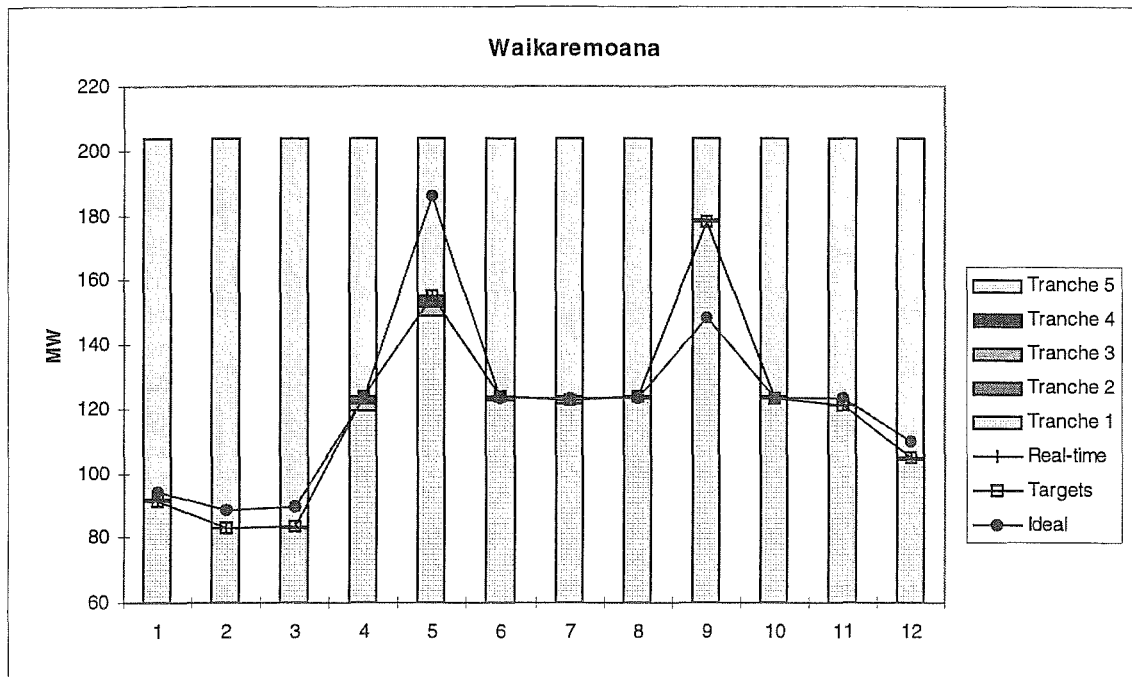


Figure 6-35: Periodic-update market energy output profile for Waikaremoana

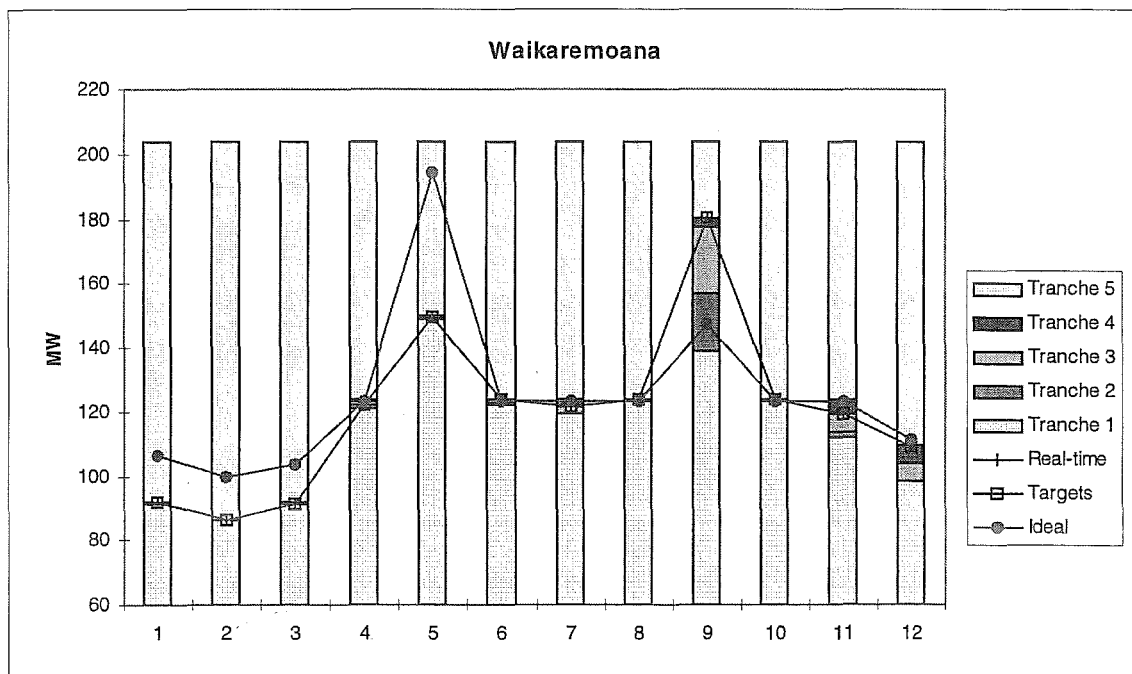


Figure 6-36: Day-ahead market energy output profile for Waikaremoana

Similar effects can be seen in the generation profiles for the Waikaremoana group, with a large amount of swapping between the equi-priced peak periods. Also the targets and

real time dispatch are lower in the initial periods, again due to non-monotonicities in the offers, but the effect is less severe with the rolling horizon market. To compensate for the reduced output from the Taupo and Waikaremoana groups in the first three periods, the thermal generators receive higher targets than in the ideal solution, causing significantly higher costs to the system.

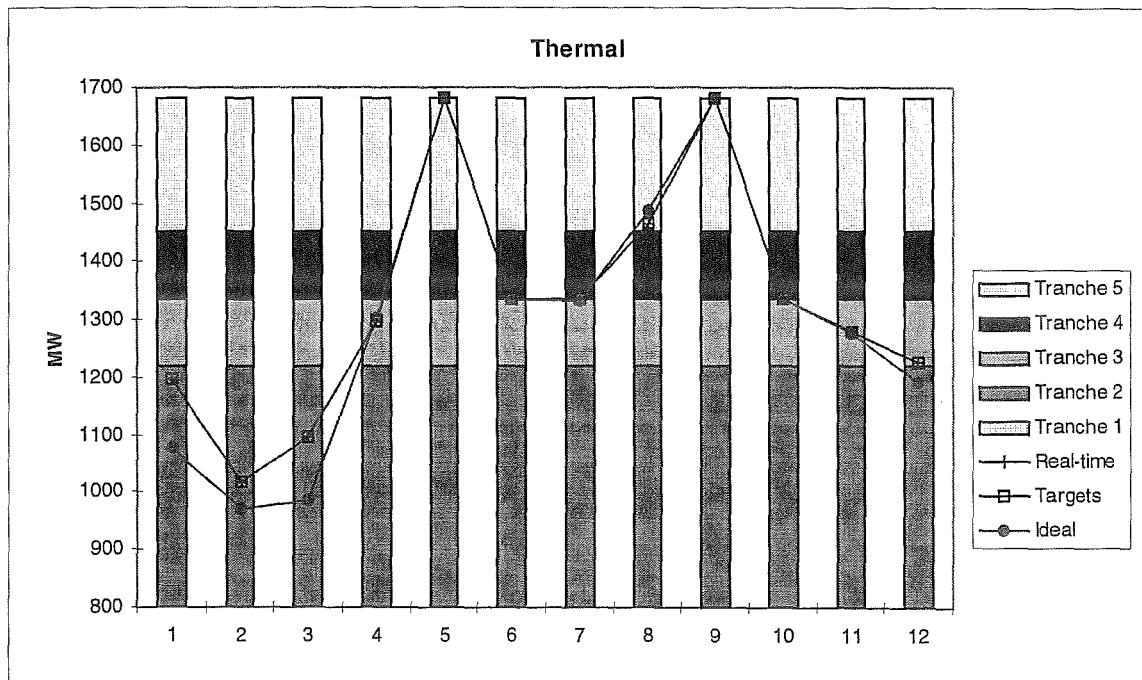


Figure 6-37: Periodic-update market energy output profile for Thermal

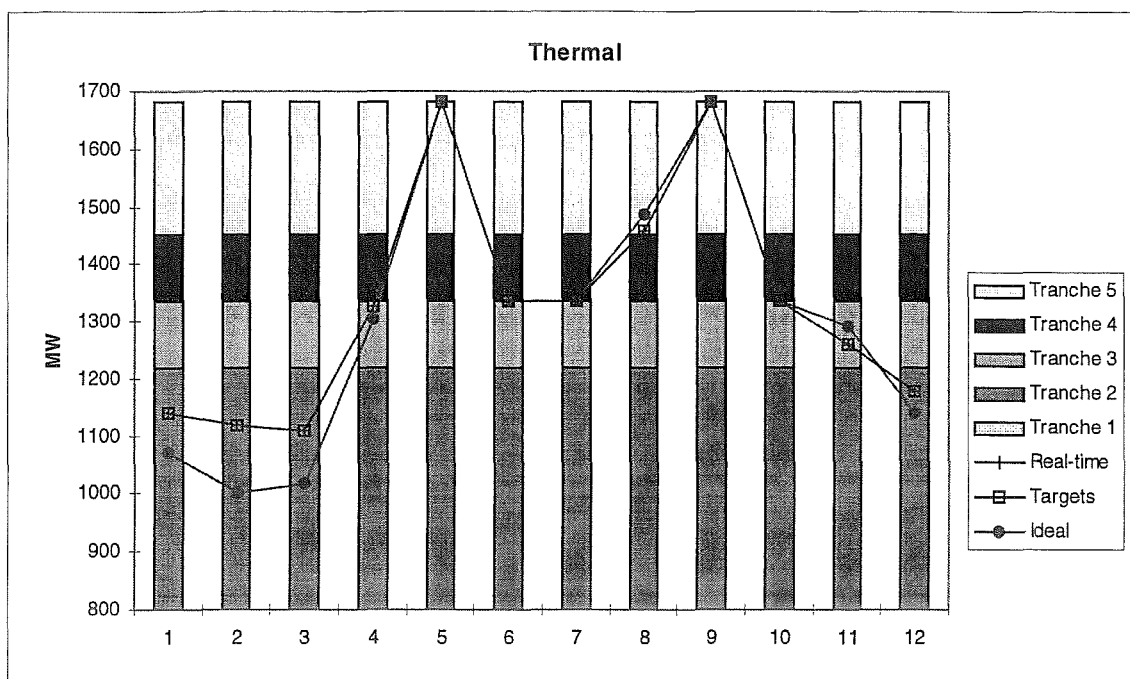


Figure 6-38: Day-ahead market energy output profile for Thermal

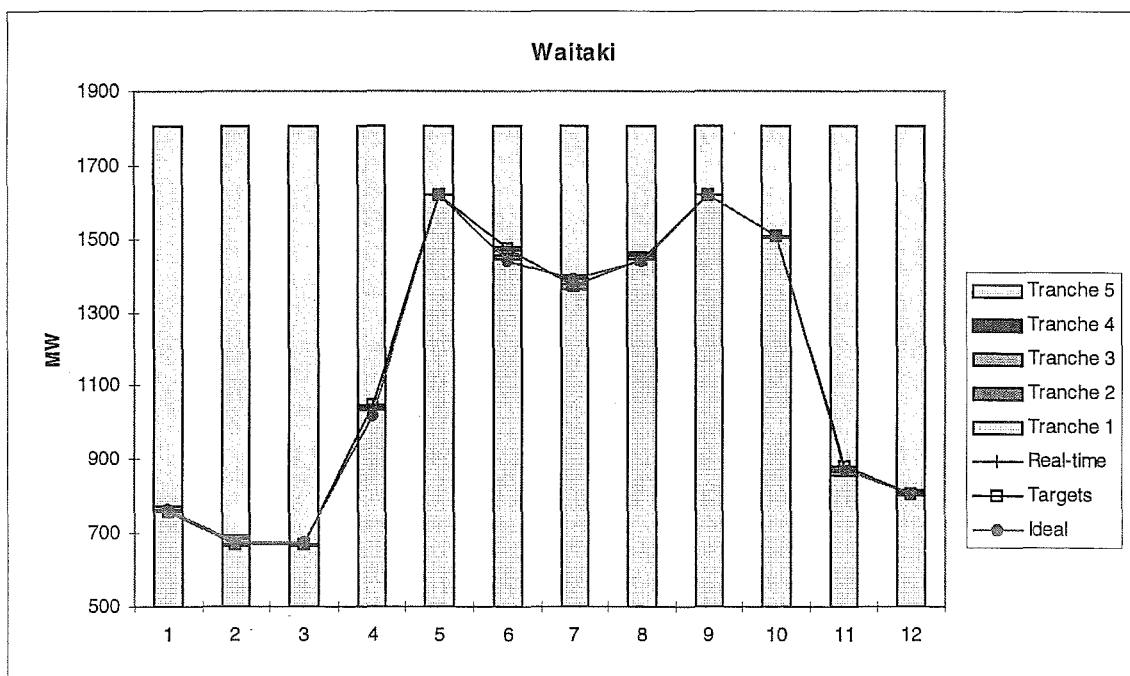


Figure 6-39: Periodic-update market energy output profile for Waitaki

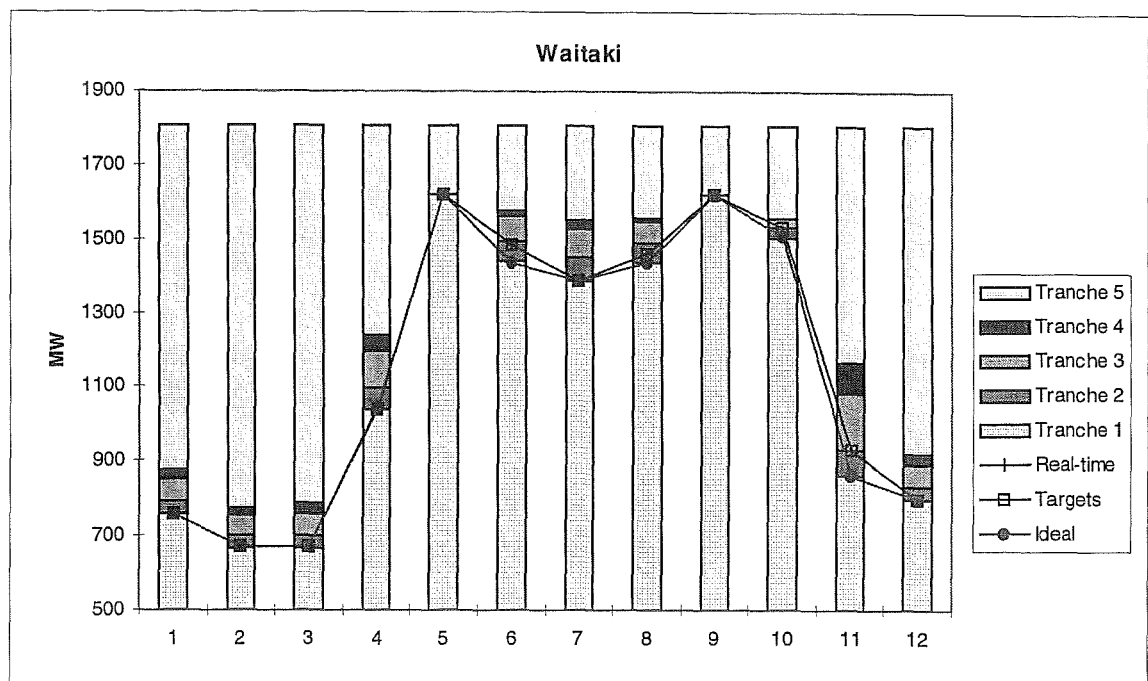
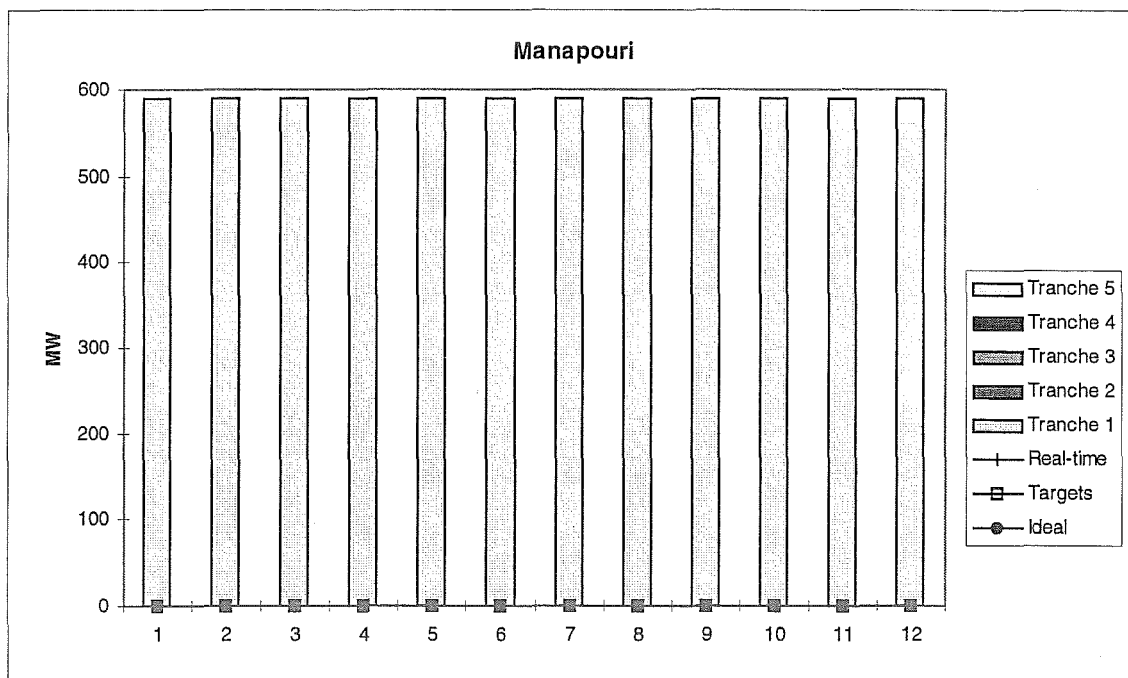


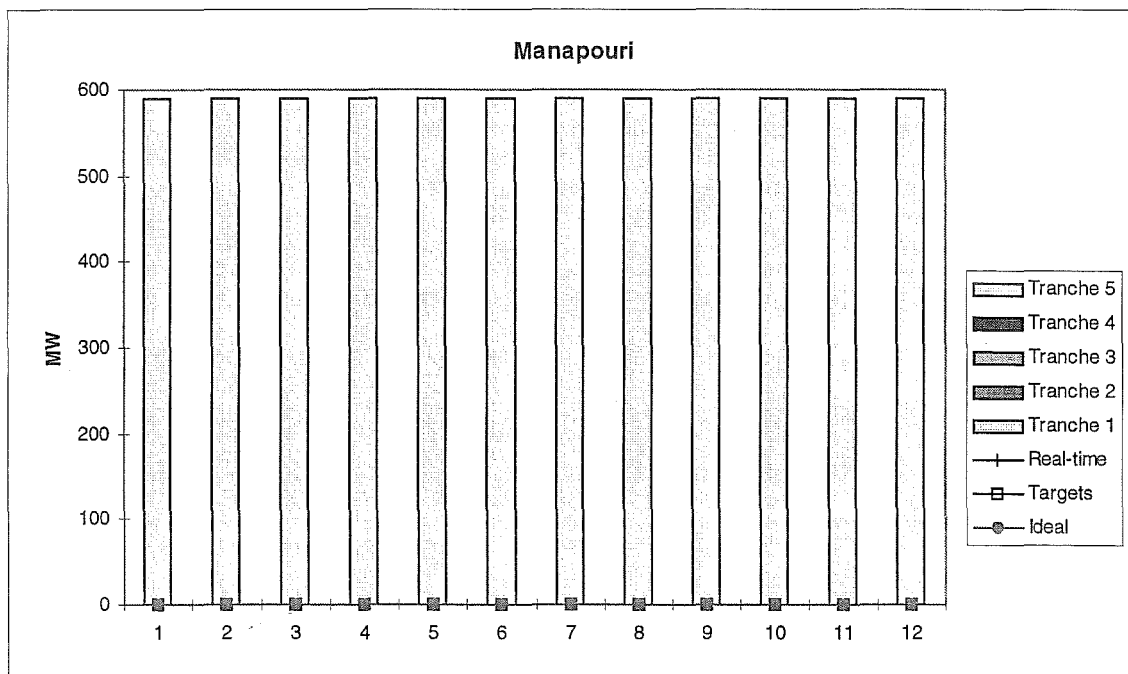
Figure 6-40: Day-ahead market energy output profile for Waitaki

The targets and real time dispatch follow the ideal dispatch very closely in the Waitaki system. In the rolling horizon model, the reduced level of uncertainty as the day progresses, allows the generator to effectively use narrow bands to ensure that the targets they receive are very close to their preferred trajectory.

The targets and real time dispatch for the day-ahead market are not quite so close to the ideal solution, reflecting the wider bands used in the offers. Notice how the targets tend to be at the lower end of the offer, since the Waitaki group was expecting a high demand day in this experiment, and so offered more generation at a higher price. Manapouri does not generate all day because the high water value assigned to its reservoir meant it did not offer any generation.



Periodic-update market energy output profile for Manapouri



Day-ahead market energy output profile for Manapouri

The Clyde group has targets and real time dispatch close to the ideal, despite the fact that there were many non-monotonicities in its offers. Because Clyde predicted a low

demand day, i.e. using the +0%, -5% and -8% demand scenarios, most of the non-monotonicities were truncated back to the +0% level, which is close to the ideal solution. Hence the net effect is to produce very low distortions.

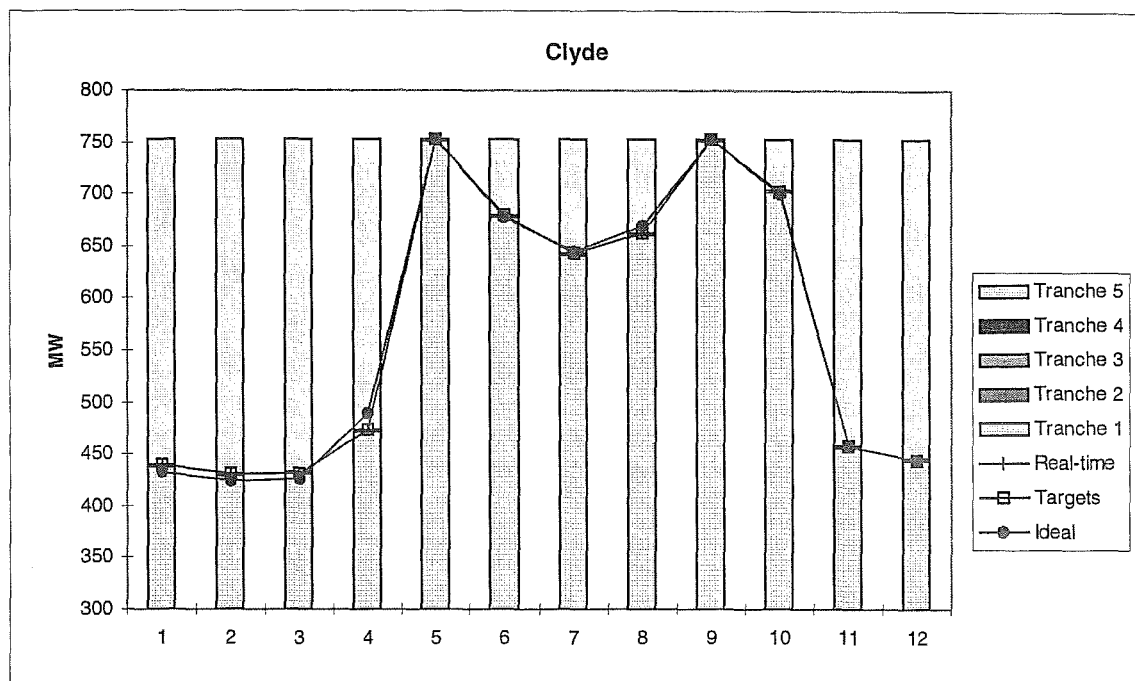


Figure 6-41: Periodic-update market energy output profile for Clyde

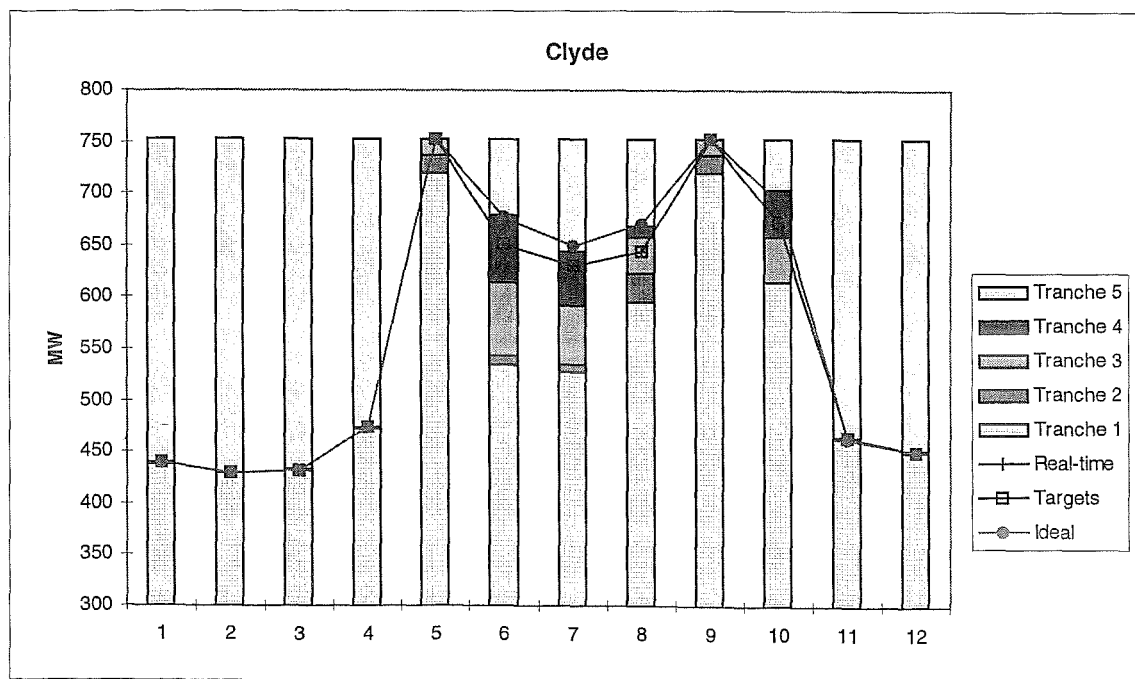


Figure 6-42: Day-ahead market energy output profile for Clyde

Again the rolling horizon market has less deviation between the real time dispatch and the ideal dispatch.

Overall, the conclusion is that the rolling horizon model produces a closer match between the actual and ideal generation than the day-ahead model.

Table 6-37 and Table 6-38 show the evolution of the market clearing prices through the day with the rolling horizon model. These are the prices, and projected prices, which the market finds from the demand forecast and the offers. The prices for the real time dispatch can not be interpreted because the optimisation has strong penalties on the output target defaults, which distort the prices. In a real market it is likely that the actual prices that revenues and other payments are based on will be determined ex post, reducing the importance of the market clearing prices. However generators, to help them determine how they should update their offers, will still use the projections of future prices.

The tables below show that, unfortunately, these price projections do not trend monotonically, and may not be very stable, especially around the peak periods. On the other hand, the variations are generally not extreme, and the initial price vector generally provides a fairly reliable indicator of the final prices.

	Actual Period											
	1	2	3	4	5	6	7	8	9	10	11	12
1	20.55											
2	20.71	20.66										
3	20.60	20.60	20.65									
4	22.00	22.05	22.00	22.04								
5	26.35	36.20	26.37	26.82	29.21							
6	22.20	22.21	22.63	22.21	22.57	22.18						
7	22.05	22.00	22.04	22.07	22.05	22.05	22.09					
8	24.07	24.07	24.07	24.07	24.07	24.07	24.07	24.03				
9	29.23	29.23	29.23	29.23	26.88	26.55	26.55	26.55	26.38			
10	22.23	22.24	22.24	22.12	22.25	22.23	22.23	22.27	22.21	22.18		
11	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	21.96	
12	20.65	20.70	20.60	20.60	20.80	20.60	20.60	20.60	20.60	20.60	20.60	21.95
1		20.65	20.60	20.60	20.60	20.60	20.60	20.60	20.60	20.60	20.60	20.70
2			20.66	20.71	20.71	20.71	20.71	20.71	20.60	20.71	20.71	20.60
3				20.55	20.60	20.60	20.60	20.60	20.71	20.60	20.60	20.60
4					21.96	22.00	22.00	22.05	22.00	22.00	22.00	22.05
5						36.17	29.18	26.49	26.35	26.37	26.35	26.76
6							22.16	22.20	22.21	22.63	22.63	22.20
7								22.04	22.00	22.70	22.05	22.05
8									24.03	24.00	24.00	24.07
9										26.38	26.41	26.82
10											22.20	22.23
11												22.04

Table 6-37: Periodic-update market North Island prices

	Actual Period											
	1	2	3	4	5	6	7	8	9	10	11	12
1	20.70											
2	20.61	20.67										
3	20.39	20.40	20.51									
4	20.56	20.61	20.56	20.80								
5	24.63	33.83	24.64	25.07	27.30							
6	20.75	20.75	21.15	20.76	21.10	20.73						
7	20.61	20.56	20.60	20.62	20.61	20.61	20.65					
8	20.61	20.61	20.61	20.62	20.62	20.61	20.61	20.64				
9	27.32	27.32	27.32	27.32	25.12	24.81	24.81	24.81	24.65			
10	20.78	20.78	20.78	20.68	20.79	20.78	20.78	20.81	20.76	20.73		
11	20.56	20.56	20.56	20.56	20.56	20.56	20.56	20.56	20.56	20.56	20.52	
12	20.49	20.45	20.43	20.44	20.45	20.45	20.45	20.45	20.52	20.44	20.57	20.52
1		20.72	20.81	20.80	20.82	20.81	20.81	20.81	20.82	20.80	20.78	20.81
2			20.56	20.60	20.62	20.61	20.61	20.61	20.62	20.61	20.61	20.61
3				20.46	20.40	20.39	20.39	20.39	20.40	20.39	20.39	20.39
4					20.52	20.56	20.56	20.61	20.56	20.56	20.56	20.61
5						33.81	27.27	24.76	24.63	24.65	24.63	25.01
6							20.71	20.75	20.75	21.15	21.15	20.75
7								20.58	20.56	20.61	20.61	20.61
8									20.58	20.61	20.61	20.61
9										24.65	24.68	25.06
10											20.75	20.78
11												20.59

Table 6-38: Periodic-update market South Island prices

The reason for the variation in projected market clearing prices is the narrow widths of the generators' offers. Recall that the market clearing prices are based solely upon the prices offered by generators, since the demand is assumed fixed over such a short time-frame. Because the bands are so narrow, it does not take much variation in the quantity to move from one band to the next. This does not matter as much for the off-peak periods, since they generally exhibit less price variation between the scenarios that form the offers. However the peak periods' scenarios show a much greater variation in price, since thermal stations with a relatively wide range of marginal costs may be on the margin. Consequently the bands of hydro generators' peak period offers may show substantial price steps, with only small quantity steps. This means that any small reductions in the amounts offered by some generators could lead to the high priced top band being accepted in order to clear the market, and consequently a high market clearing price. This problem affects both the day-ahead, and rolling horizon models, as can be seen in

the graphs of the price profiles below. In this case, the rolling horizon model, while predicting high prices for the peaks in some periods, finds reduced prices when the peak periods actually arrive. This is not always the case though, and some of the rolling horizon experiments do have high peak market clearing prices.

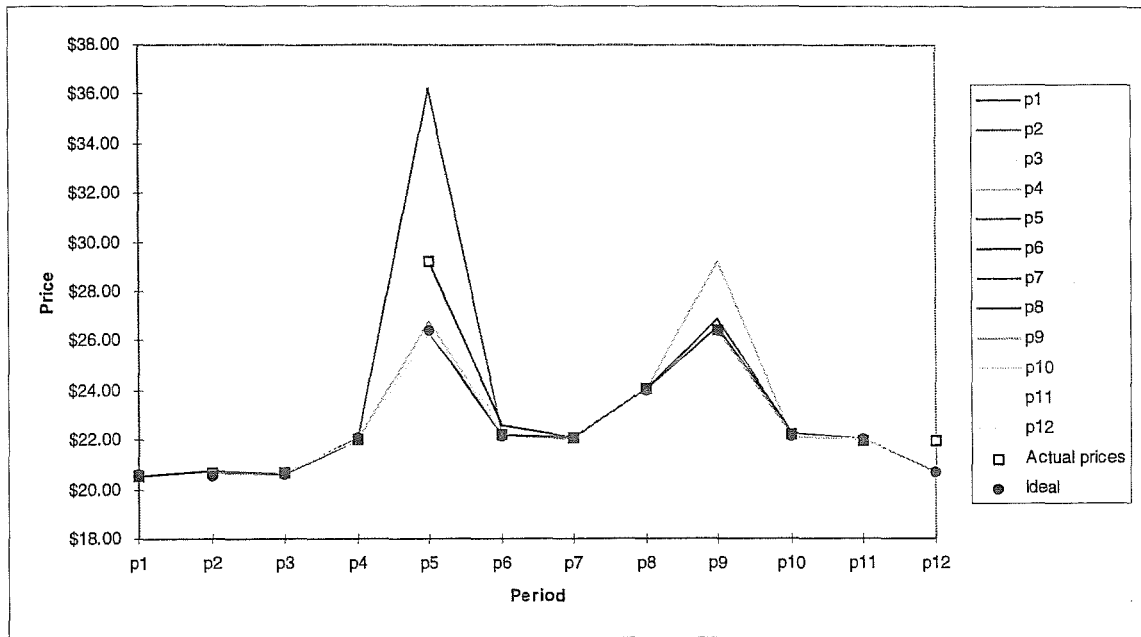


Figure 6-43: Periodic-update market North Island energy prices

The lines on the rolling horizon graph represent the prices for that period as projected at some previous period.

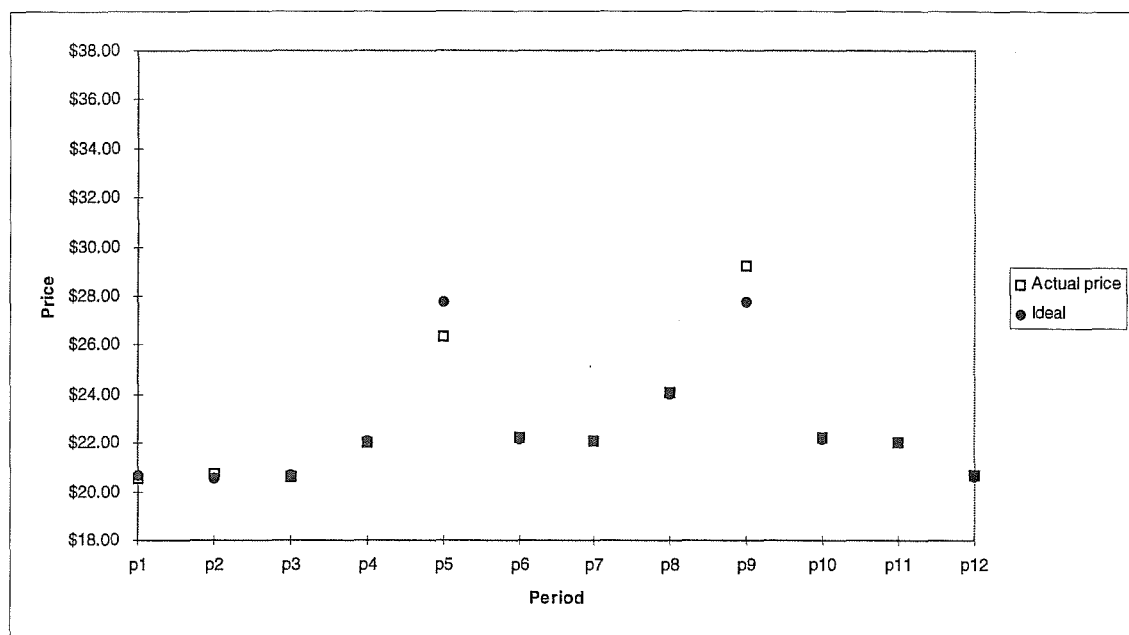


Figure 6-44: Day-ahead market North Island energy prices

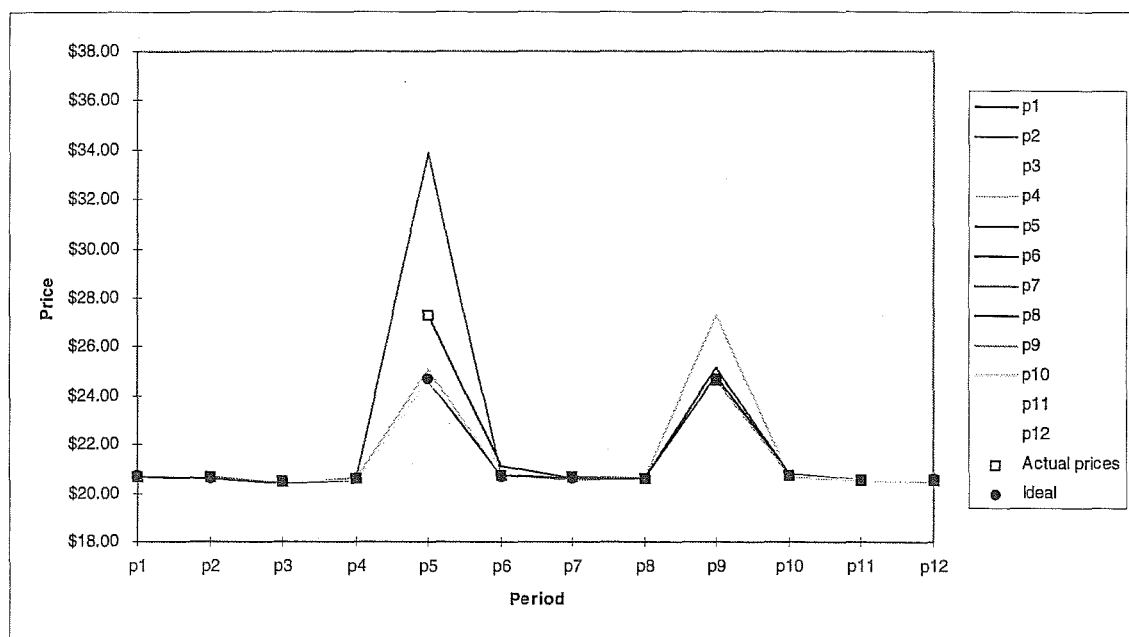


Figure 6-45: Periodic-update market South Island energy prices

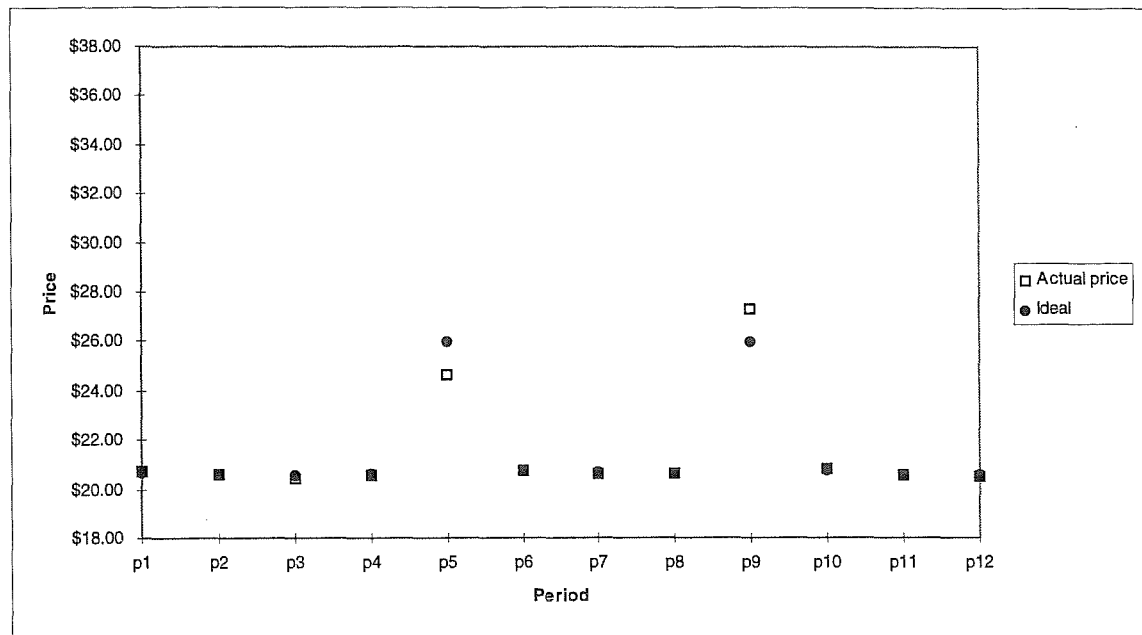


Figure 6-46: Day-ahead market South Island energy prices

The graphs below show the trajectories of the aggregated storage in each hydro group. The rolling horizon and day-ahead results are quite similar. In the Waikato system, the storage shows a cyclical pattern, dropping at each peak, and then recovering. The overall downward trend indicates that the water value at Lake Taupo is such that it is being allowed to run down over the day.

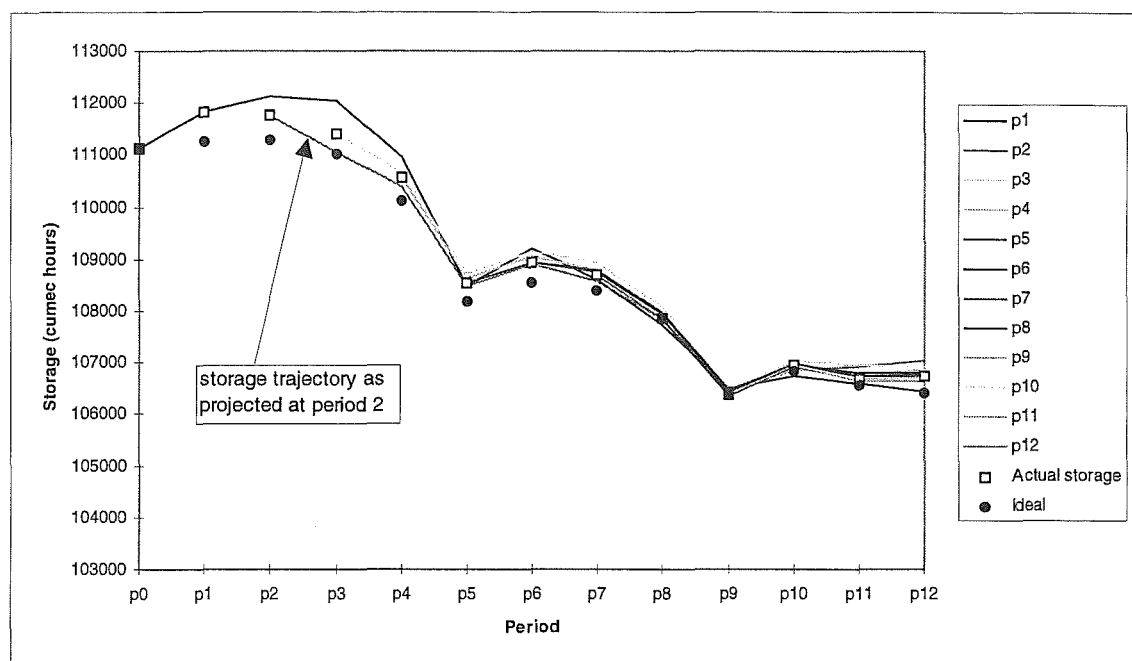


Figure 6-47: Periodic-update market aggregated storage profile for Waikato

The lines on the rolling horizon graphs represent the projected storage levels in various periods.

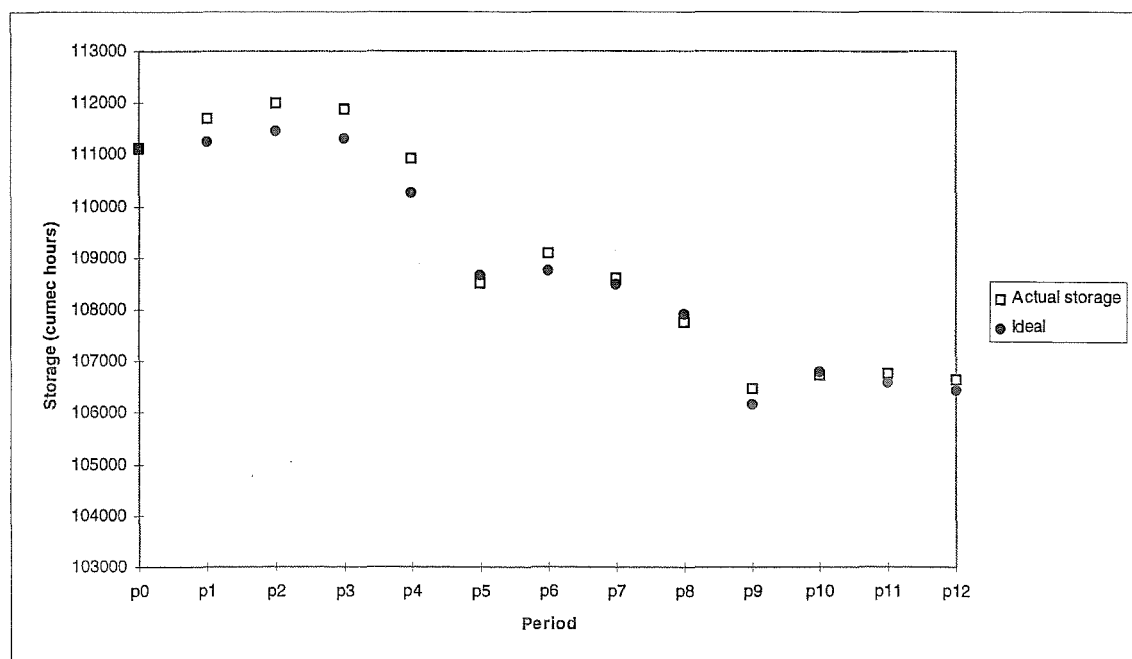


Figure 6-48: Day-ahead market aggregated storage profile for Waikato

The Waikato group storage is higher in the market solution than in the ideal solution, because of the truncation of generation in its initial offers, which results in the use of less water in these periods.

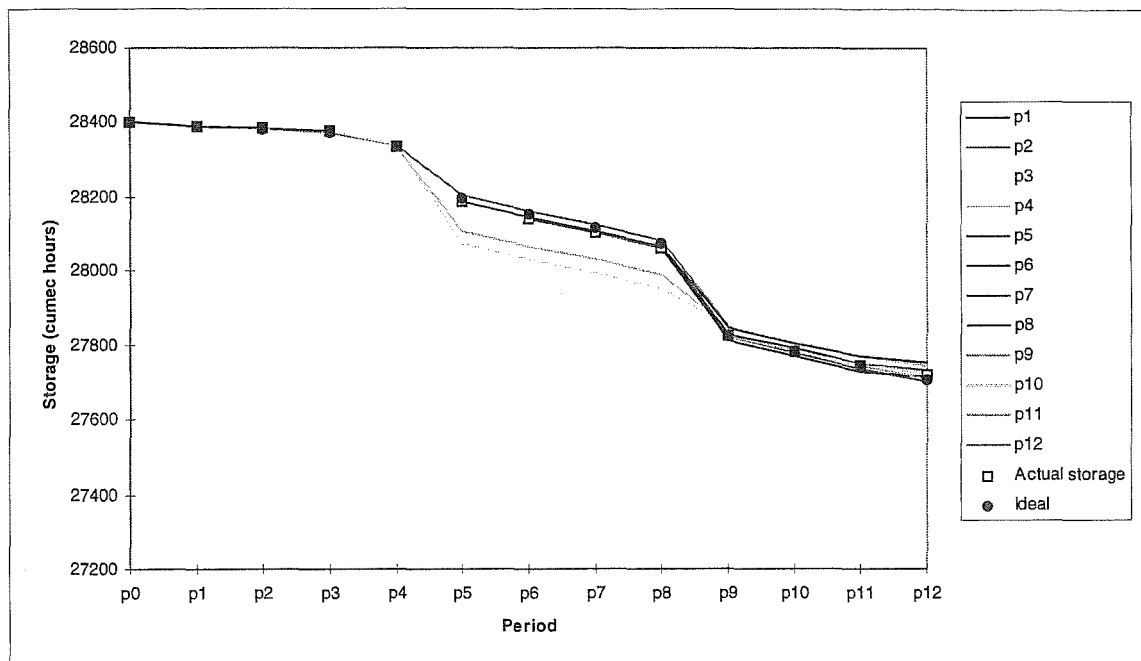


Figure 6-49: Periodic-update market aggregated storage profile for Waikaremoana

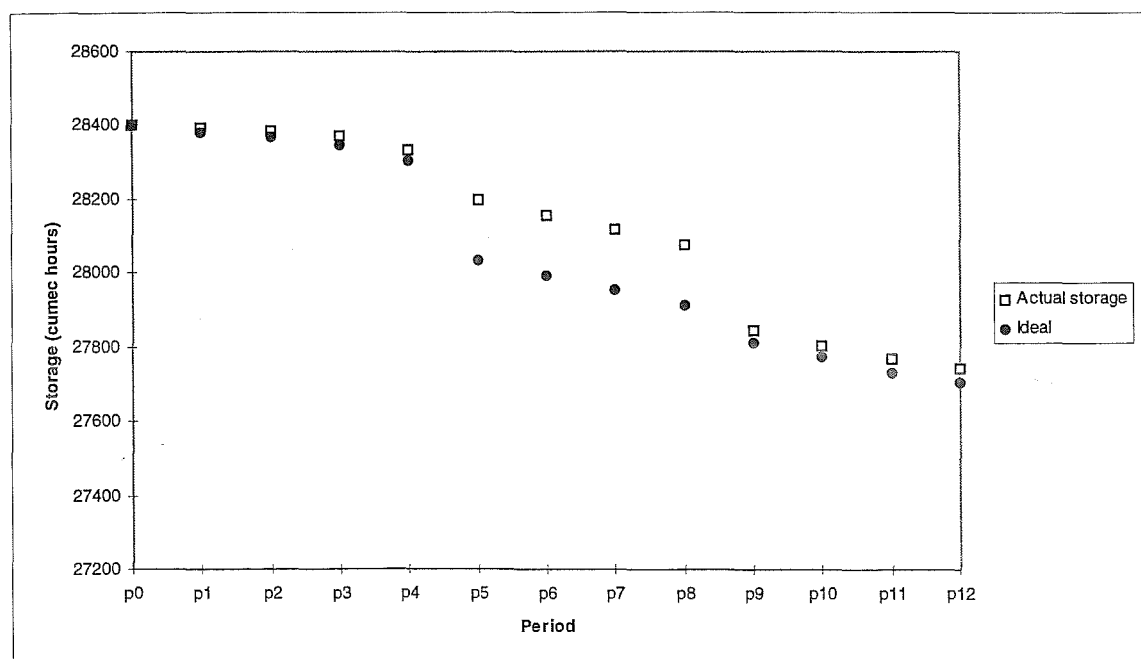


Figure 6-50: Day-ahead market aggregated storage profile for Waikaremoana

In the rolling horizon model, the Waikaremoana aggregated storage shows how the scenarios can swap generation between the peak periods. The lower lines represent periods in which the higher level of generation is expected in the first peak, hence there is less water in the middle of the day. The upper lines represent the periods in which more generation is expected at the second peak (including those that actually occur). Similar swapping is evident in the day-ahead market results.

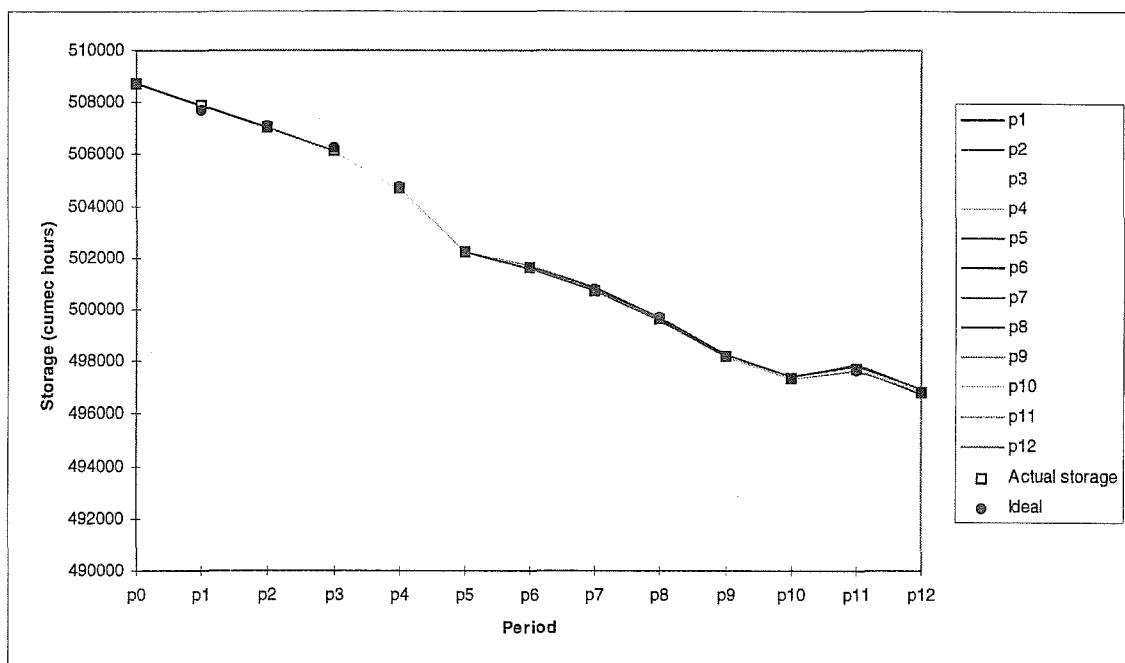


Figure 6-51: Periodic-update market aggregated storage profile for Waitaki

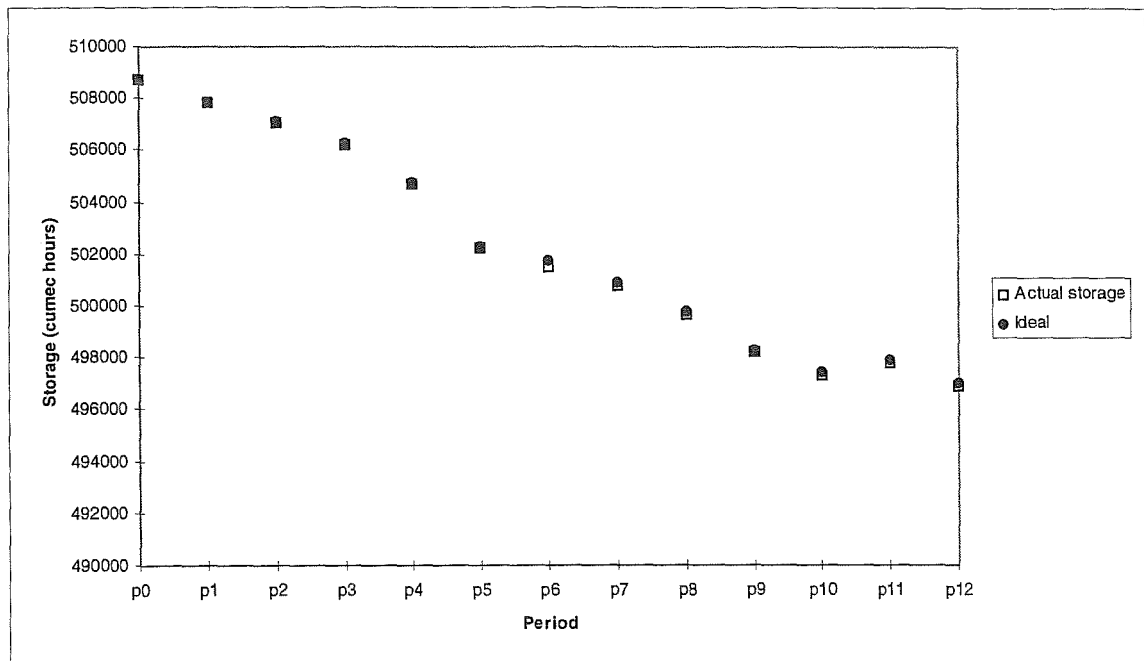


Figure 6-52: Day-ahead market aggregated storage profile for Waitaki

The storage in the Waitaki system exhibits a cyclic pattern, as storage dips and then recovers after the peak periods. The downward trend reflects the fact that the Pukaki reservoir is being run down over the day, since its water value is less than the system price.

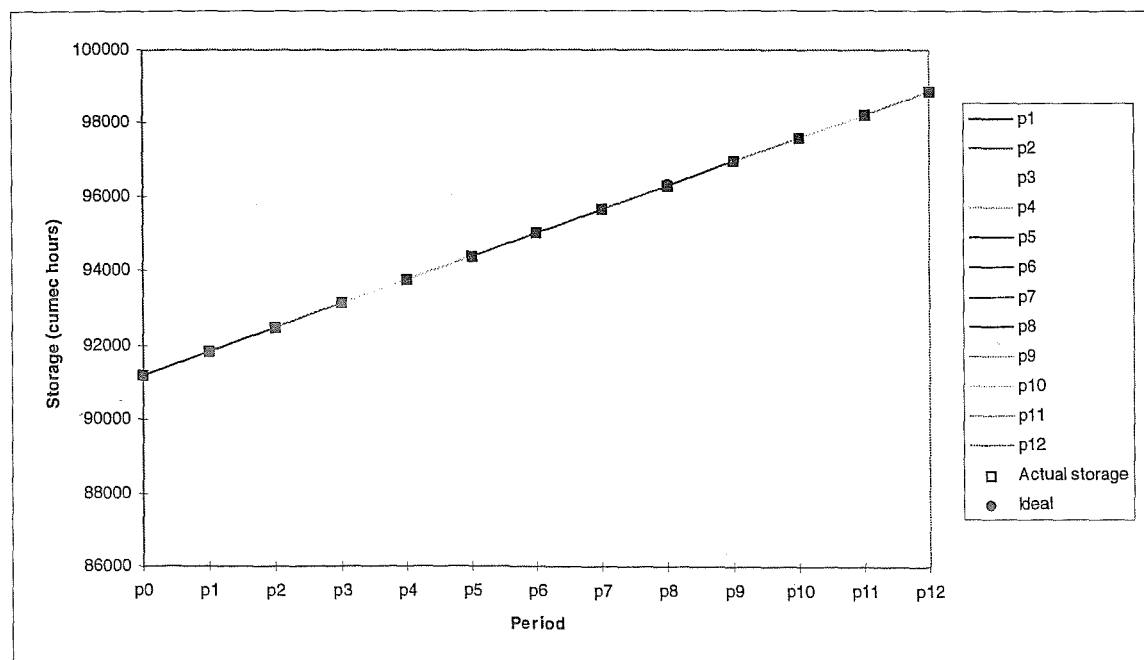


Figure 6-53: Periodic-update market aggregated storage profile for Manapouri

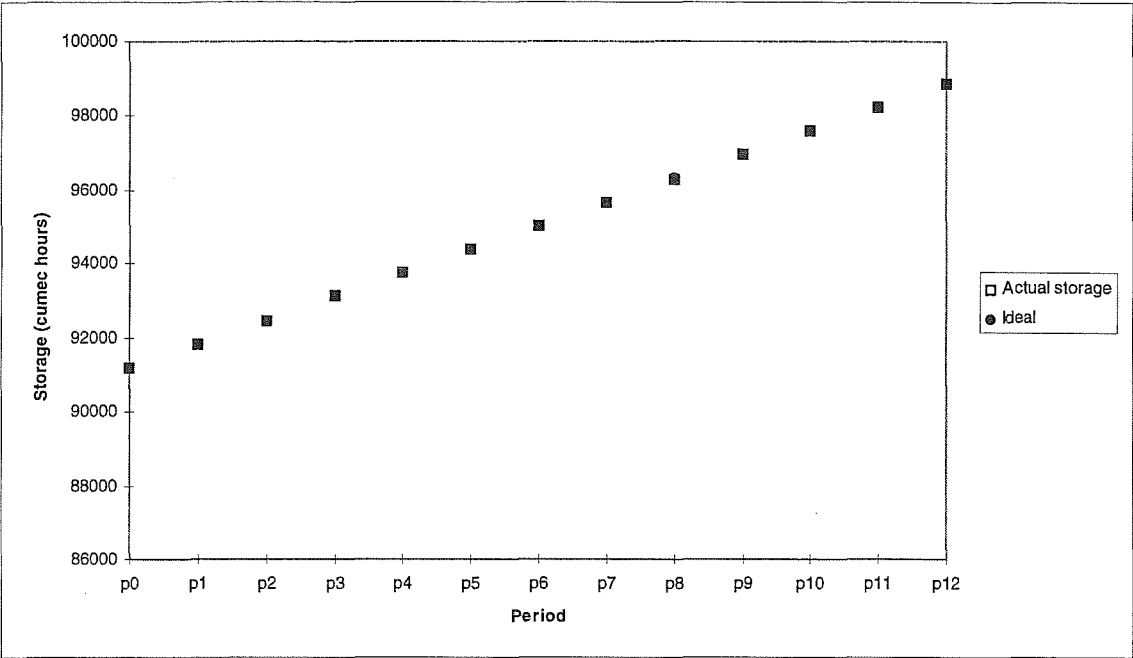


Figure 6-54: Day-ahead market aggregated storage profile for Manapouri

The Manapouri reservoir does not release all day due to the high water value assigned to it, and its storage level is increased by inflows.

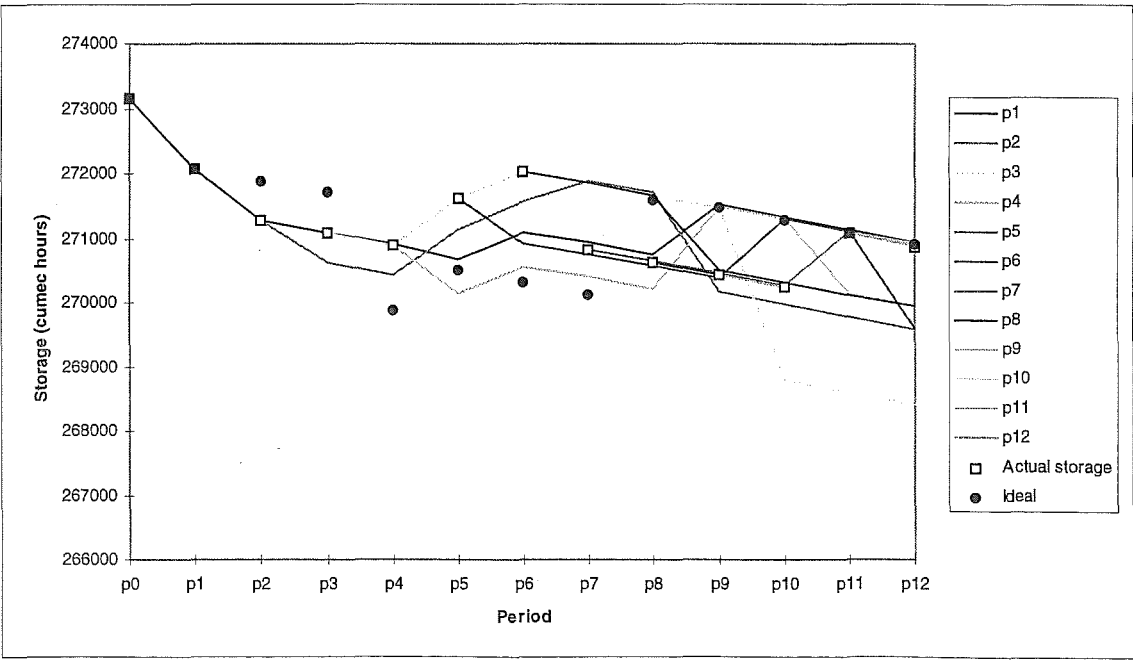


Figure 6-55: Periodic-update market aggregated storage profile for Clyde

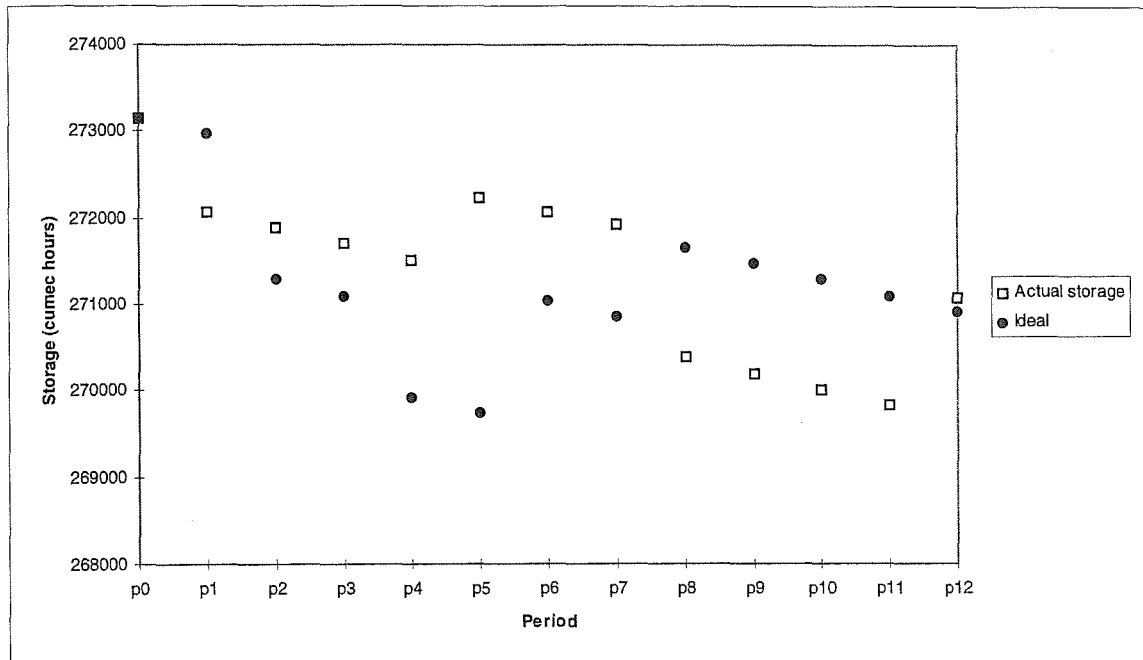


Figure 6-56: Day-ahead market aggregated storage profile for Clyde

The aggregated storage in the Clyde system is complicated by the long delay between the Hawea reservoir and the Clyde station's head-pond. Hence in the day-ahead model when the storage appears to decrease sharply in the early periods, and then rise again, it is simply that the releases in the early periods from the Hawea reservoir do not start arriving at the Clyde head-pond for 9 hours. The tangled web of storage projections in the rolling horizon model indicates the many alternative choices about when to release water from the Hawea reservoir, and the way in which changes to the perceived situation can cause the solution to switch releases between periods.

Because of the reduced uncertainty generators face when they make their offers several hours ahead of real-time, rather than the previous day:

- the offers can be targeted more closely around their preferred trajectory,
- the scale of non-monotonicities is reduced, meaning less approximation is required to make supply curve offers;
- hence the targets are generally closer to the ideal dispatch than the equivalent day-ahead targets, and so the cost distortion caused by the market process is lessened

The projected market clearing prices can vary considerably as the day progresses:

- price projections for peak periods are especially poor due to large price steps and the narrow offer bands;
- the market clearing price in the peak periods may not give a good indication of what the real-time price or ex post prices will be, since it is based purely upon the generator offers

The market models used less water than the ideal solution:

- conservative offers were used, which resolve non-monotonicities by truncating the higher level of generation back to the lower level of generation;
- hence overall, the market models were biased to offer less water than the ideal solution, resulting in higher thermal costs;
- however, this effect will be gradually counter balanced as the unused water will mean lower water values, resulting in higher releases (eventually the system will reach an equilibrium in which releases equal inflows, on average)

C.4.2 Sequential Days

A major focus of this research project is to produce estimates of the loss in productive efficiency of a simple market for generation, when compared to a centrally optimised system. The long-term average efficiency is the important issue in this respect, rather than the efficiency on any particular day. For this reason, it is important that the experiments are performed with representative days. Significant factors in this respect include the setting of appropriate boundary conditions, in terms of the beginning and ending storage and flows in transit.

The solutions found should also be sustainable in the longer term, so the trends in storage levels and the trends in price levels are also of interest. With this in mind, experiments have been performed which model several sequential days, in order to examine whether the solutions found represent equilibrium.

The rolling horizon modelling system was modified to enable it to simulate sequential days. This entailed ensuring all of the ending conditions of one day could be input as

the starting conditions for the next day. The last period storage level and flows in transit and releases from the previous day are recorded and used as input for the current day.

Three generator forecasts were used in these experiments:

- 1) All generators forecast demand accurately, i.e. use (+5%, 0%, -5%) scenarios.
- 2) All generators forecast a high demand day, i.e. use (0%, +5%, +8%) scenarios.
- 3) All generators forecast a low demand day, i.e. use (0%, -5%, -8%) scenarios.

Each of the three generators forecast situations, above, was run for 14 sequential days, using the same demand on each day. While it might seem unrealistic for a generator to continue to overestimate demand by 5% every day over a week or more, the intention is to assess if the solutions found in the first day are representative of an equilibrium position. The percentage cost distortions for each of the simulated days is shown in the table below.

<i>Day</i>	<i>All low</i>	<i>All accurate</i>	<i>All high</i>
<i>1</i>	0.20	0.14	0.25
<i>2</i>	0.14	0.20	0.22
<i>3</i>	0.07	0.14	0.21
<i>4</i>	0.08	0.10	0.12
<i>5</i>	0.07	0.09	0.15
<i>6</i>	0.06	0.15	0.17
<i>7</i>	0.02	0.10	0.19
<i>8</i>	0.03	0.11	0.17
<i>9</i>	0.02	0.07	0.18
<i>10</i>	0.27	0.22	0.18
<i>11</i>	0.78	0.84	0.94
<i>12</i>	0.27	0.39	0.48
<i>13</i>	0.26	0.41	0.55
<i>14</i>	0.25	0.42	0.45
<i>Average 0.05</i>	0.11	0.17	

Table 6-39: The periodic-update market: direct cost distortions (%)

After the first two days, the solutions seem to settle down to a kind of equilibrium, which is perhaps a little lower in cost distortion than the results from day 1. From day 10 the solution changes because the Manapouri reservoir reaches its upper bound, as shown in Figure 5.1. This occurs because of the limitations of the simple research model

used, which has only a single water value for all of the storage in a reservoir. A more sophisticated model would have several bands of storage with decreasing water values, and this would result in the model finding some equilibrium away from the storage bounds of the reservoir.

In light of this, it does not seem unreasonable to use the results from days three to nine to form estimates of the equilibrium average cost increase compared with an idealised centralised solution.

The average of these three results is 0.11% loss in productive efficiency, compared with an average of 0.14% for the results on day 1. Hence it appears that the initial distortion estimate may be slightly higher than for the equilibrium situation. Boundary Conditions

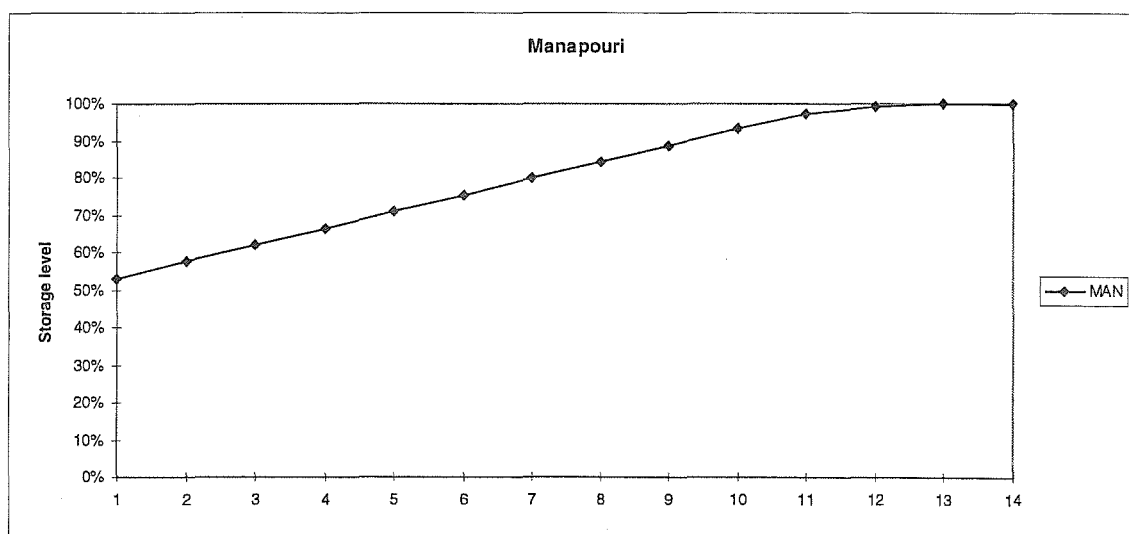


Figure 6-57: Boundary conditions for Manapouri

The Manapouri reservoir collects all of its inflows because of the higher water value assigned to it. Because the model used for this research has a single water value for all of the water in a reservoir, and this value was not changed as time went by, the reservoir filled up. A more sophisticated model would either use different water values for different bands of storage, or update the water value as it ran, and the model would find some equilibrium away from the reservoirs bounds.

The upper storage bound is reached in day 11, but the model results begin to be affected in day 10 because the rolling horizon model looks forward into the next day. Hence the results from day 10 onwards are inconsistent. The Manapouri station is forced to generate, or it must spill any new inflows. Yet its water value is still set at a high level, and so the calculated overall national cost rises as this 'expensive' generation displaces cheaper generation. However the prices actually fall, because the marginal costs of generation are reduced, as the other generators reduce their generation in response to the generation at Manapouri. Because of these inconsistencies, the experimental results from days 10 to 14 should be disregarded, but the prior results stand.

The graphs below show the starting levels for each day (of course, these are the ending levels from the previous day) for the headponds in each river system. This set of graphs is the set of results from the 'all accurate' run. The other two runs have similar results.

The storage levels are given as a percentage of the maximum storage, and include any flows in transit into that head-pond at the beginning of the day. Hence values above 100% imply that there are flows in transit, and that there will be releases from that head-pond in the initial periods of the day to cope with these flows.

Of primary interest here are any trends in the head-pond storage, as the storage reservoirs will be increasing or decreasing depending on the water value assigned to them. It might be expected that the head-pond storage to move down over several days, because the initial levels were as high as possible to provide maximum head operation under perfect foresight. In reality, allowance must be made for spill risk. Eventually, though, equilibrium must be found.

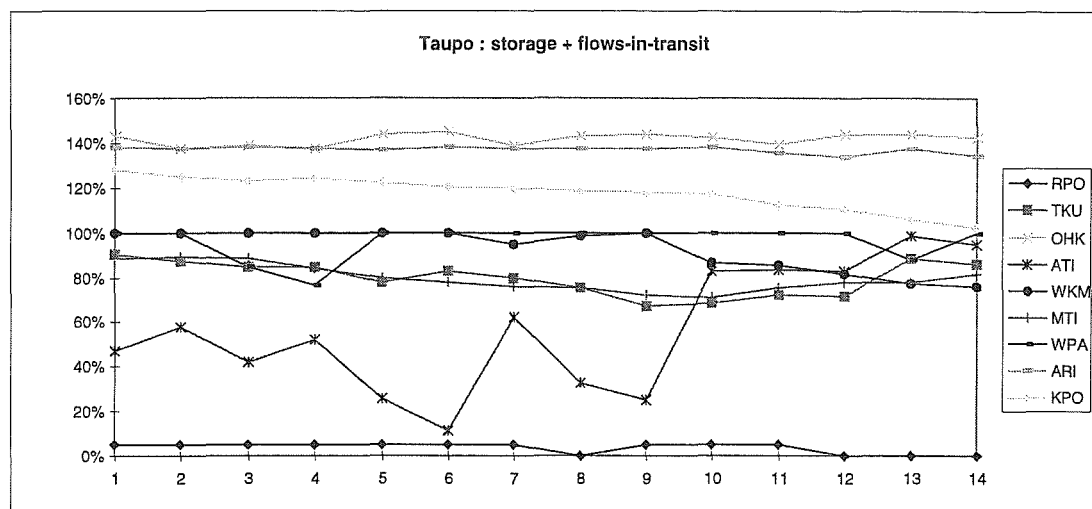


Figure 6-58: Boundary conditions for Taupo head-pond

Most of the head-ponds show no clear trend, but the Tokaanu, Atiamuri, Maraetai and Karapiro headponds display a slight decrease in combined storage and flows over the first nine days. This might increase their daily operating range, but it will also reduce the head.

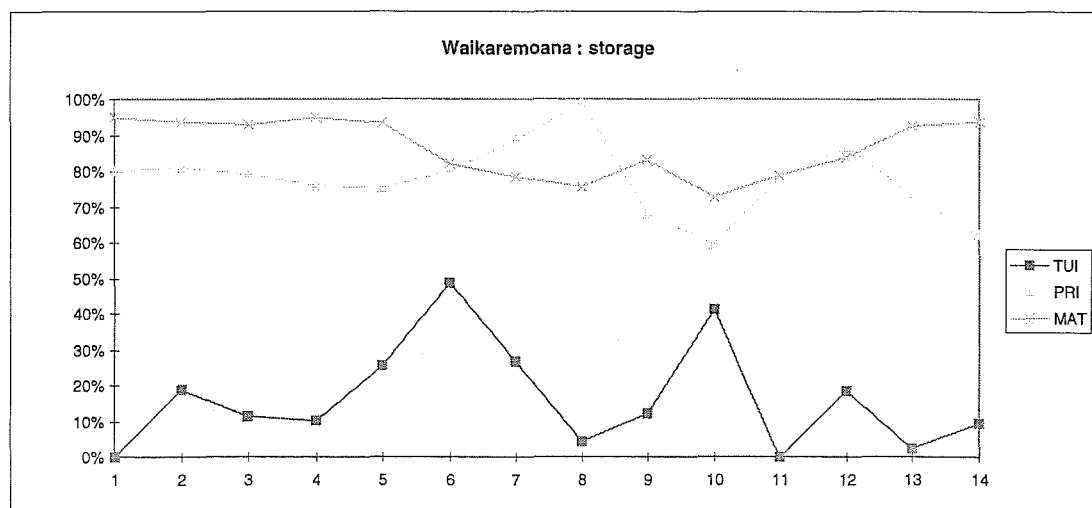


Figure 6-59: Boundary conditions for Waikaremoana

The Piripaua head-pond displays an increase over time, but this is unlikely to be significant because of its small size.

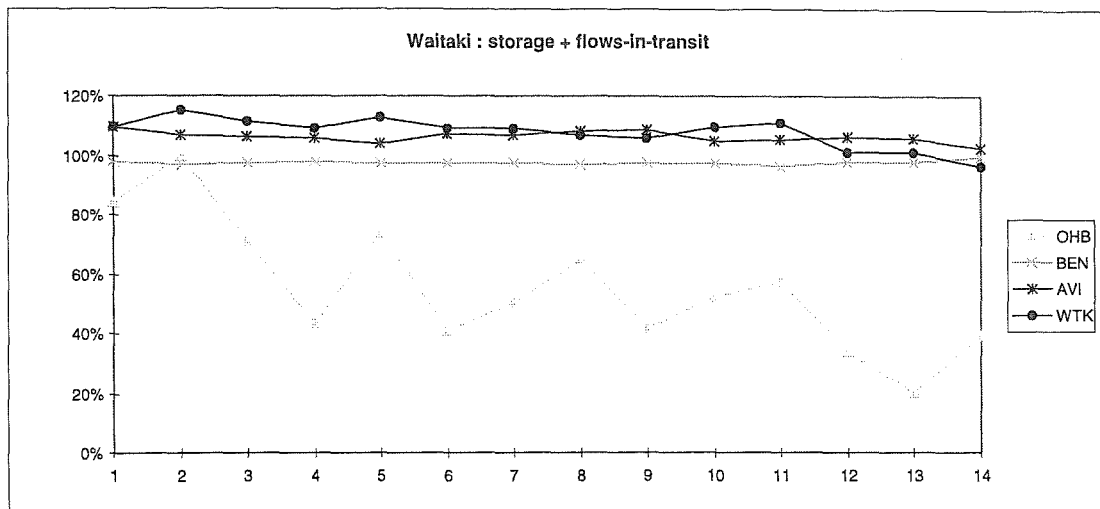


Figure 6-60: Boundary conditions for Waitaki

The Ohau B head-pond trends downward, and Ohau B operates over a wider range of storage levels in the later days. The Aviemore and Waitaki reservoirs' storage levels are out of phase, with an increase in storage at one resulting in a decrease in the other. The river chain is probably almost indifferent about whether the water is in one or the other of these reservoirs, or in transit between them, at the beginning of the day.

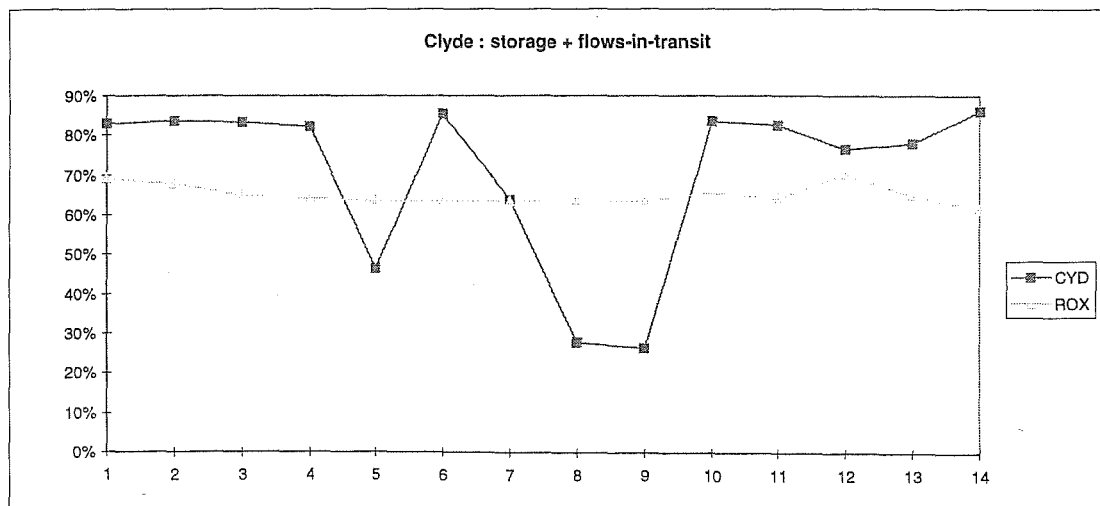


Figure 6-61: Boundary conditions for Clyde

The Roxburgh head-pond shows a slight decrease in storage initially, but then flattens out. The Clyde head-pond storage shows considerable variation, but this is probably due

to its dependence upon the exact timing of releases from the Hawea reservoir. See the Appendix for a more detailed record of the storage behaviour.

The graphs below show the price profiles in each island for each of the three model runs. They show quite stable behaviour, particularly between day 4 and day 10, when the effects of the Manapouri reservoir filling up are felt. Because the increased Manapouri generation causes the other generators to back off, they can offer cheaper marginal generation, and lower price result.

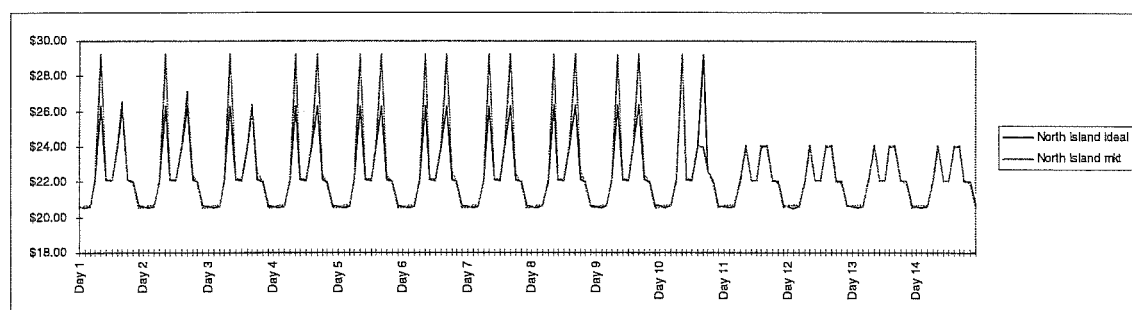


Figure 6-62: All accurate North Island price profiles

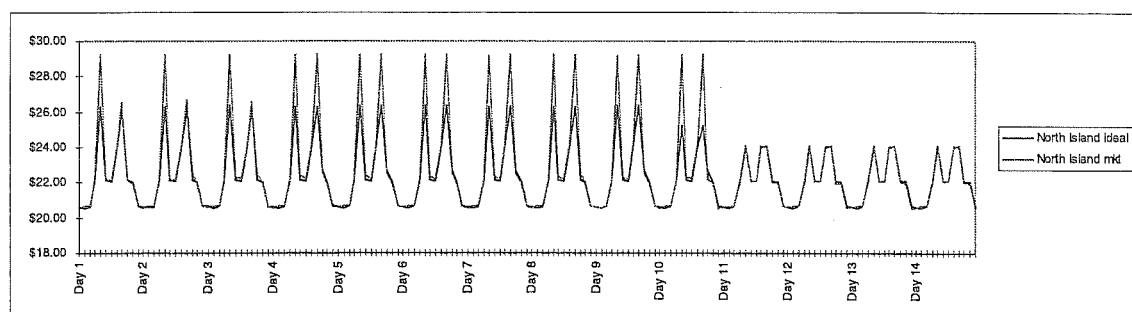


Figure 6-63: All high North Island price profiles

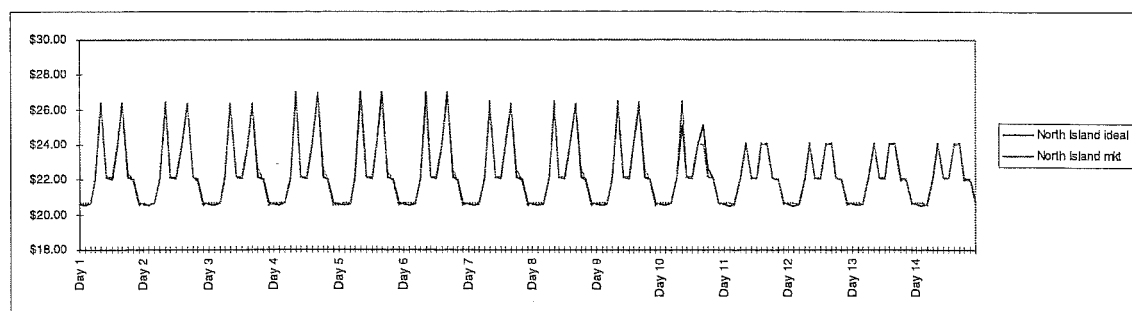


Figure 6-64: All low North Island price profiles

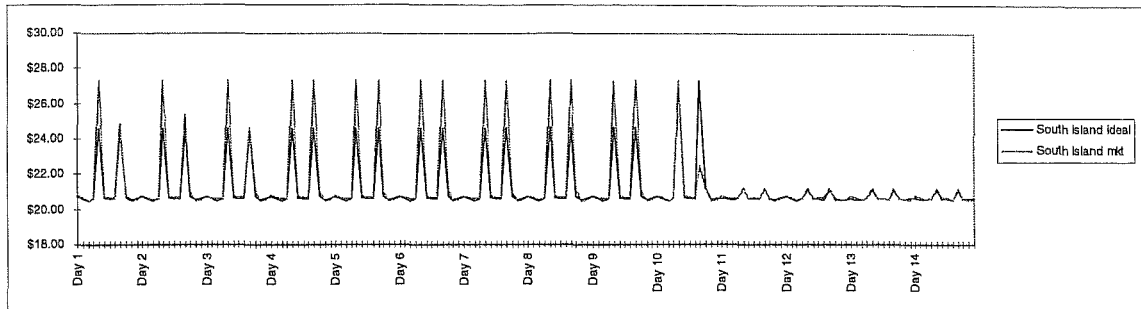


Figure 6-65: All accurate South Island price profiles

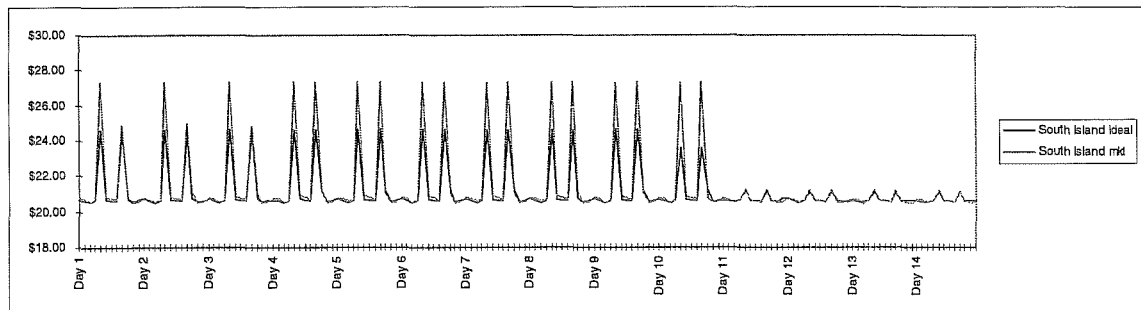


Figure 6-66: All high South Island price profiles

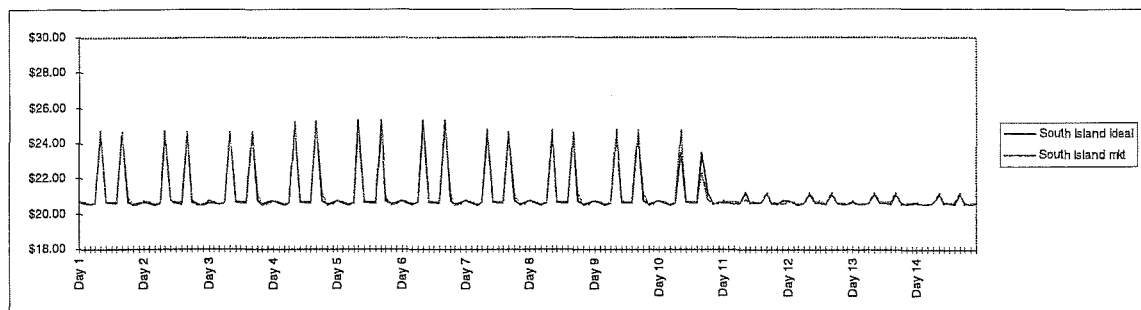


Figure 6-67: All low South Island price profiles

The next set of graphs shows the generation profiles of each generation group for the 'all accurate' model run. These also show stable behaviour until day 10, when Manapouri starts generating. The other generators, especially Waitaki, reduce their generation in response. Also note the discrepancy between the market and ideal generation at the beginning of each day in the Taupo system, due to the non-monotonicities in its offer, and the corresponding discrepancy in the thermal generation.

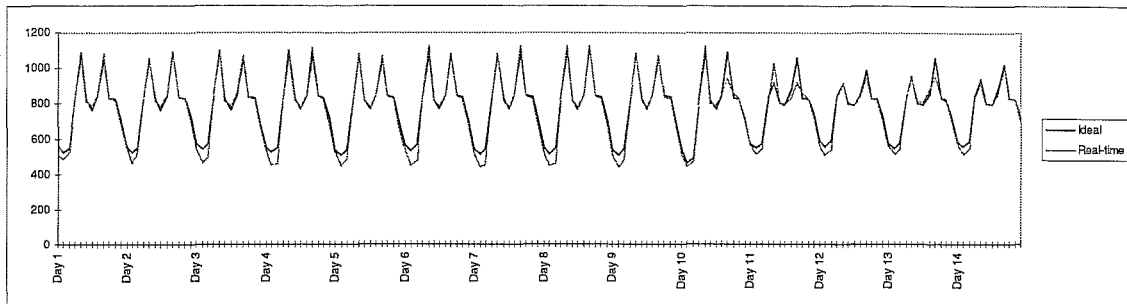


Figure 6-68: Energy output profile for Waikato

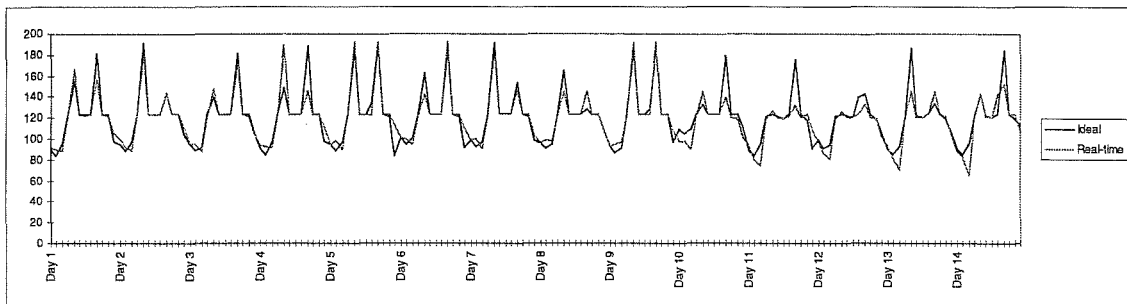


Figure 6-69: Energy output profile for Waikaremoana

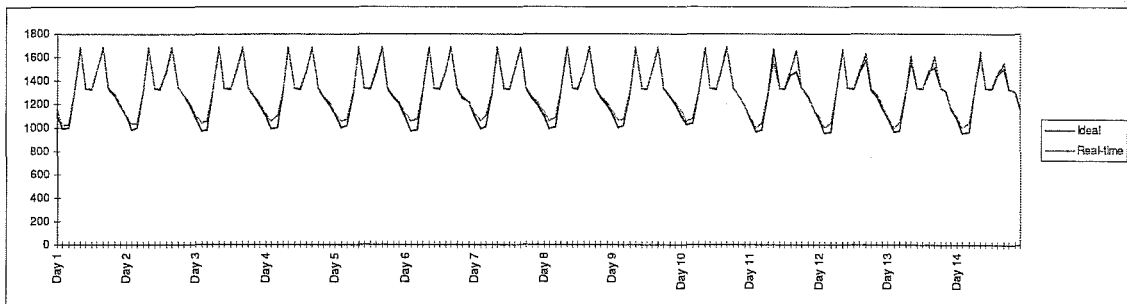


Figure 6-70: Energy output profile for Thermal

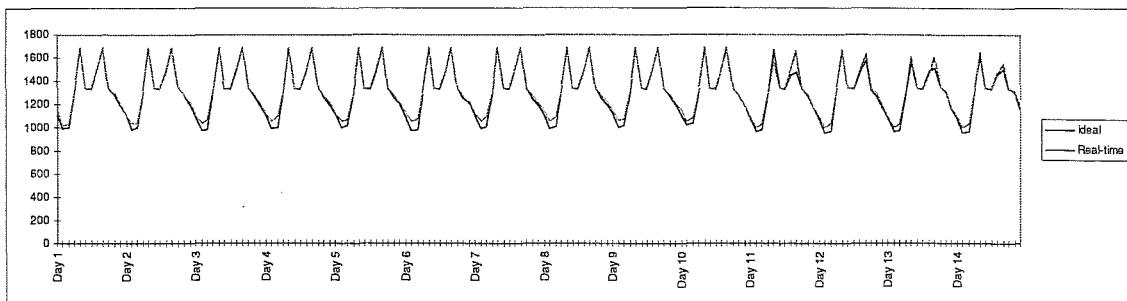


Figure 6-71: Energy output profile for Waitaki

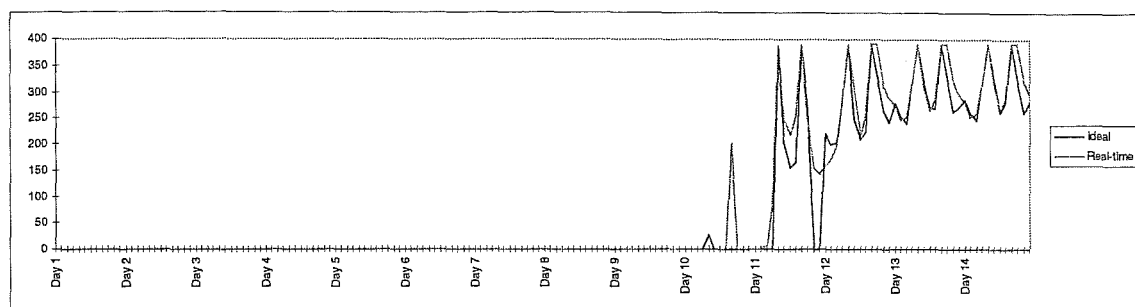


Figure 6-72: Energy output profile for Manapouri

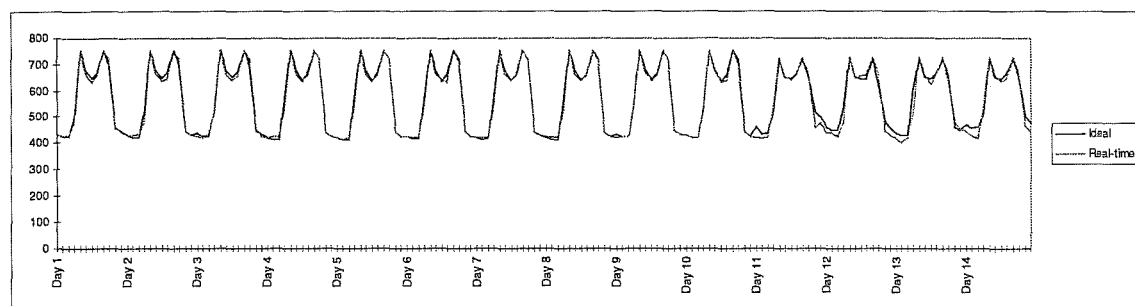


Figure 6-73: Energy output profile for Clyde

The results of these experiments seem generally to be fairly stable until day 10 when the Manapouri reservoir fills up. The boundary conditions adjust slightly in the first few days, so that by about day 3 equilibrium is set up which produces slightly lower cost distortions than the day 1 results previously reported.

These estimates are optimistic in the sense that the real generation system will not be at equilibrium, but will experience varying demands and inflows. The flows in transit and storage, which are optimal for the end of one day, may not be good starting points for the following day. However, this effects both the centralised and decentralised solutions. These estimates are pessimistic in that a simple autonomous process creates the offers. In a real system, the generating company would react in a more intelligent manner, and such a long sequence of over/under estimates is unlikely in practice.

For future experiments using the boundary conditions from the first day seems adequate.

C.5 The Impact of Forecast Uncertainty

The results of experiments like those reported in so far may be expected to exhibit a degree of random variation, or ‘noise’. The results of individual experiments will not always conform precisely to expectations because they are drawn from a distribution of all possible results. In a similar way to taking a sample of observations from some physical experiment, these computational experiments could be expected to occasionally produce results that are not really representative. Further, differences between the results of individual experiments may not be significant for this reason.

One of the reasons that such behaviour takes place in these experiments is that there are mathematically alternative optima. These occur when an optimisation problem, such as that solved to find the least cost dispatch of generation from an electricity system, has several solutions with the same cost but with different settings of the decision variables.

For example, the linear programs used in this thesis might propose that some hydro station generate at a low level in one period and a high level in the next, while another station generates at a high level in the first period and a low level in the next. It may be the case that the generation profiles could be exchanged to some extent without altering the overall cost, i.e. the first station generates high then low and the second generates low then high. These two distinct solutions could have approximately the same total cost and hence both are optimal, as would any linear combination of the two.

In two places, such alternative optima may cause variation in the cost penalties reported:

- the market clearing phase in which there are several alternative optimal sets of output targets;
- and the centralised optimisation when it is used to form generating company market forecasts for offer creation

Investigation of the former is reported later in the sensitivity analysis. Alternative optima in the offer formation process is dealt with by introducing a moderation factor—a slight penalty on changing the level of generation at a station relative to system load.

Additional variation in the solutions may be introduced by sensitivity of the model to its input data. Later sections of this appendix investigate the sensitivity of the model's results to random changes in tributary and demand forecasts as well as storage policies.

C.5.1 Alternative Optima in Market Clearing

When more than one generator offers the same price, alternative optima can occur in the market-clearing phase. This may happen if a tie occurs at the market clearing price, then all of the generators offering this price will be on the margin. The market solution will be indifferent about which of the offers is taken up and the decision made within the model is arbitrary. This is the reason behind the strategies for forming multi-band fixed offers, to guide the market's solution toward more moderate decisions by adding intermediate price bands. Consequently, different targets can result from random optimal selections of the same set of offers, each distinct sets of targets produces different decentralised dispatches and hence cost distortions.

Experiments in this section test the scale of this variation with experiments that slightly perturb the prices offered by different generators, to encourage the market to consistently favour one generator. This situation is likely to be observed in a real market situation too, as some generators offer slightly lower prices than others because of marginally different short-term forecasts and/or long-term strategies. This can result in cases in which one generator's offers are accepted and another's are declined even though the difference in offer price is minute.

One strategy to remedy this situation in a real market is to limit the precision to which offer prices may be specified, then some rule could cover situations where two or more generators offer at the same price. For example, each offer could be partially accepted in proportion to its size.

The following sections examine the impact of alternative optima on multi-band fixed offers and multi-band sculpted offers, ignoring this latter issue, and focusing on the range of variation in the results reported earlier.

The set of experiments detailed in this section uses multi-band fixed offers. It is likely the results for the three-band sculpted offers will be more stable when using narrow width central bands because, in these cases the market does not have much leeway when making decisions.

The five hydro groups are ranked in a variety of merit orders:

- 1) North-South order: Taupo, Waikaremoana, Clyde, Manapouri, Waitaki.
- 2) A South-North order: Waitaki, Manapouri, Clyde, Waikaremoana, Taupo.
- 3) A small to large order: Waikaremoana, Manapouri, Clyde, Waitaki, Taupo.
- 4) A large to small order: Taupo, Waitaki, Clyde, Manapouri, Waikaremoana.

For each merit order, the prices offered are perturbed slightly so that the market will prefer generators lower in the merit order. For example, the first merit order has all offers from Taupo decreased in price by 2 cents per MW, Waikaremoana decreased by 1 cent per MW, Clyde at its normal price, Manapouri increased by 1 cent per MW and Waitaki increased by 2 cents per MW. Note that these merit orders represent extreme points and hence might be expected to show the limit of the range of variation. Arbitrary choices between marginal offers might be expected to show less variation. Hence these results probably represent a worst case. However, this may not be unrealistic since offer prices are likely to be subtly different due to different perspective and strategies.

The results in Table 6-40 to Table 6-43 show significant variation. For example the cost distortions for the \$148.00 per MW penalty rate and the \$(0,0.6)\$ parameter setting are shown below—this parameter setting shows the lowest cost distortions reported earlier. The last column in these tables gives the cost distortion for those earlier experiments. This set of experiments is a similar to, but larger than those performed here except that the choice between the marginal offers is arbitrary rather than driven by the merit order cost perturbations described above.

<i>Forecast</i>	<i>Grouping</i>				
	<i>SI low</i>	<i>NI low</i>	<i>Small low</i>	<i>Large low</i>	<i>Previous</i>
<i>Over</i>	4.0	1.4	3.4	2.1	2.8
<i>Accurate</i>	2.2	0.7	1.5	1.8	1.8
<i>Under</i>	1.3	3.3	3.2	1.6	1.6
<i>Average</i>	2.5	1.8	2.7	2.1	2.1

Table 6-40: Multi-band fixed offers: direct cost distortion (%)

The range of results from these experiments is a 1.8%–2.7% cost distortion. The previously reported value of 2.1% falls within this range. Note that the methods used in the previous experiments could have produced any of the results in the bottom row, simply as a result of the way in which the market set the targets. It might be expected, on average, the cost distortion would fall in the middle of the range. However, this is not necessarily true, as the extreme points tested above may all be worse than a more moderate solution.

The range of average results for \$18.00 per MW and \$1.12 per MW penalty rates are 1.1%–2.2% and 0.04%–0.07% respectively.

<i>Forecast</i>	<i>Grouping</i>				
	<i>SI low</i>	<i>NI low</i>	<i>Small low</i>	<i>Large low</i>	<i>Previous</i>
<i>Over</i>	4.0	1.4	3.4	2.1	2.8
<i>Accurate</i>	2.2	0.7	1.5	1.8	1.8
<i>Under</i>	2.9	5.9	3.4	4.5	7.9
<i>Average</i>	3.0	2.7	2.8	2.8	4.2

Table 6-41: Multi-band fixed offers: transaction cost inclusive distortion (%)

The variation in total costs seen in these experiments is less than that for the direct costs, with a range of 2.7–3.0%. This is due to the model optimising the total cost, including penalties—one would expect these values to remain quite stable. The direct costs vary more because the model can trade off between a low direct cost but high penalty payments as generators default on targets, or a higher direct cost but lower penalties as generators meet the targets.

Surprisingly, the value previously reported, of 4.2% cost distortion, lies outside the range of results from this experiment. This is a consequence of the one extreme result of 7.9% total cost distortion when all generators underestimate the demand. It could be

concluded that the previous result is an outlier and a better estimate of the increase in total costs over the centralised solution would be 2.7–3.0%. The range of average results for \$18.00 per MW and \$1.12 per MW penalty rates are 1.6–2.5% and 0.29–0.33% respectively.

It is very difficult to estimate standard deviations from the data reported here, because the experiments deliberately set out to produce extreme results, rather than a really representative sample. Hence, any confidence interval based upon sample standard deviations is likely to be too wide. A very rough, rule-of-thumb estimate of the range of uncertainty in the cost results for multi-band fixed offers in this report might be +/- 1.0%.

This level of variation does mean that differences apparent in individual experiments may not prove to be very significant. However, the large number of experiments performed means that the overall conclusions reached still appear to be valid after this type of error has been accounted for. Each experiment performed involves several runs of the market clearing model and this helps to reduce the variation in the averages calculated, also, the distributions of the cost and defaults results appear symmetrical.

One might conclude that fixed offers are highly sensitive to perturbation in offer prices. A facet that makes them undesirable both in terms of average cost distortion and the range of such distortions.

The following reports the results of two sets of experiments, with different constraints on ending storage and flows in transit:

- 1) Beginning and ending storage are set to the same values as the centrally coordinated model, for reservoirs without an assigned water value. End of day flows-in-transit are looped by constraining them to be the same as the flows-in-transit at the beginning of the day but this level is allowed to be optimised in the market dispatch. The experiments use factors of \$8.00 and \$300.00 to moderate the centralised dispatch.
- 2) The sum of end of period storage and flows-in-transit for each reservoir in the de-centralised dispatch is constrained to be the same as the sum of these

quantities in the centrally coordinated dispatch. This is a more realistic assumption than the first model that allows flows-in-transit to be optimised in the de-centralised model. A factor of \$300.00 is used to moderate the centralised dispatch.

It is somewhat unclear exactly what level of moderation factor should be used. If set very low it may not have enough stabilising effect on the solutions, but, if set high, even without altering the cost of the solution, it might distort the prices. The values of \$300.00 and \$8.00 are selected for these experiments after some preliminary trials, with \$300.00 being the highest the model can tolerate before the cost of the dispatch is affected and \$8.00 being small enough to have some moderating effect without distorting prices.

These experiments are similar to those shown earlier for multi-scenario multi-band sculpted offers. Of five possible demand scenarios, each generator chooses three to form their offer. Five different forecast scenarios have different generators forecasting high, low, or accurate demand scenarios. The results in Table 6-42 to Table 6-49 show the averages across these five forecast scenarios.

The extrapolation parameters used are 0.2% and 2.0% of the maximum possible generation, as these are the best settings found earlier. The set of penalty rates trialed is \$148.00, \$37.00, \$18.00 and \$4.50.

Table 6-42 to Table 6-45 shows the results for the first case with a moderation factor of \$300.00 in the top half of each table and a moderation factor of \$8.00 below. The results show a significant reduction in cost distortion compared with the earlier model. The best results for the latter, at a high penalty rate are a 0.6% rise in direct costs, with no penalty payments. A similar pattern to earlier experiments is evident, with the distortions remaining approximately constant until the penalty rate falls below \$18 per MW and offers with minimal extrapolation outside the range of the scenarios performing best.

<i>(Moderation, Width)</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$4.50</i>
<i>(\$300.00, 0.002)</i>	0.27	0.27	0.26	0.04
<i>(\$300.00, 0.02)</i>	0.44	0.44	0.33	0.06
<i>(\$8.00, 0.002)</i>	0.32	0.32	0.32	0.05
<i>(\$8.00, 0.02)</i>	0.45	0.45	0.42	0.08

Table 6-42: Multi-band sculpted offers with three scenarios: direct cost distortion (%)

<i>(Moderation, Width)</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$4.50</i>
<i>(\$300.00, 0.002)</i>	0.27	0.27	0.26	0.12
<i>(\$300.00, 0.02)</i>	0.44	0.44	0.41	0.18
<i>(\$8.00, 0.002)</i>	0.32	0.32	0.32	0.15
<i>(\$8.00, 0.02)</i>	0.45	0.45	0.44	0.21

Table 6-43: Multi-band sculpted offers with three scenarios: transaction cost inclusive distortion (%)

<i>(Moderation, Width)</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$4.50</i>
<i>(\$300.00, 0.002)</i>	0.0	0.0	6.0	197.0
<i>(\$300.00, 0.02)</i>	0.0	0.0	50.0	296.0
<i>(\$8.00, 0.002)</i>	0.0	0.0	0.0	256.0
<i>(\$8.00, 0.02)</i>	0.0	0.0	12.0	328.0

Table 6-44: Multi-band sculpted offers with three scenarios: target defaults (MWh)

In both cases, the target defaults are zero for high penalties and a small extrapolation width. For \$4.50 penalties, the quantity of target defaults show a slight improvement over the results found earlier.

<i>(Moderation, Width)</i>	<i>Penalty</i>			
	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$4.50</i>
<i>(\$300.00, 0.002)</i>	-12.0	-12.0	-12.0	-29.0
<i>(\$300.00, 0.02)</i>	28.0	28.0	-1.0	-25.0
<i>(\$8.00, 0.002)</i>	-39.0	-39.0	-39.0	-49.0
<i>(\$8.00, 0.02)</i>	0.0	0.0	-7.0	-36.0

Table 6-45: Multi-band sculpted offers with three scenarios: increase in spills (MWh)

The centralised solution results in 206.0 MW of spill occurring on the Waikato chain with spill past Whakamaru down to Maraetai before and during the second peak of the day. The average spill in the market dispatch is less than the centrally coordinated dispatch in many cases because the sum of the targets set in market clearing for the Wai-

kato chain is often slightly below the sum of the centrally coordinated responses hence, the river chain is not driven quite so hard over the day and less spill is needed.

Experiments with a more realistic set of boundary conditions, i.e. end of day storage aggregated with flows-in-transit and constrained, shown in Table 6-46 to Table 6-49, resulted in slightly higher cost distortions.

	<i>Penalty</i>			
<i>(Moderation, Width)</i>	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$4.50</i>
<i>(\$300.00, 0.002)</i>	0.30	0.30	0.28	0.06
<i>(\$300.00, 0.02)</i>	0.47	0.47	0.47	0.10

Table 6-46: Multi-band sculpted offers with three scenarios: direct cost distortion (%)

	<i>Penalty</i>			
<i>(Moderation, Width)</i>	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$4.50</i>
<i>(\$300.00, 0.002)</i>	0.30	0.03	0.28	0.09
<i>(\$300.00, 0.02)</i>	0.47	0.47	0.47	0.15

Table 6-47: Multi-band sculpted offers with three scenarios: transaction cost inclusive distortion (%)

Target defaults and spill are also generally slightly worse than for the looped and optimised end conditions.

	<i>Penalty</i>			
<i>(Moderation, Width)</i>	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$4.50</i>
<i>(\$300.00, 0.002)</i>	0.0	0.0	0.5	267.0
<i>(\$300.00, 0.02)</i>	0.0	0.0	0.0	373.0

Table 6-48: Multi-band sculpted offers with three scenarios: target defaults (MWh)

	<i>Penalty</i>			
<i>(Moderation, Width)</i>	<i>\$148.00</i>	<i>\$37.00</i>	<i>\$18.00</i>	<i>\$4.50</i>
<i>(\$300.00, 0.002)</i>	9.0	9.0	9.0	-2.0
<i>(\$300.00, 0.02)</i>	51.0	51.0	51.0	10.0

Table 6-49: Multi-band sculpted offers with three scenarios: increase in spills (MWh)

From the results above it appears that the occurrence of alternative optima in the unmoderated models causes the model to over estimate the minimum cost distortion. This is probably because the unmoderated models tend to produce extreme solutions in both the offer formation and the market-clearing phase. The more moderate solutions produced by the new model have a lower mean as well as variation of their cost distortion. A more accurate estimate of the minimum achievable distortion might be 0.25–0.35%

increase in direct fuel costs, with virtually no defaults on targets by generators under a high penalty rate.

C.5.2 Demand Forecast Sensitivity

In all the experiments with multiple scenario offers performed to date, the generators form their offers using three out of a set of five scenarios. Although some account is taken of generators' uncertainty by varying the demand forecasts, given that they also used the two scenarios on either side of their expected load profile, one of these scenarios is always the day that actually happens. Hence, the results of these experiments may be optimistic, since in reality, generators will not guess the exact form of the following day's demand.

To validate the results presented thus far one might hypothesise that if generators include the actual day inside the 'envelope' of forecast days it is possible to replicate cost distortions as low as if generators exactly forecast the actual day in one of their scenarios.

The scenarios used earlier are formed simply by scaling the whole day's demand up or down by a percentage, whereas there may also be uncertainty about the shape of the load profile. For example, peak periods might be earlier or later than expected or last for longer or be more or less extreme. To test the effects of generators' uncertainty about the form of the load profile, this set of experiments uses demand profiles for the market clearing and market dispatch phases different from those in the simulations used to form offers.

Two types of discrepancy between the demand profiles the generators expect and those that actually occur are trialed. They are chosen for simplicity and ease of implementation in the experiments and do not necessarily represent the range of misforecasting actually experienced.

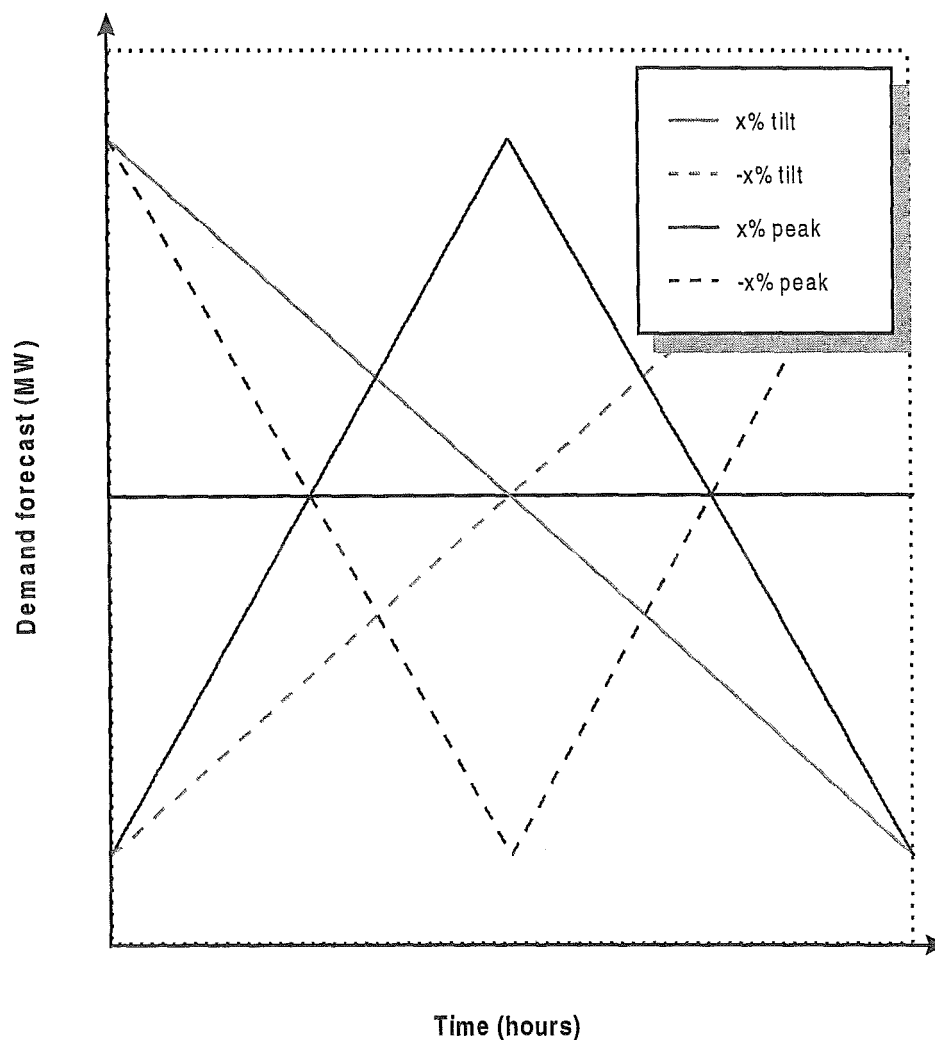


Figure 6-74: Deviation of demand from generators' expectation

Figure 6-74 illustrates the two kinds of differences between generators' expectations and the real load with the central line in the diagram representing generators' expectations about the form of the demand—they scale this line up or down to give their three scenarios:

- 1) The 'tilted' demand profile represents a day in which the load is initially higher than generators expect but deviations decrease as the day progresses until by the end of the day it is less than generators expect. With a negative

tilt, the converse is true—the demand begins the day lower and finishes higher than expected.

- 2) The 'peaked' demand profile is used to make peak/off-peak differences greater or smaller. A positive value of x gives the valley profile seen above, relative to the base demand day. This is used to generate a profile in which the demand is more evenly spread across the day. A negative value of x gives a more peaked profile, and represents a day in which the demand for the day is concentrated more in the central periods than generators expect. This has the effect of making the peak periods more extreme, and the off-peak periods even lower in load.

A set of experiments is presented for both types of demand deviation for deviations of +12.0%, +4.0%, -4.0% and -12.0%. Each set is carried out using a variety of extrapolation widths for the offers and penalty rates. They also involve different groups of generators forecasting high, medium or low demand days using their expectations of the form of the demand that differs from the actual demand profile that occurred.

The figures reported in Table 6-50 to Table 6-53 are the average results across the five forecast scenarios. As before these are:

- 1) All accurate
- 2) All high
- 3) All low
- 4) Waikato and Waitaki high with the rest low
- 5) Waikato and Waitaki low with the rest high

Consistent with the hypothesis, the experiments where generators correctly forecast the shape of the demand profile, though not necessarily its scale, yield lower costs than situations where generators mis-forecast the shape of the demand profile across the day.

(Penalty, Width)	Tilt				
	-12.0%	-4.0%	0.0%	4.0%	12.0%
(\$148.00, 0.002)	1.54	0.43	0.30	0.38	1.03
(\$148.00, 0.02)	1.38	0.50	0.47	0.45	0.80
(\$18.00, 0.002)	1.09	0.43	0.28	0.35	0.88
(\$18.00, 0.02)	0.95	0.50	0.47	0.45	0.69
(\$4.50, 0.002)	0.19	0.08	0.06	0.13	0.34
(\$4.50, 0.02)	0.19	0.12	0.10	0.14	0.34

Table 6-50: Multi-band sculpted offers with three scenarios: direct cost distortion (%)

In previous experiments the lowest cost results when using a very narrow extrapolation parameter. This is not surprising, since the actual demand is always one of the scenarios chosen by the generator hence, any widening of the offer bands beyond the scenarios will only allow the market room to push the generator away from their preferred trajectory.

In contrast, the above results show the wider extrapolation parameter, with offers which extend 2% of the maximum generation beyond the market simulation observations, yielding lower costs for the +12% and -12% tilts. This indicates that for situations in which generators' forecasts deviate significantly from the load which actually occurs, some extrapolation beyond the points resulting from the market simulations may be desirable. This is probably an even better solution, in cases with high uncertainty, will be to use a wider range of scenarios in the offer formation process.

(Penalty, Width)	Tilt				
	-12.0%	-4.0%	0.0%	4.0%	12.0%
(\$148.00, 0.002)	1.58	0.43	0.30	0.38	1.03
(\$148.00, 0.02)	1.38	0.50	0.47	0.45	0.80
(\$18.00, 0.002)	1.10	0.43	0.28	0.35	0.97
(\$18.00, 0.02)	0.95	0.50	0.47	0.45	0.77
(\$4.50, 0.002)	0.23	0.10	0.09	0.16	0.54
(\$4.50, 0.02\$)	0.23	0.16	0.15	0.19	0.48

Table 6-51: Multi-band sculpted offers with three scenarios: transaction cost inclusive distortion (%)

(Penalty, Width)	Tilt				
	-12.0%	-4.0%	0.0%	4.0%	12.0%
(\$148.00, 0.002)	3.0	0.0	0.0	0.0	0.0
(\$148.00, 0.02)	0.0	0.0	0.0	0.0	0.0
(\$18.00, 0.002)	81.0	0.0	5.0	13.0	72.0
(\$18.00, 0.02)	73.0	0.0	0.0	0.0	50.0
(\$4.50, 0.002)	1117.0	390.0	267.0	262.0	729.0
(\$4.50, 0.02)	946.0	398.0	373.0	295.0	472.0

Table 6-52: Multi-band sculpted offers with three scenarios: target defaults (MWh)

The target defaults have a similar pattern to the costs with the worst results coming from the situations in which the worst generator misestimation of the form of the demand profile occurs. Note that except for the case of -12% demand discrepancy with 0.2% extrapolation, the targets set are still physically feasible, being met for a penalty rate of \$148.00 per MW.

(Penalty, Width)	Tilt				
	-12.0%	-4.0%	0.0%	4.0%	12.0%
(\$148.00, 0.002)	-26.0	-21.0	9.0	-12.0	63.0
(\$148.00, 0.02)	-34.0	26.0	51.0	4.0	24.0
(\$18.00, 0.002)	-26.0	-21.0	9.0	-12.0	61.0
(\$18.00, 0.02)	-34.0	26.0	51.0	4.0	24.0

Table 6-53: Multi-band sculpted offers with three scenarios: increase in spills (MWh)

No clear pattern emerges from the spill figures with the results all being reasonably close to the level of spill occurring in the ideal solution. The apparent consistently lower spill in the -12% case is misleading since the centralised solution has more spill in this case.

Patterns for peaked demand profiles are similar except that the -12% results are much worse. The average results for the cost distortion increase mainly in the forecast scenarios in which all generators predict a low day and the one in which Waikato and Waitaki predict a low day. Both scenarios result in substantially less generation being offered however, a -12% peaked demand gives an actual demand 12% greater than generators' 0% scenario in the middle of the day. The first peak of the day is far more extreme than the hydro generators' offers make allowance for and so more thermal generation is scheduled resulting in a higher cost.

(Penalty, Width)	Peak				
	-12.0%	-4.0%	0.0%	4.0%	12.0%
(\$148.00, 0.002)	3.49	0.39	0.30	0.66	1.32
(\$148.00, 0.02)	3.74	0.47	0.47	0.69	1.08
(\$18.00, 0.002)	2.14	0.38	0.28	0.65	1.24
(\$18.00, 0.02)	1.80	0.47	0.47	0.68	1.07
(\$4.50, 0.002)	0.30	0.18	0.06	0.09	0.58
(\$4.50, 0.02)	0.29	0.20	0.10	0.11	0.55

Table 6-54: Multi-band sculpted offers with three scenarios: direct cost distortion (%)

(Penalty, Width)	Peak				
	-12.0%	-4.0%	0.0%	4.0%	12.0%
(\$148.00, 0.002)	3.62	0.39	0.30	0.66	1.32
(\$148.00, 0.02)	4.96	0.47	0.47	0.69	1.08
(\$18.00, 0.002)	2.23	0.38	0.28	0.65	1.25
(\$18.00, 0.02)	1.96	0.47	0.47	0.68	1.07
(\$4.50, 0.002)	0.55	0.21	0.09	0.11	0.80
(\$4.50, 0.02)	0.51	0.24	0.15	0.16	0.73

Table 6-55: Multi-band sculpted offers with three scenarios: transaction cost inclusive distortion (%)

(Penalty, Width)	Peak				
	-12.0%	-4.0%	0.0%	4.0%	12.0%
(\$148.00, 0.002)	10.0	0.0	0.0	0.0	0.0
(\$148.00, 0.02)	96.0	0.0	0.0	0.0	0.0
(\$18.00, 0.002)	224.0	7.0	5.0	5.0	34.0
(\$18.00, 0.02)	265.0	0.0	0.0	1.0	7.0
(\$4.50, 0.002)	2176.0	238.0	267.0	682.0	878.0
(\$4.50, 0.02)	1987.0	279.0	373.0	641.0	654.0

Table 6-56: Multi-band sculpted offers with three scenarios: target defaults (MWh)

(Penalty, Width)	Peak				
	-12.0%	-4.0%	0.0%	4.0%	12.0%
(\$148.00, 0.002)	533.0	-29.0	9.0	3.0	-36.0
(\$148.00, 0.02)	316.0	-17.0	51.0	55.0	-37.0
(\$18.00, 0.002)	426.0	-29.0	9.0	3.0	-36.0
(\$18.00, 0.02)	302.0	-17.0	51.0	55.0	-37.0
(\$4.50, 0.002)	75.0	-34.0	-2.0	0.0	-45.0
(\$4.50, 0.02)	77.0	-34.0	10.0	21.0	-42.0

Table 6-57: Multi-band sculpted offers with three scenarios: increase in spills (MWh)

Clearly, the previous estimate of cost distortion of around 0.25–0.35% is optimistic if the kind of misestimation of demand explored here occurs in practice. Cost distortions

as high as 3.7% direct cost and 5.0% total cost are observed in these experiments, when the actual demand falls outside the range of the generators' scenarios however, several factors should be noted in connection with the results. The offers are formed in a fairly simple manner from a few scenarios—performance will likely be improved by the following measures:

- Widening the range of the three offers for example, using an expected scenario and $\pm 10.0\%$ scenarios around this. This will cope better with a wider range of possible demand patterns. However it would probably produce inferior results to closely spaced scenarios for the cases in which scenarios are accurate, since this would allow the market to set targets away from the generators' desired trajectories. There is a trade-off here between what the generators know, and what the market knows
- A better solution would be to use more scenarios in the offer formation process—in this way a wider variety of scenarios will allow the expected scenarios to be catered for and a few extreme scenarios could hedge against extreme outcomes

The long term average cost distortion produced by a market dispatch regime is dependent on how often various misforecasting situations occur. The worst case results above, of 3.7% increase in direct costs, caused by generators not offering enough to cover a peak which is higher than projected may be a situation that occurs relatively infrequently

The misestimates presented above probably do not represent the most extreme situations may occur. Both the tilted and the peaked discrepancies have generators estimating the total demand for the day approximately correctly. If this is not the case, the results could well be worse.

These conclusions largely support the hypothesis stated at the beginning of this section, i.e. if generators include the real-time demand inside the envelope of their demand forecasts and their demand forecasts are not too widely spaced cost distortions close to the best obtained from accurate forecasting may be achieved.

C.5.3 Water Value Balance

Earlier experiments assume a balanced set of water values throughout the system. This will be the case if the inflows are moderate enough to allow the entire generation system to be kept in equilibrium, since it implies attempting to equalise the marginal value of water in all reservoirs. However, for whatever reason this may not be the case in practice.

One might hypothesise that if water values in major reservoirs become significantly unbalanced the flexibility of the generation system with regards to dispatch and in particular to any re-dispatch required to work around inappropriate targets may be seriously affected increasing the costs of a de-centralised regime.

Conversely, it is possible that the difference between the efficiency of the de-centralised and that of the centralised regime will decrease as water values become unbalanced because the optimal solution becomes 'clear cut' with such an obvious merit-order that both regimes produce the same solution.

To test these hypotheses a set of experiments with the following parameters is presented:

- South Island water values half their base case values the North Island water values are twice their base case values;
- South Island water values twice their base case values and the North Island water values half their base case values

The centrally coordinated dispatch is solved for each of the scenarios and these solutions are used as a basis for a variety of offers which are then used in market clearing and dispatch phases. These offers are a selected subset of the types trialed earlier, including the parameter settings found to be most promising in addition to some points bounding the range in which reasonable performance is reported:

- Multi-band, fixed offers with (β_{HI}, β_{LO}) pairs of (0.0,0.2), (0.0,0.6), (0.0,1.0) and (0.4,0.6)
- Three-band sculpted offers with band widths of 2.0% and 10.0%

Three different penalty rates of \$148.00 per MW, \$18.00 per MW and \$1.12 per MW for defaulting on output targets are tested with three of the demand forecast scenarios used earlier:

- 1) All accurately estimate demand.
- 2) All over estimate demand by 5.0%.
- 3) All under estimate demand by 5.0%.

The rest of the data: the actual demand, the tributary flows, etc. are the same as in the base scenarios tested earlier.

For the high penalty case of \$148.00 per MW, the results generally conform to the first hypothesis with the original balanced water value scenario out-performing the un-balanced cases. Also, the cost penalties are lower for the South Island low/North Island high scenario than for the North Island low/South Island high scenario. This is probably because the former gives a cheaper overall solution and probably allows dispatch deviations to be met more cheaply on the margin.

Penalty (β_{HI}, β_{LO})	Balance		
	SI high, NI low	Water value balanced	SI low, NI high
\$148.00 (0.0,0.2)	7.3	3.2	8.9
\$148.00 (0,0.6.0)	7.9	2.8	7.6
\$148.00 (0.0,1.0)	7.3	3.9	5.7
\$148.00 (0.4,0.6)	4.8	3.5	6.0
\$18.00 (0.0,0.2)	0.7	2.1	3.0
\$18.00 (0.0,0.6)	0.5	2.0	2.0
\$18.00 (0.0,1.0)	0.6	2.7	1.5
\$18.00 (0.4,0.6)	0.6	2.5	2.5

Table 6-58: Multi-band fixed offers: direct cost distortion (%)

Penalty (β_{HI}, β_{LO})	Balance		
	SI high, NI low	Water value balanced	SI low, NI high
\$148.00 (0.0,0.2)	12.3	4.7	8.9
\$148.00 (0.0,0.6)	11.5	3.8	7.6
\$148.00 (0.0,1.0)	12.6	5.4	6.3
\$148.00 (0.4,0.6)	10.0	5.2	6.1
\$18.00 (0.0,0.2)	5.4	3.2	7.2
\$18.00 (0.0,0.6)	5.5	2.6	5.8
\$18.00 (0.0,1.0)	5.3	3.7	4.4
\$18.00 (0.4,0.6)	3.7	3.2	4.9

Table 6-59: Multi-band fixed offers: transaction cost inclusive distortion (%)

For penalties of \$18.00 per MW the pattern is not as clear as \$148.00 per MW penalties. As may be seen from Table 6-58 to Table 6-61 the first hypothesis is confirmed with respect to total costs including penalties but not for the direct cost excluding penalties. It would appear that, for the unbalanced cases with lower penalties, defaulting from output targets is much more prevalent. This can mean lower direct costs since the de-centralised solution can be closer to the centralised one if targets are not met but, the increased penalty cost means that total costs may still be higher than the balanced water values scenario.

Penalty (β_{HI}, β_{LO})	Balance		
	SI high, NI low	Water value balanced	SI low, NI high
\$148.00 (0.0,0.2)	190.0	58.0	0.0
\$148.00 (0.0,0.6)	138.0	39.0	2.0
\$148.00 (0.0,1.0)	207.0	58.0	21.0
\$148.00 (0.4,0.6)	200.0	64.0	3.0
\$18.00 (0.0,0.2)	1459.0	344.0	1305.0
\$18.00 (0.0,0.6)	1582.0	177.0	1211.0
\$18.00 (0.0,1.0)	1497.0	298.0	909.0
\$18.00 (0.4,0.6)	958.0	212.0	754.0

Table 6-60: Multi-band fixed offers: target defaults (MWh)

Generators are more willing to default on targets at a penalty rate of \$18.00 per MW under the unbalanced water value cases because in these scenarios the high-water-value generators water values are set to around \$40.00 per MW. So, buying in some generation

at the market price and paying the penalty might still be cheaper than using their own water—this is obviously not the case if the penalties are very high.

The results indicate that the balanced water value scenarios used in previous experiments can be expected to yield optimistic cost estimates especially for cases with high penalties. In terms of direct costs it would seem that previous estimates of around 2.0% increase may still be reasonable on average for the \$18.00 penalty case but for higher penalties direct costs could rise by perhaps 3.0–6.0% on average when the range of reservoir balance situations are accounted for. Including penalties these estimates rise to perhaps 6.0–10.0% for high penalties and 4.0–5.0% for medium penalties.

<i>Penalty</i> (β_{HI}, β_{LO})	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>Water value balanced</i>	<i>SI low, NI high</i>
<i>\$148.00 (0.0,0.2)</i>	-923.0	674.0	-3359.0
<i>\$148.00 (0.0,0.6)</i>	164.0	619.0	-2661.0
<i>\$148.00 (0.0,1.0)</i>	542.0	758.0	-745.0
<i>\$148.00 (0.4,0.6)</i>	209.0	662.0	-2405.0
<i>\$18.00 (0.0,0.2)</i>	-679.0	207.0	-2168.0
<i>\$18.00 (0.0,0.6)</i>	-161.0	204.0	-1479.0
<i>\$18.00 (0.0,1.0)</i>	145.0	236.0	-677.0
<i>\$18.00 (0.4,0.6)</i>	-75.0	251.0	-1797.0

Table 6-61: Multi-band fixed offers: increase in spills (MWh)

Referring to Table 6-61, the unbalanced water value cases show much-reduced spill relative to the centralised solution compared with the balanced water value case however, in absolute terms the amount of spill rises for both the unbalanced cases. The centralised solution in each unbalanced water value case spills large amounts of water down river chains in which ever island has cheap water. The de-centralised solutions also spill large amounts of water but not as much—this is probably because the offers moderate the targets and so moderate the de-centralised solution.

Three-band sculpted offers clearly show the balanced scenario performing better—see Table 6-62 to Table 6-65.

<i>Penalty, Width</i>	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>balanced</i>	<i>SI low, NI high</i>
<i>\$148.00, 0.02</i>	3.6	1.8	2.7
<i>\$148.00, 0.10</i>	3.1	2.7	4.5
<i>\$18.00, 0.02</i>	1.1	1.5	0.7
<i>\$18.00, 0.10</i>	0.9	1.2	1.0

Table 6-62: Three-band sculpted offers: direct cost distortion (%)

<i>Penalty, Width</i>	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>balanced</i>	<i>SI low, NI high</i>
<i>\$148.00, 0.02</i>	9.4	1.8	3.7
<i>\$148.00, 0.10</i>	10.8	3.5	4.9
<i>\$18.00, 0.02</i>	3.2	1.6	1.5
<i>\$18.00, 0.10</i>	3.1	2.1	2.9

Table 6-63: Three-band sculpted offers: transaction cost inclusive distortion (%)

<i>Penalty, Width</i>	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>balanced</i>	<i>SI low, NI high</i>
<i>\$148.00, 0.02</i>	225.0	1.0	40.0
<i>\$148.00, 0.10</i>	295.0	30.0	15.0
<i>\$18.00, 0.02</i>	651.0	30.0	237.0
<i>\$18.00, 0.10</i>	670.0	268.0	589.0

Table 6-64: Three-band sculpted offers: target defaults (MW)

<i>Penalty, Width</i>	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>balanced</i>	<i>SI low, NI high</i>
<i>\$148.00, 0.02</i>	1329.0	160.0	149.0
<i>\$148.00, 0.10</i>	1108.0	1199.0	-163.0
<i>\$18.00, 0.02</i>	1288.0	172.0	-170.0
<i>\$18.00, 0.10</i>	1108.0	1199.0	-163.0

Table 6-65: Three-band sculpted offers: increase in spill (MW)

Direct cost distortions are relatively higher when the penalties for defaulting on targets are high—presumably because defaulting can be a more attractive option when the penalties are more moderate. Overall, the previous estimate of around 1.8% (direct cost) distortion for the 2.0% band width might rise to 2.5–3.0% in the high penalty case but fall to around 1.2–1.5% in the moderate penalty case. For the 10% band cost distortions of around 3.0–4.0%, for high penalties, and 0.5–1.5% for moderate penalties seem reasonable.

One might conclude that using balanced water values gives superior results when compared with unbalanced water value scenarios because of the greater flexibility the system has to adapt to real-time demand in the balanced case.

In particular, these effects could add perhaps 2.0–4.0% to the previous estimates of cost distortion under regimes with high penalties—or equivalently on regimes that allow little flexibility once targets have been set.

In summary, it would appear that:

- the balance of water values between reservoirs has a significant impact on the relative efficiency of a market based dispatch system—this means the results presented here may prove to be optimistic in practice in this respect;
- water value imbalance makes narrow band widths relatively less attractive from a system perspective when penalties are high

The results from the previous section, for three-band independent period offers, are now supplemented by results from the two additional water value scenarios used in the previous water value balance experiments:

- one in which the South Island water values are half their base case values and the North Island water values were twice their base case values;
- one in which the South Island water values are twice their base case values and the North Island water values were half their base case values

For each of these scenarios the centralised solution is found across five demand scenarios and the generators use three of the five solutions to form five band sculpted offers—the following scenarios are simulated:

- 1) All forecast accurately, i.e. use (-5.0%, +0.0%, +5.0%) demand scenarios to form their offers.
- 2) All forecast a low demand day, i.e. use (-8.0%, -5.0%, +0.0%) to form their offers.
- 3) All forecast a high demand day, i.e. use (+0.0%, +5.0%, +8.0%) to form their offers.
- 4) The Taupo and Waitaki forecast a high day, the rest a low day.

5) The Taupo and Waitaki generators forecast a low day, the rest a high day.

Sets of experiments are presented for penalty rates of \$148.00 per MW, \$37.00 per MW, \$18.00 per MW and \$4.50 per MW on target defaults and across a selection of extrapolation parameters: 0.2%, 2.0%, 10.0%, 20.0% of the maximum possible generation. The results of 160 experiments are presented here in addition to the 80 previous results from the balanced water value case.

Referring to Table 6-66 to Table 6-69, the results show considerable improvement over those for scenarios with unbalanced water values but, as is consistent with the first hypothesis, they are worse than the results for similar offers with balanced water values. The numbers given are averages of five forecast scenarios for each water value scenario, parameter setting and penalty rate. The cost results are the percentage increase in costs over the centralised solution with the same water value scenario but given as a percentage of the balanced water value solution.

<i>Penalty, Width</i>	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>balanced</i>	<i>SI low, NI high</i>
<i>\$148.00, 0.002</i>	1.0	0.6	1.4
<i>\$148.00, 0.020</i>	1.1	0.7	1.6
<i>\$148.00, 0.100</i>	1.7	1.1	2.5
<i>\$148.00, 0.200</i>	2.4	2.4	4.6
<i>\$37.00, 0.002</i>	0.9	0.6	1.2
<i>\$37.00, 0.020</i>	1.1	0.7	1.5
<i>\$37.00, 0.100</i>	1.4	0.9	2.3
<i>\$37.00, 0.200</i>	1.9	1.4	4.3
<i>\$18.00, 0.002</i>	0.3	0.6	0.5
<i>\$18.00, 0.020</i>	0.3	0.7	0.5
<i>\$18.00, 0.100</i>	0.6	0.7	0.9
<i>\$18.00, 0.200</i>	0.7	0.7	1.3
<i>\$4.50, 0.002</i>	0.04	0.05	0.02
<i>\$4.50, 0.020</i>	0.05	0.08	0.03
<i>\$4.50, 0.100</i>	0.08	0.11	0.03
<i>\$4.50, 0.200</i>	0.08	0.15	0.03

Table 6-66: Multi-band sculpted offers with three scenarios: direct costs distortion (%)

These results show the same pattern as the earlier water value balance experiments with the \$148.00 per MW penalties giving higher direct costs for the unbalanced water

value cases. However, the \$18.00 per MW penalties yields lower direct costs in these cases because a higher level of defaulting on targets occurs.

<i>Penalty, Width</i>	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>balanced</i>	<i>SI low, NI high</i>
<i>\$148.00, 0.002</i>	1.0	0.6	1.4
<i>\$148.00, 0.020</i>	1.2	0.7	1.6
<i>\$148.00, 0.100</i>	5.2	1.7	2.5
<i>\$148.00, 0.200</i>	7.9	3.8	4.8
<i>\$37.00, 0.002</i>	0.9	0.6	1.4
<i>\$37.00, 0.020</i>	1.1	0.7	1.6
<i>\$37.00, 0.100</i>	2.5	1.2	2.5
<i>\$37.00, 0.200</i>	3.7	2.5	4.5
<i>\$18.00, 0.002</i>	0.7	0.6	1.2
<i>\$18.00, 0.020</i>	0.9	0.7	1.3
<i>\$18.00, 0.100</i>	1.7	1.0	2.1
<i>\$18.00, 0.200</i>	2.4	1.7	3.7
<i>\$4.50, 0.002</i>	0.31	0.32	0.57
<i>\$4.50, 0.020</i>	0.37	0.35	0.63
<i>\$4.50, 0.100</i>	0.66	0.48	0.99
<i>\$4.50, 0.200</i>	0.88	0.68	1.47

Table 6-67: Multi-band sculpted offers with three scenarios: transaction cost inclusive distortion (%)

The increase in total cost is always greater in the imbalance water value cases also, note that, although the results are close together for varying penalties in the balanced case they show more variance for the unbalanced cases. This is because a higher level of defaults occurs in the unbalanced cases, since inappropriate targets are more expensive to reach for the generators with higher water values hence, with a moderate penalty it is often cheaper for such a generator to default, pay the penalty and buy in cheap power from the other island than to generate themselves.

<i>Penalty, Width</i>	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>balanced</i>	<i>SI low, NI high</i>
<i>\$148.00, 0.002</i>	1.0	0.0	0.0
<i>\$148.00, 0.020</i>	4.0	2.0	0.0
<i>\$148.00, 0.100</i>	136.0	22.0	0.0
<i>\$148.00, 0.200</i>	212.0	54.0	7.0
<i>\$37.00, 0.002</i>	9.0	0.0	28.0
<i>\$37.00, 0.020</i>	7.0	2.0	18.0
<i>\$37.00, 0.100</i>	167.0	36.0	27.0
<i>\$37.00, 0.200</i>	267.0	169.0	35.0
<i>\$18.00, 0.002</i>	132.0	5.0	220.0
<i>\$18.00, 0.020</i>	167.0	4.0	281.0
<i>\$18.00, 0.100</i>	347.0	91.0	385.0
<i>\$18.00, 0.200</i>	524.0	316.0	760.0
<i>\$4.50, 0.002</i>	284.0	339.0	590.0
<i>\$4.50, 0.020</i>	328.0	345.0	644.0
<i>\$4.50, 0.100</i>	613.0	462.0	1032.0
<i>\$4.50, 0.200</i>	845.0	671.0	1537.0

Table 6-68: Multi-band sculpted offers with three scenarios: target defaults (MWh)

Low defaults are shown for all water value cases at high penalties. For a penalty rate of \$18.00 per MW the defaults for the unbalanced cases are considerably higher. These defaults occur in whichever island has high water values. They are the result of the generators in that island choosing to buy in cheap generation from the other island and pay the penalty rather than generate them.

<i>Penalty, Width</i>	<i>Balance</i>		
	<i>SI high, NI low</i>	<i>balanced</i>	<i>SI low, NI high</i>
<i>\$148.00, 0.002</i>	-61.0	8.0	-135.0
<i>\$148.00, 0.020</i>	240.0	61.0	-195.0
<i>\$148.00, 0.100</i>	1203.0	429.0	-204.0
<i>\$148.00, 0.200</i>	1814.0	1268.0	-257.0
<i>\$37.00, 0.002</i>	-61.0	8.0	-180.0
<i>\$37.00, 0.020</i>	238.0	61.0	-235.0
<i>\$37.00, 0.100</i>	1155.0	363.0	-306.0
<i>\$37.00, 0.200</i>	1723.0	757.0	-437.0
<i>\$18.00, 0.002</i>	-109.0	6.0	-53.0
<i>\$18.00, 0.020</i>	64.0	60.0	-81.0
<i>\$18.00, 0.100</i>	967.0	297.0	-151.0
<i>\$18.00, 0.200</i>	1408.0	486.0	-150.0
<i>\$4.50, 0.002</i>	29.0	5.0	0.0
<i>\$4.50, 0.020</i>	87.0	20.0	-6.0
<i>\$4.50, 0.100</i>	503.0	58.0	-11.0
<i>\$4.50, 0.200</i>	456.0	143.0	-13.0

Table 6-69: Multi-band sculpted offers with three scenarios: increase in spills (MWh)

The amount of spill produced by each of the three scenarios is more difficult to interpret. The centralised solutions of each of the unbalanced cases show large increases in spill over the balanced case—this is due to the low water value generators running their river system close to its physical limits for example, many of the South Island low experiments had the Clutha river system running at its maximum generation all day.

The de-centralised solutions show quite a lot of variation in their spill results. As an example, the average across the five demand forecast scenarios for the case with South Island water values high/North Island water values low 0.2% extrapolation and \$148.00 per MW penalties is shown above as -61.0 MW however, because of the variation across the forecast scenarios a rough 90.0% confidence interval for this would be approximately (-320.0, 200.0)—as a consequence, the significance of differences shown in the graphs above is dubious.

The South Island low/North Island high case does seem to show that the de-centralised solutions produce less spill on average. This is because the targets for the South Island generators are sometimes less than the centralised dispatch generation lev-

els and so the generators do not push their systems as hard in the de-centralised dispatch.

From these results it would appear that using several scenarios to construct offers decreases the cost distortions in cases with unbalanced water values as well as the balanced case previously reported. Direct cost distortions of around 1.0–2.0%, with a low incidence of defaulting, seem likely for unbalanced water values when high penalties apply on defaults. With lower penalties on defaults the direct costs seem reasonably comparable across water value scenarios at around 0.3–0.8% but with generators defaulting more in the unbalanced cases. The unbalanced cases give more incentive for high water value generators to default from inappropriate targets. The higher defaults give total costs estimates of 0.5–1.5% for unbalanced water values at moderate penalty rates.

C.5.4 Tributary Uncertainty

The factors modelled in previous sections are generators' uncertainty about the market's demand and their water values. Such uncertainty could stem from uncertainty about the total market demand but also from uncertainty about their competitors' offers. Thus, although this latter aspect is not explicitly modelled in previous experiments it might be expected to have a very similar effect to uncertainty about total market demand since both influence the demand for an individual generator's offer.

There are sources of uncertainty that do not relate to the market. Uncertainty about a generator's physical situation for example, tributary flows could also prove significant. The aim of this set of experiments is to determine if this is the case.

To investigate this issue a set of experiments is shown which combines tributary uncertainty with the demand uncertainty of previous experiments in order to gauge the effects of this combined uncertainty using a set of tributary forecast scenarios.

The base case is the same as that used in previous experiments, with accurate estimation by generators of the tributary flows for themselves and other generators—the other five tributary cases are:

- 1) All expect the base case tributary flows but these flows turn out to be 20.0% less across the entire country.
- 2) All expect the base case tributary flows but the flows into the Taupo river-chain are 20.0% greater than expected.
- 3) All expect the base case tributary flows but the flows into the Taupo river-chain are 50.0% greater than expected.
- 4) All expect the base case tributary flows but the flows into the Clyde river-chain are 20.0% greater than expected.
- 5) All generators expect the base case tributary flows but the flows into the Clyde river-chain are 50.0% greater than expected.

These five cases are run using the same three demand forecast scenarios as previously used:

- 1) All hydro groups forecast demand accurately.
- 2) All hydro groups over estimate demand by 5.0%.
- 3) All hydro groups under estimate demand by 5.0%.

Each of these fifteen experiments is run for penalty rates for defaults on targets of \$148.00 and \$18.00, and the variety of offer types and parameter values.

It could be expected that solutions with accurate estimation of forecast flows would be superior but the pattern of results does not reflect this. Instead, for high penalties there is a slight decrease in both direct and total costs as tributaries increase. However, these differences are quite possibly due to random variation. For moderate penalties there is very little difference between the tributary forecast scenarios although on average the accurate forecasts are slightly better—see Table 6-70 to Table 6-73.

Penalty,	Tributary forecast					
	All +20.0%	All +0.0%	WKO -20.0%	TPO -50.0%	CLT -20.0%	CLT -50.0%
\$148.00 (0.0,0.2)	3.3	3.2	3.0	2.9	3.1	3.1
\$148.00 (0.0,0.6.0)	2.8	2.8	2.6	2.6	2.8	2.7
\$148.00 (0.0,1.0)	4.1	3.9	3.8	3.7	3.9	3.9
\$148.00 (0.4,0.6)	3.4	3.5	3.4	3.4	3.5	3.4
\$18.00 (0.0,0.2)	2.3	2.1	2.3	2.7	2.1	2.1
\$18.00 (0.0,0.6)	2.1	2.0	2.1	2.4	1.9	2.0
\$18.00 (0.0,1.0)	2.6	2.7	2.8	3.1	2.7	2.7
\$18.00 (0.4,0.6)	2.4	2.5	2.5	2.7	2.6	2.5

Table 6-70: Multi-band fixed offers: direct cost distortion (%)

Penalty,	Tributary forecast					
	All +20.0%	All +0.0%	WKO -20.0%	TPO -50.0%	CLT -20.0%	CLT -50.0%
\$148.00 (0.0,0.2)	5.1	4.7	4.5	4.4	4.7	4.6
\$148.00 (0.0,0.6.0)	4.0	3.8	3.6	3.6	3.8	3.7
\$148.00 (0.0,1.0)	5.8	5.4	5.3	5.2	5.4	5.4
\$148.00 (0.4,0.6)	5.7	5.2	4.8	4.4	5.1	5.1
\$18.00 (0.0,0.2)	3.0	3.2	2.8	3.3	2.9	2.8
\$18.00 (0.0,0.6)	2.7	2.6	2.5	3.0	2.6	2.6
\$18.00 (0.0,1.0)	3.9	3.7	3.6	3.9	3.7	3.6
\$18.00 (0.4,0.6)	3.2	3.2	3.1	3.5	3.3	3.2

Table 6-71: Multi-band fixed offers: transaction cost inclusive distortion (%)

Penalty, (β_{HI}, β_{LO})	Tributary forecast					
	All +20.0%	All +0.0%	WKO -20.0%	TPO -50.0%	CLT -20.0%	CLT -50.0%
\$148.00 (0.0,0.2)	70.0	58.0	58.0	58.0	58.0	58.0
\$148.00 (0.0,0.6.0)	44.0	39.0	39.0	39.0	39.0	39.0
\$148.00 (0.0,1.0)	63.0	58.0	58.0	58.0	58.0	58.0
\$148.00 (0.4,0.6)	88.0	64.0	51.0	39.0	64.0	64.0
\$18.00 (0.0,0.2)	222.0	344.0	175.0	180.0	230.0	238.0
\$18.00 (0.0,0.6)	206.0	177.0	136.0	172.0	213.0	180.0
\$18.00 (0.0,1.0)	405.0	298.0	248.0	266.0	296.0	290.0
\$18.00 (0.4,0.6)	263.0	212.0	213.0	255.0	211.0	209.0

Table 6-72: Multi-band fixed offers: target defaults (MWh)

The increase in spill over the centralised solution is less for tributary under estimates. The reason for this is unclear but appears to be a combination of more water being

available to meet targets and more spill in the centralised solution especially in the case of Taupo tributaries being under estimated by 50.0%.

Penalty, (β_{HI}, β_{LO})	Tributary forecast					
	All +20.0%	All +0.0%	WKO -20.0%	TPO -50.0%	CLT -20.0%	CLT -50.0%
\$148.00 (0.0,0.2)	654.0	674.0	546.0	429.0	678.0	673.0
\$148.00 (0.0,0.6)	607.0	619.0	493.0	412.0	647.0	618.0
\$148.00 (0.0,1.0)	809.0	758.0	618.0	482.0	745.0	754.0
\$148.00 (0.4,0.6)	575.0	662.0	590.0	501.0	664.0	662.0
\$18.00 (0.0,0.2)	201.0	207.0	219.0	134.0	225.0	203.0
\$18.00 (0.0,0.6)	206.0	204.0	223.0	89.0	192.0	200.0
\$18.00 (0.0,1.0)	145.0	236.0	199.0	118.0	228.0	237.0
\$18.00 (0.4,0.6)	142.0	251.0	149.0	-51.0	282.0	257.0

Table 6-73: Multi-band fixed offers: increase in spills (MWh)

Here, the results in each period are quite uniform, as may be seen in Table 6-74 to Table 6-77. They show little difference in cost between the tributary forecast scenarios with slightly worse performance in the over estimation scenarios. This is not surprising since over-estimating supply capability leaves the generator or the system with a difficult task to meet any dispatch variations with less than expected resources.

Penalty, Width	Tributary forecast			
	All +20.0%	All +0.0%	TPO -50.0%	CLT -50.0%
\$148.00, 0.02	1.9	1.8	1.7	1.8
\$148.00, 0.10	2.8	2.7	2.5	2.5
\$18.00, 0.02	1.6	1.5	1.5	1.5
\$18.00, 0.10	1.4	1.2	1.3	1.3

Table 6-74: Multi-band sculpted offers: direct cost distortion (%)

Penalty, Width	Tributary forecast			
	All +20.0%	All +0.0%	TPO -50.0%	CLT -50.0%
\$148.00, 0.02	2.2	1.8	1.8	1.8
\$148.00, 0.10	3.9	3.5	3.3	3.3
\$18.00, 0.02	1.8	1.6	1.6	1.6
\$18.00, 0.10	2.3	2.1	2.0	2.2

Table 6-75: Multi-band sculpted offers: transaction cost inclusive distortion (%)

The target defaults are also fairly consistent although it appears that over estimation of tributary flows leads to higher defaults. This could be expected since over estimating tributary inflows would result in overly optimistic offers, which could become unattainable targets whereas, river chains with sufficient flexibility are able to absorb unexpected flows in the real-time and will have sufficient time to adjust their offers for the next day.

<i>Penalty, Width</i>	<i>Tributary forecast</i>			
	<i>All +20.0%</i>	<i>All +0.0%</i>	<i>TPO -50.0%</i>	<i>CLT -50.0%</i>
<i>\$148.00, 0.02</i>	12.0	1.0	1.0	1.0
<i>\$148.00, 0.10</i>	43.0	30.0	30.0	30.0
<i>\$18.00, 0.02</i>	47.0	30.0	37.0	42.0
<i>\$18.00, 0.10</i>	277.0	268.0	223.0	260.0

Table 6-76: Three-band sculpted offers: target defaults (MWh)

The increase in aggregate spill over the centralised solution across the whole generation system falls as the tributaries increase. The amount of spill rises as the tributaries increase but the centralised solution spills rise faster than the de-centralised ones—the latter could be anchored somewhat by the targets.

<i>Penalty, Width</i>	<i>Tributary forecast</i>			
	<i>All +20.0%</i>	<i>All +0.0%</i>	<i>TPO -50.0%</i>	<i>CLT -50.0%</i>
<i>\$148.00, 0.02</i>	145.0	160.0	70.0	24.0
<i>\$148.00, 0.10</i>	1197.0	1199.0	1054.0	1000.0
<i>\$18.00, 0.02</i>	143.0	172.0	82.0	31.0
<i>\$18.00, 0.10</i>	551.0	522.0	553.0	525.0

Table 6-77 Three-band sculpted offers: increase in spills (MWh)

These preliminary results suggest that hydro generators' uncertainty about their tributary inflows does not have a particularly significant effect on the relative efficiency of a de-centralised generation dispatch system of the type studied. This is slightly surprising because it might be expected that over estimating tributary flows would result in over generous offers and hence inappropriate targets. Conversely, underestimating inflows might also be expected to exhibit poor performance relative to the centralised solution because the additional water cannot be easily utilised—since the targets have al-

ready been set. These two effects do not appear to have been significant in the experiments performed however, a slight increase in defaults is noted for the over estimation case.

The reasons for tributary uncertainty having little effect in the experiments could include having not modelled extreme enough scenarios. In cases where the tributary forecast is not too different from what actually happens—the generator can modify the releases from the top reservoir to compensate for the shortfall or surplus of tributary flows. For this reason, systems with large capacity reservoirs could probably handle errors in tributary forecasts more easily. In addition, the level of tributary inflows is always assumed to be constant throughout the day. In reality the inflows could peak across a number of periods making the effects more extreme.

In conclusion it appears that generators' uncertainty about their own physical condition is a relatively minor factor in the relative efficiency of the de-centralised dispatch system. Differences of about 0.5% are seen in the experiments but given the small number of experiments this is probably not significant.

C.5.5 Uncertain Tributary Flows in the Periodic-update Market

Previous experiments with the rolling horizon market model have modelled generators' uncertainty about the market demand, but not uncertainty about their own physical situations. Experiments with the day-ahead market indicated that uncertainty about the level of tributary flows into the Clyde system had a major impact on the relative efficiency of the market, when compared with a centrally optimised regime. The inefficiency of the market in these circumstances was caused by the inability to effectively use additional, unexpected, inflows because the day-ahead targets effectively precluded additional generation from the Clyde group. Consequently, a market which allows generators to update their offers and receive revised targets should perform much better under these conditions, and experiments have been performed to test this hypothesis.

The situation modelled in these experiments was one where the day-ahead forecasts of the Clyde tributaries were inaccurate, but by the time the day begins, the forecast has

been revised and is accurate. Thus the set of experiments was similar to those performed with the day-ahead model.

For these experiments the model was adjusted so that the sets of offers for the first period, which would be submitted some hours ahead of real time, were based upon the base level tributary inflows. The market was cleared, and the targets set using these offers. However by the time the real time dispatch occurs the actual tributary inflows are known, and so this phase was run using the actual tributary flows into the Clyde system. For the rest of the day the offer scenarios and the real time dispatch are also run using the correct tributary levels, which are assumed to continue into the next day. The comparison centrally optimised dispatch also uses the correct tributary inflows.

As in the day-ahead experiments, sets of runs were performed with the true tributary levels at 80%, 100%, 120%, 150%, and 200% of the base levels expected on the previous day. Each of these sets of runs consisted of three experiments, with the generators forecasting high, accurate and low demand days in the normal manner. Hence a total of 12 experiments were performed, in addition to the 3 results already obtained for accurate Clyde tributary forecasts.

The increases in costs compared with the centralised dispatch are shown in Table 6-78.

	<i>Actual tributaries as % of expected</i>				
	80%	100%	120%	150%	200%
<i>All accurate</i>	0.16	0.14	0.26	0.45	1.62
<i>All high</i>	0.28	0.25	0.26	0.60	1.50
<i>All low</i>	0.06	0.02	0.15	0.44	0.91
<i>Average</i>	0.17	0.14	0.22	0.50	1.34

Table 6-78: Multi-band sculpted offers in the periodic-update market: direct cost distortions (%)

The results showed a similar pattern to the day-ahead experiments, but the cost distortions were reduced considerably, as can be seen in Table 6-79.

	<i>Actual Clyde tributaries as % of expected</i>				
	80%	100%	120%	150%	200%
<i>Periodic-update</i>	0.17	0.14	0.22	0.50	1.34
<i>Day-ahead</i>	0.18	0.18	0.71	2.27	6.58

Table 6-79: Multi-band sculpted offers in the periodic-update market: direct cost distortions (%)

Considering as an example one of the 200% tributary experiments, one can see the reason for the improvement over the day-ahead market. The other experiments show similar behaviour, but on a smaller scale. The table below shows the way in which the Clyde group's initial offer changes when they become aware of the extra tributary flows in period 2. The offers increase substantially, and the Waitaki group reduces their offers in response.

	<i>Actual Period</i>											
	1	2	3	4	5	6	7	8	9	10	11	12
1	426											
2	415	593										
3	418	600	600									
4	502	704	705	705								
5	752	752	752	752	752							
6	698	728	728	721	721	721						
7	609	752	721	711	709	714	713					
8	633	752	724	721	721	721	721	721				
9	752	752	752	752	752	752	752	752	752			
10	718	752	752	736	736	721	721	721	721	721		
11	454	752	721	721	721	721	721	721	721	721	721	
12	443	741	721	721	721	721	721	721	721	718	719	720
1		728	721	716	716	719	721	720	721	721	721	719
2			721	676	668	686	687	686	688	687	671	670
3				676	668	673	674	674	676	679	666	666
4					721	721	721	721	721	721	721	721
5						752	752	752	752	752	752	752
6							721	721	721	721	721	721
7								714	707	707	707	707
8									721	721	721	721
9										752	752	752
10											736	736
11												721

Table 6-80: Central band of the Clyde offer

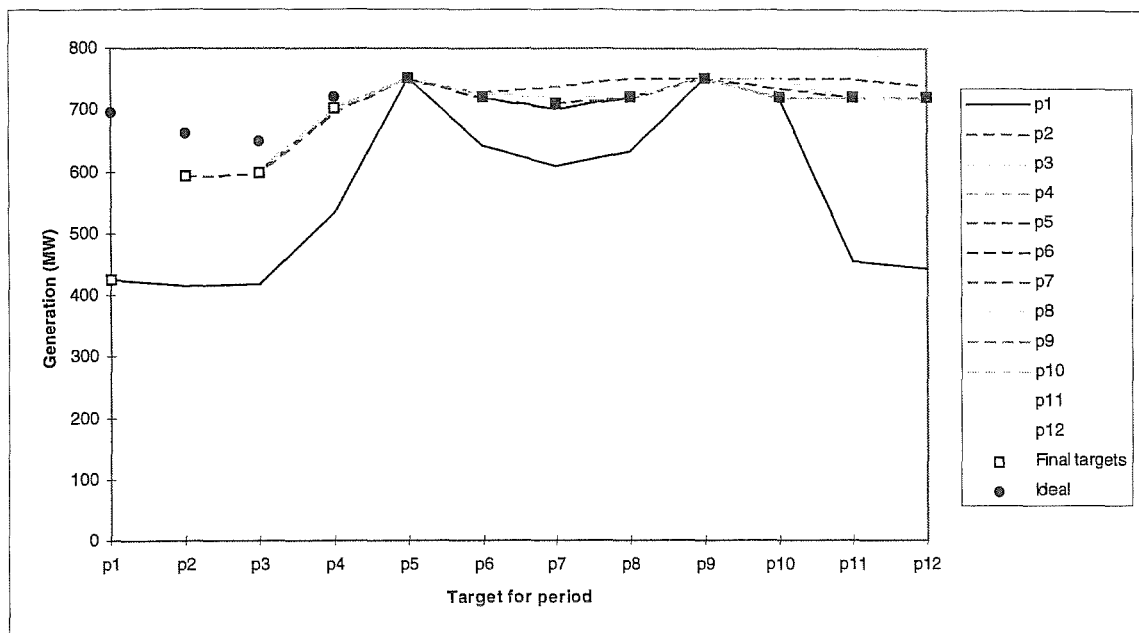


Figure 6-75: Periodic-update market: energy output targets for Clyde

The results are shown in Figure 6-75, which shows a profile of the way in which the projected targets for the Clyde group evolve over the day. The solid line shows the projected targets from the market clearing in period 1, using the offers that were formed assuming the base level tributary inflows. In the day-ahead market, these would have been the final targets. The ideal dispatch is much higher than this level through the day, and this deviation is the cause of the large cost distortions seen in the day-ahead market. However, the rolling horizon model allows generators to update their offers, and so the Clyde group offers more generation from the second period on.

This results in the revised targets seen in Figure 6-75, which is much closer to the ideal dispatch. Periods 2 and 3 form a transition period as the Clyde system switches to the new trajectory, since the ramp rate restrictions, and the storage levels inherited from the periods when the tributary forecast was inaccurate, prevent the river chain from obtaining the maximum benefit from the additional water immediately. In the situation demonstrated in this set of experiments, similar benefits as those enjoyed by the rolling horizon model could be obtained in a day-ahead market by allowing a single update at the beginning of the day if the situation requires it. However, the benefits of such a sys-

tem would be minimal if the correct forecasts were not available until the second or some subsequent period.

The impact of underestimating tributary inflows to the Clyde system in a simple market is reduced very significantly for a market that allows generators to update their offers as the day progresses. The market which allows such revised offers creates the ability to react effectively to unexpected events, such as a higher level of inflows, and the results can be a dramatic improvement in the efficiency of the market, when compared with a rigid day-ahead market. For example, the experiments indicate that if the tributary inflows are 50.0% higher than expected into the Clyde system, the day-ahead market results in costs 2.23% higher than an idealised centralised solution, but the rolling horizon market costs are only 0.50% higher. The long-term significance of such a difference would depend upon the frequency with which such misforecasting occurs. It is clear, however, that the market which allows updated offers gives considerably more flexibility to respond to unexpected events.

Appendix D

Reserve Offers in Practice

This appendix provides a more technical discussion of the reserve offer framework described in Chapter 5. Section D.1 examines aspects of thermal unit offers. Section D.2 shows how the framework can be used to model multi-unit hydro (or thermal) stations. Finally, Section D.3 provides the formulae for calculating the components of a reserve offer and shows an example.

D.1 Offers for Thermal Units

Chapter 5 described the reserve capability of a unit assuming it could provide reserves over its entire operating range. This section shows how aspects such as minimum unit loading points and efficiency curves can be represented in the general offering framework described in that chapter.

D.1.1 Minimum Unit Loading Points

Figure 6-1 shows a continuous approximation to a unit's feasible operating points for instantaneous reserves. There are three key points labelled on this figure:

- A. The origin, with zero output and zero reserve.
- B. The unit's minimum loading point (MULP).
- C. Which represents operation of the unit at full output.

In reality, operation below the MULP is not possible, however a linear program can not exclude this possibility because the feasible region must include operation at zero and hence it must also include operation between zero and the MULP. The linear model can be discouraged from choosing points below the MULP by inserting an upward sloping section from zero to some point vertically above the MULP. This linearisation gives maximum disincentive to operation below the MULP without artificially limiting reserve provision at the MULP or above. Thus, if point *B* is feasible, the ray *AB* is the tightest linear constraint that can be applied to discourage solutions implying operation below the MULP.

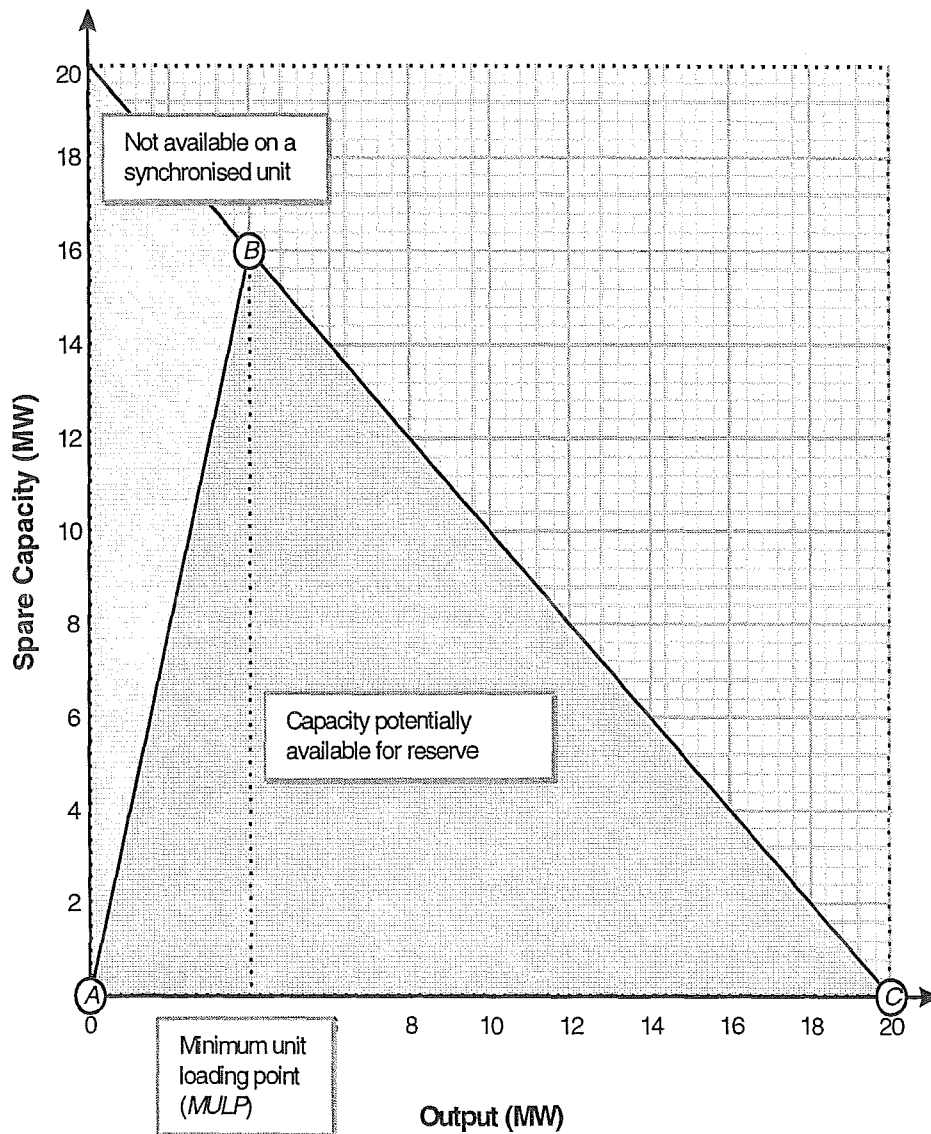


Figure 6-1: A linear approximation to a unit's minimum loading point

Figure 6-1 shows all possible operating points for the unit; however, the amount of instantaneous reserve actually available is usually less than the unit's spare capacity. Figure 6-2 shows how the ABC region may be restricted to $AB^x D^x C$ to represent the reserve available in x -seconds (x might be 6 or 60 seconds). Note how the point B^x is the projection of B (a product of the approximation to MULP) onto the time constrained reserve feasible region.

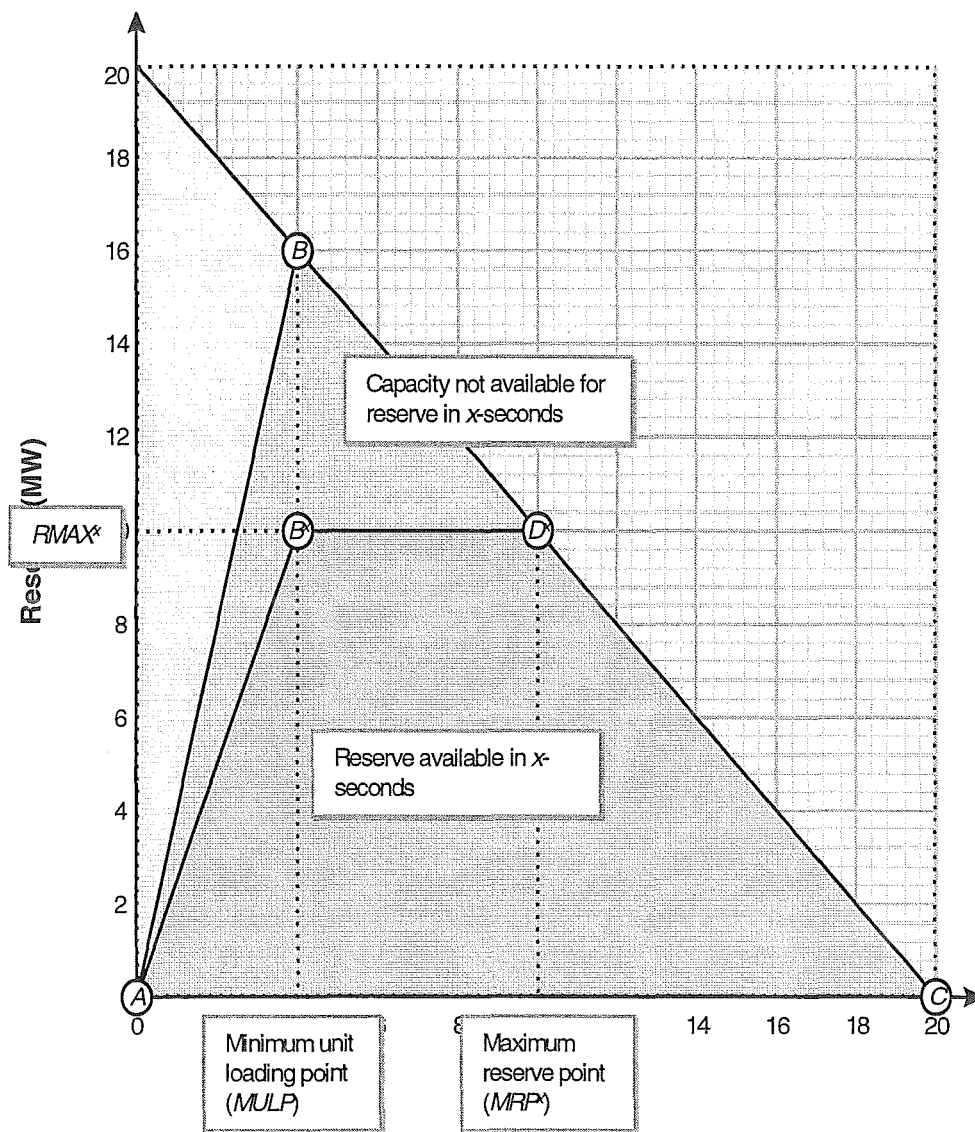


Figure 6-2: Projection of the linear approximation of the MULP onto the time limited reserve capability curve

The boundary of the region AB^xD^xC has a characteristic 'inverted bath-tub' shape composed of:

- a section from point D^x to point C along which each extra unit of spare capacity provides an extra unit of reserve in the x -second time-frame;
- a horizontal section (B^xD^x) along which increasing spare capacity yields no more x -second reserve (this maximum will be denoted the x -second PLSR $RMAX^x$);

- a section from A to B^x representing the MULP constraint, derived in the manner discussed above

If more time is allowed, the bathtub has a higher profile and vice versa. If the timeframe is large enough, the reserve ‘cap’ disappears so that the region ABC as in Figure 6-1 defines the reserve capability.

In the reserve-offering framework, a single wedge can be used to represent this form of reserve capability. The parameters defining this wedge are given by:

- IRP : the slope of the line from A to B^x
- \bar{r}^w : the maximum reserve $RMAX^x$
- CUF : not required

D.1.2 Fixed Costs and Declining Marginal Cost Curves

In most cases there is a fixed cost incurred by operating a unit and/or the unit’s marginal cost curve declines over much of the units operating range. In these cases, the most efficient loading point, MELP occurs at some output level greater than zero. The optimal operating policy for such a unit, in response to an energy price alone, was characterised by Read [1979] who showed that no energy price less than the average cost of operating the unit at the MELP would encourage it to operate at all. When the energy price reaches this average cost, the unit begins to operate at the MELP. Once the energy price is raised beyond this point, the optimal response will increase until, eventually, the energy price will be greater than or equal to the marginal cost of operating at full capacity. Hence, ignoring incentives to provide reserves, or any other constraint that might require operation at low loading, the unit would not normally operate economically below the MELP.

The extent to which reserve requirements affect the unit's economic operating policy depends on the position of the x -second maximum reserve point in relation to the MELP, and on the energy price. Consider these three cases:

1. Where the energy price alone gives sufficient incentive to operate at an energy output level (e^*) between the MELP and full capacity, and maximum reserve (MRP^x) is provided at an output point in the same range.
2. As for case 1, but where the maximum reserve is at a lower energy output point than MELP.
3. Where the energy price, alone, does not give incentive to operate.

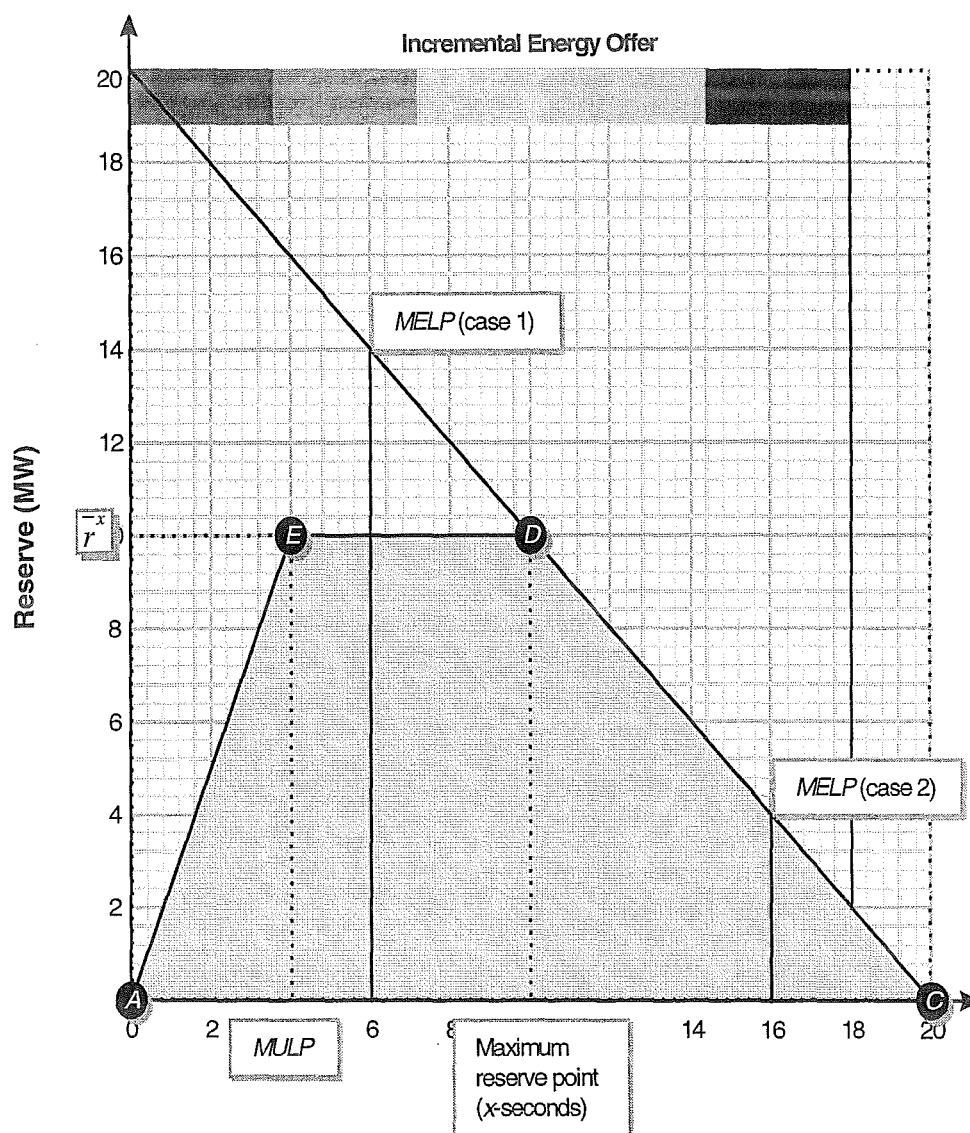


Figure 6-3: Three possible positions of the most efficient loading point

In the first case, no reserve price would induce the generator to consider running the unit below the MELP—although it is conceivable that a high enough reserve price may make operating at some point between MRP^x and e^* more attractive than operating at e^* . In the second case, given a high enough price for reserve, the generating unit could be economically operated between MRP^x and e^* and possibly at a lower output level than the MELP. In the third case, any reserve price high enough to cover the cost of bringing the unit on to provide reserve would result in the unit generating at the MULP, while providing maximum reserve, as defined by the x -second bath-tub curve.

Note that it is not possible to reflect a decreasing marginal cost profile with increasing energy, as occurs below MELP, with a simple multi-band energy offer, because the LP model will internally reorder the bands so that they are of increasing price. This implies that the cost of operating inefficiently to provide reserve cannot be expressed using the energy offer alone. However, this cost can be expressed by dividing the reserve region radially, using two or more wedges, as in Figure 6-4.

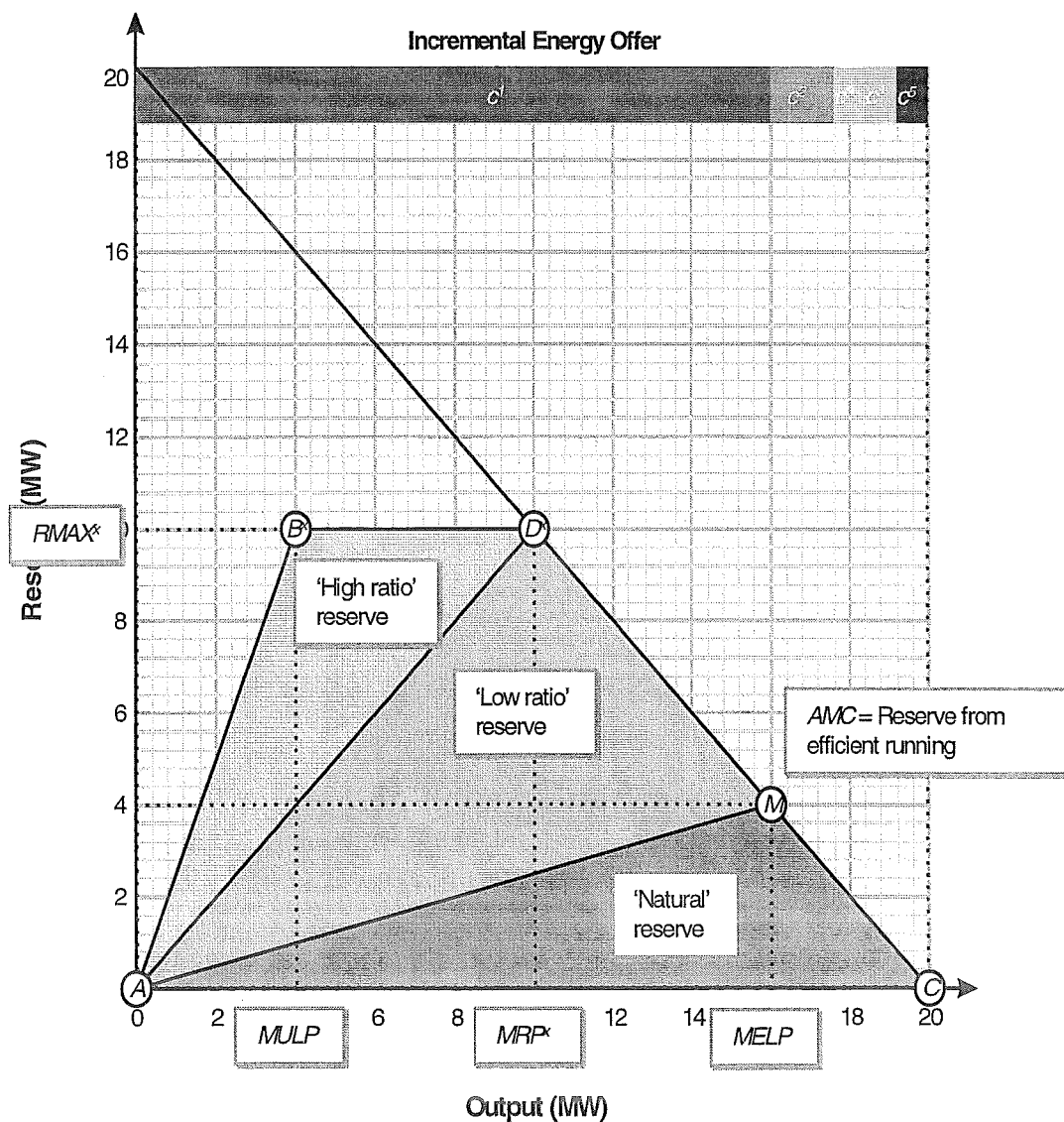


Figure 6-4: Dividing the feasible region into efficiency wedges

First, consider the wedge AMC . Even from an energy perspective, operation within the interior of this wedge is undesirable. In fact, only the point A and the line MC represent efficient operating points; however, the wedge serves to delineate the minimum LP feasible region enclosing these three points, and the reserve supplied by that wedge could be thought of as ‘natural’ reserve arising automatically out of the economics of generation.

Next consider the point D^x . At this point the unit is still producing as much energy as it can for that level of reserve. Although operating at point D^x is not ‘efficient’ in energy terms, it is in terms of the combined energy/reserve output, if both are profitable, on the margin, at that point. Thus, if the energy point is high enough, and e^* lay along the line MC , the optimal energy/reserve point may lie along the line D^xM , and the wedge AD^xM allows for this. The generator would clearly want a fee for operating in this inefficient region to provide ‘additional’ reserve over and above any compensation for lost energy output.

Finally, consider the wedge AB^xD^x . It would never be economic to *constrain off* a unit into this region, because it provides no more reserve and less energy. However, if energy production at point D^x was not economic, it may become economic to have a unit *constrained on* along B^xD^x to generate between the $MULP$ and MRP^x . The reserve in the wedge AB^xD^x is referred to as ‘high ratio’ reserve and that in AD^xM as ‘low ratio’ (The ratio referred to is the reserve to energy ratio, which will be unity along the ray AD^x if reserve and energy are scaled by dividing by $RMAX^x$ and MRP^x respectively). The high ratio reserve can then be charged at a higher fee than the low ratio reserve. (If only the inefficiency cost is considered, the fee for the natural reserve wedge could be set to zero because it lies in the region of increasing marginal cost which can be expressed in the energy offer. In the long run, equilibrium prices should also recover the capital costs involved, both for TWD and PLSR. However the market may be out of equilibrium (e.g. with too much reserve capacity) for quite long periods, and there is no guarantee of cost recovery in the medium term. Reserve providers may set fees in an attempt to recover such costs, though.)

D.2 Hydro Station Offers

This section describes the creation of reserve offers for multi-unit hydro stations.

D.2.1 Station PLSR Capability

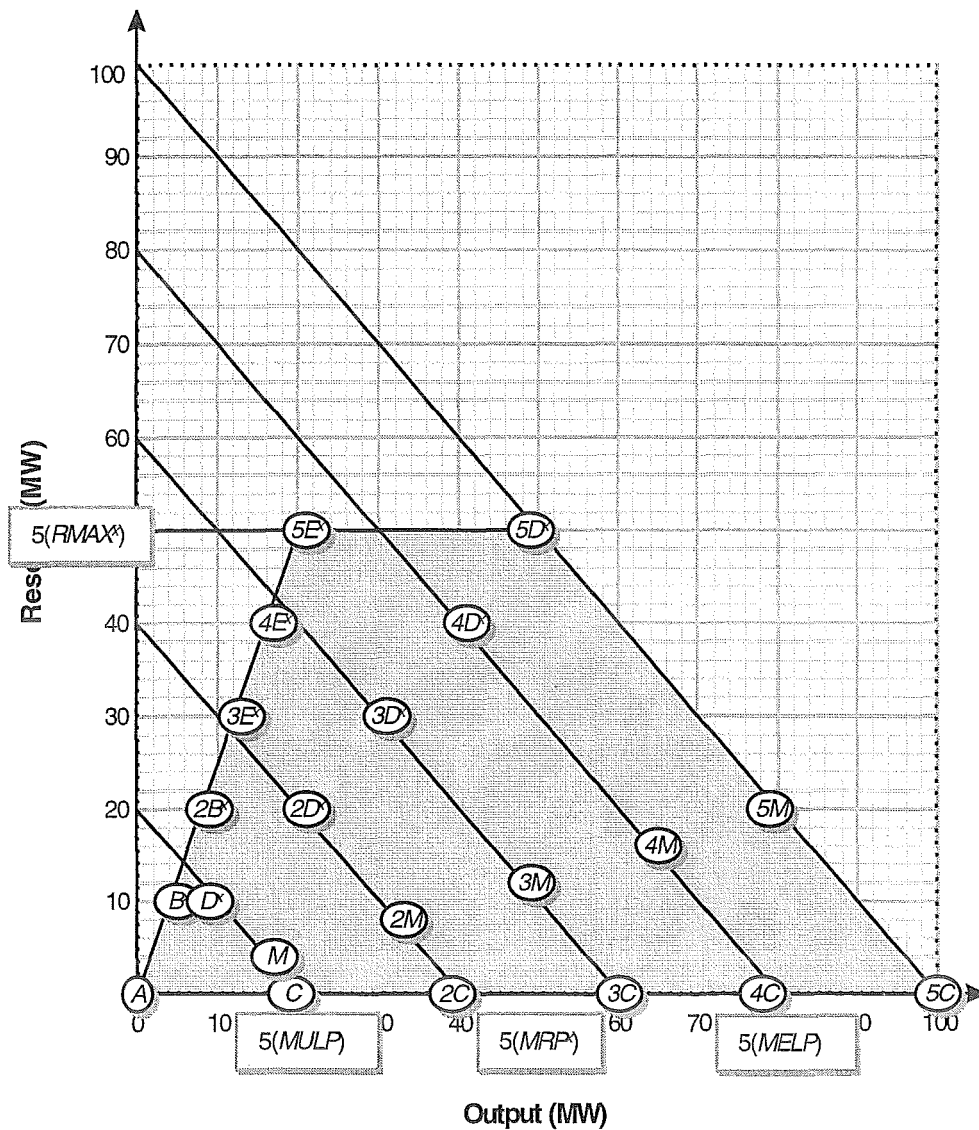


Figure 6-1: Unit reserve capability scaled to the station level

Assuming all station units are identical, the unit energy and reserve optimal operating policy can be readily extended to a station basis. Figure 6-1 shows the reserve capability

plot for a five-unit station. The falling diagonal lines mark the station's capacity limit for increasing unit commitment. The unit limit on reserve delivered in a particular time frame is also scaled up to a station limit.

At the station level, each energy output point could imply a range of reserve levels, depending on the unit commitment. For example, consider the point C^3 in Figure 6-1. The station could produce C^3 MW in three ways by operating:

- three units at full output, providing no reserve;
- or four units to provide some reserve;
- or five units to provide maximum reserve

The average cost of the reserve provided in these three cases is not the same. This is because the cost of running units is lowest, on average, at the MELP, and increasingly expensive away from that point. But, accepting as always in this market, the validity of a linear approximation to an integer situation, the average cost *is the same along the rays* from the origin through the marked points for increasing unit commitment.

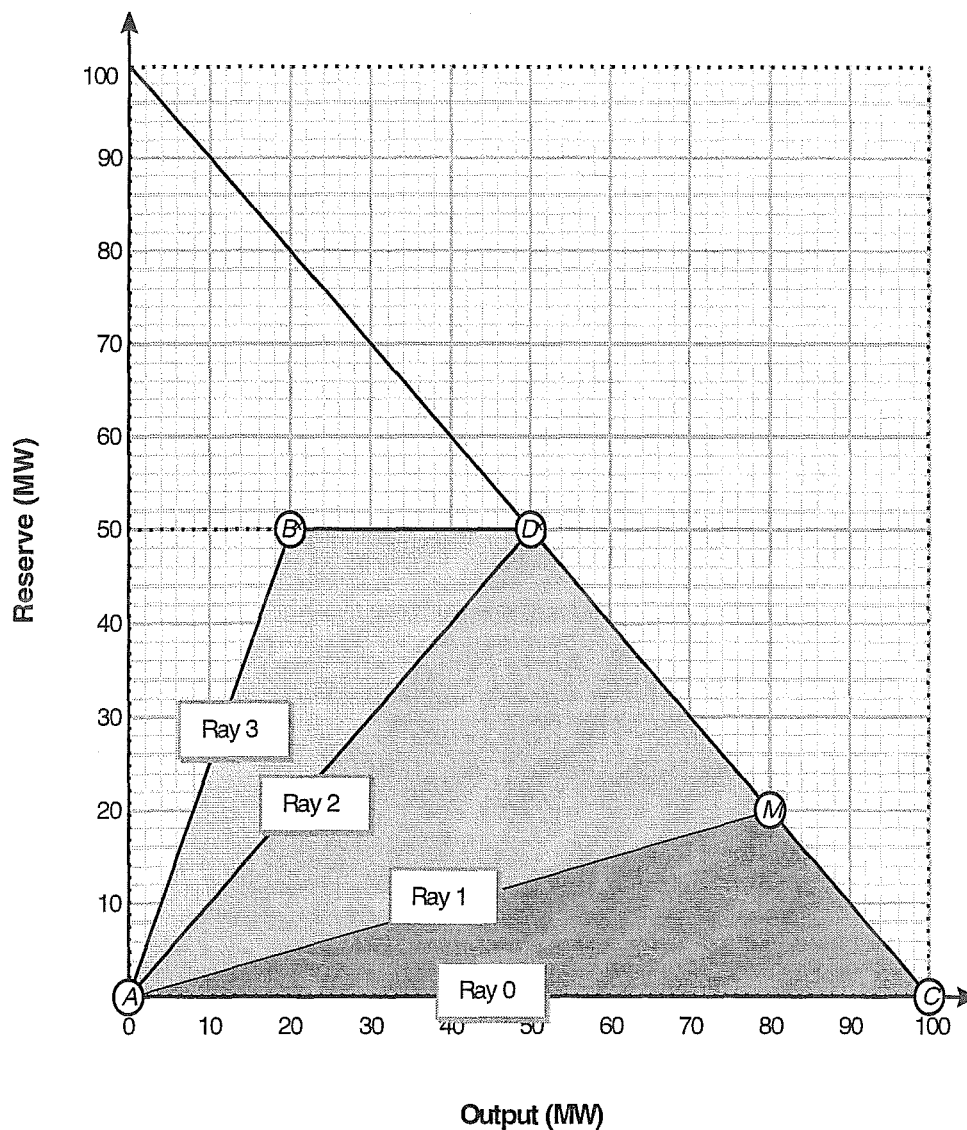


Figure 6-2: Station reserve capacity divided into efficiency wedges

Figure 6-2 divides the available x -second reserve into three 'wedges'. Observe from this graph that:

- along any ray from the origin, in this figure, there is a constant ratio of reserve to output, corresponding to a constant unit loading regime but with an increasing number of units committed;
- the first wedge, AMC , or more exactly, its boundaries, AM and MC represents 'natural' reserve, some provided by operating units at MELP along AM or between MELP and capacity, along MC ;

- the second wedge, AD^xM , represents additional ‘low ratio’ reserve, provided by operating units below their MELP (one unit of reserve is made available for every unit of energy ‘backed off’, along D^xM , but the generator will want extra compensation for operating their units inefficiently in this way);
- the third wedge, AB^xD^x , represents additional ‘high ratio’ reserve, and if the energy dispatch suggests output between M and D^x , then the station will naturally produce reserve at the corresponding point on AM , or, for a fee on AD^x (otherwise, the station will not be backed off below D^x for reserve purposes, but it could be brought on to provide extra reserve up to point D^x)

Adding additional wedges within any of the regions shown can increase the accuracy of this approximation.

The offer framework specifies each ray by the associated increase in slope over the previous ray—with the x -axis being called ‘ray zero’ and having a slope of zero (this specification is consistent with the incremental style of the energy offer). These slope increments are the Incremental Reserve Proportions or (*IRP*) for the wedges in the offer. Fees for the wedges should increase monotonically as the unit loading increases and the ray slope increases (if this condition is violated, the LP will simply reorder the wedges appropriately, just as it does with bands of the energy offer). The calculation of the wedges for this reserve offer is discussed in Section D.3.

D.2.2 Combining PLSR and TWD

Units running in tail-water depressed (TWD) mode for reserve purposes respond to falling frequency by ‘tripping’ to generate once the frequency has been below a certain point for a measurable length of time. Because of this required time delay, TWD reserve is likely to be most important in the 60-second reserve market as a supplement to PLSR.

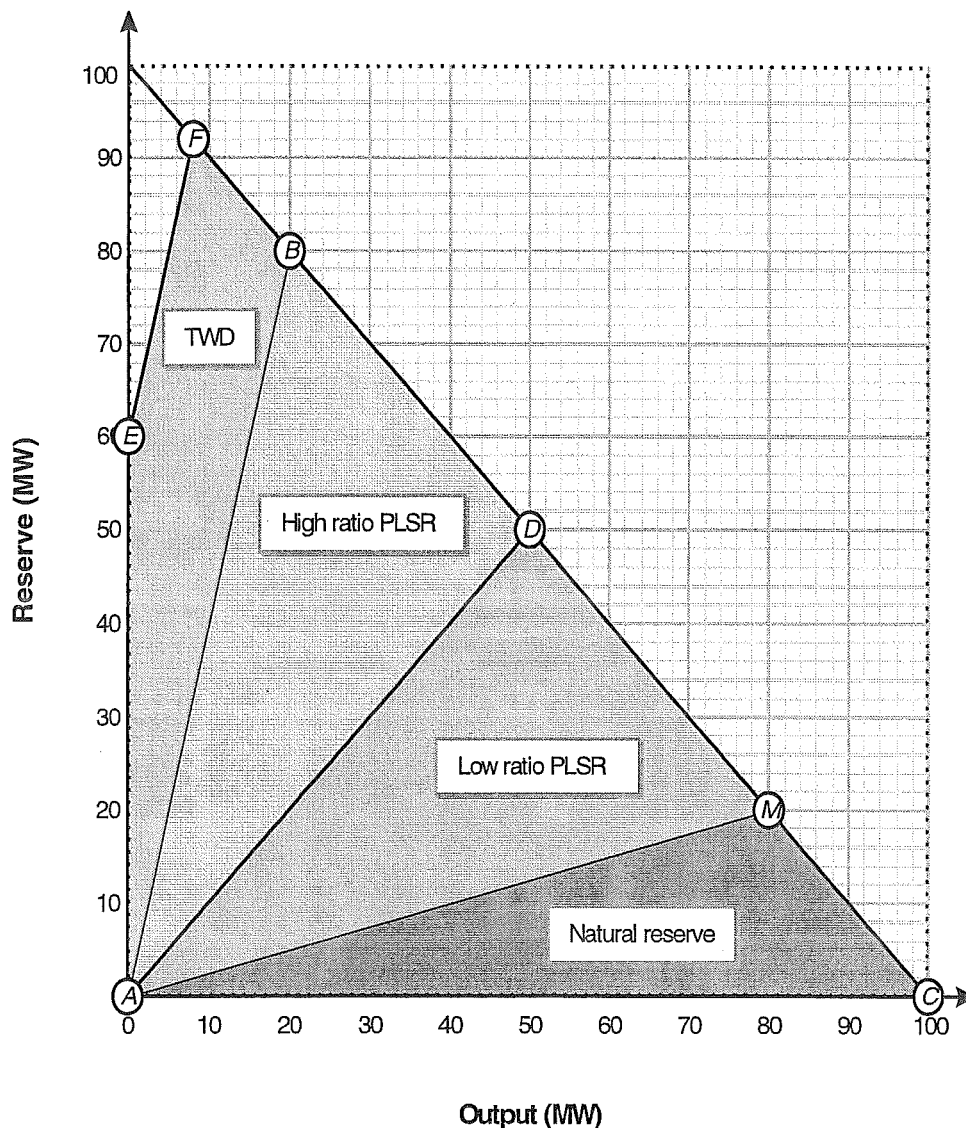


Figure 6-3: Energy, PLSR, and TWD capability

Assume first, that each MW of capacity assigned to PLSR and TWD provides one MW of reserve, as in Figure 6-3, and that the response time allowed is so great that there is no 'cap' ($RMAX^x$) on the PLSR. (The fact that TWD can only be supplied in whole units will be ignored and assume that some practical rule will be adopted to round the continuous LP solution to the nearest integer number of units.) Suppose that a block of TWD, $AEFB$, is available at some cost greater than that for PLSR along ray AB . If generation is low, it will be possible to commit all TWD units for that purpose, along the line EF . Otherwise, the availability of units will be restricted by the combined en-

ergy/PLSR requirements, falling along the line FB . Beyond B , no TWD is available. (This figure implicitly assumes that not all units can be used in TWD mode. In the special case where all units can provide TWD, $F = G$ will represent all units committed to TWD, and reserve availability will fall continuously from there.)

Figure 6-3, however, assumes that all committed capacity provides reserve on a MW for MW basis. TWD, though, responds rather like PLSR except that it tends to have less impact because of its triggering mechanism and subsequent delay. Thus, the PLSR that can be delivered in a particular timeframe is limited to $RMAX^x$ say, and a similar, or perhaps lower, limit should apply to the TWD block as well. This is represented in Figure 6-4.

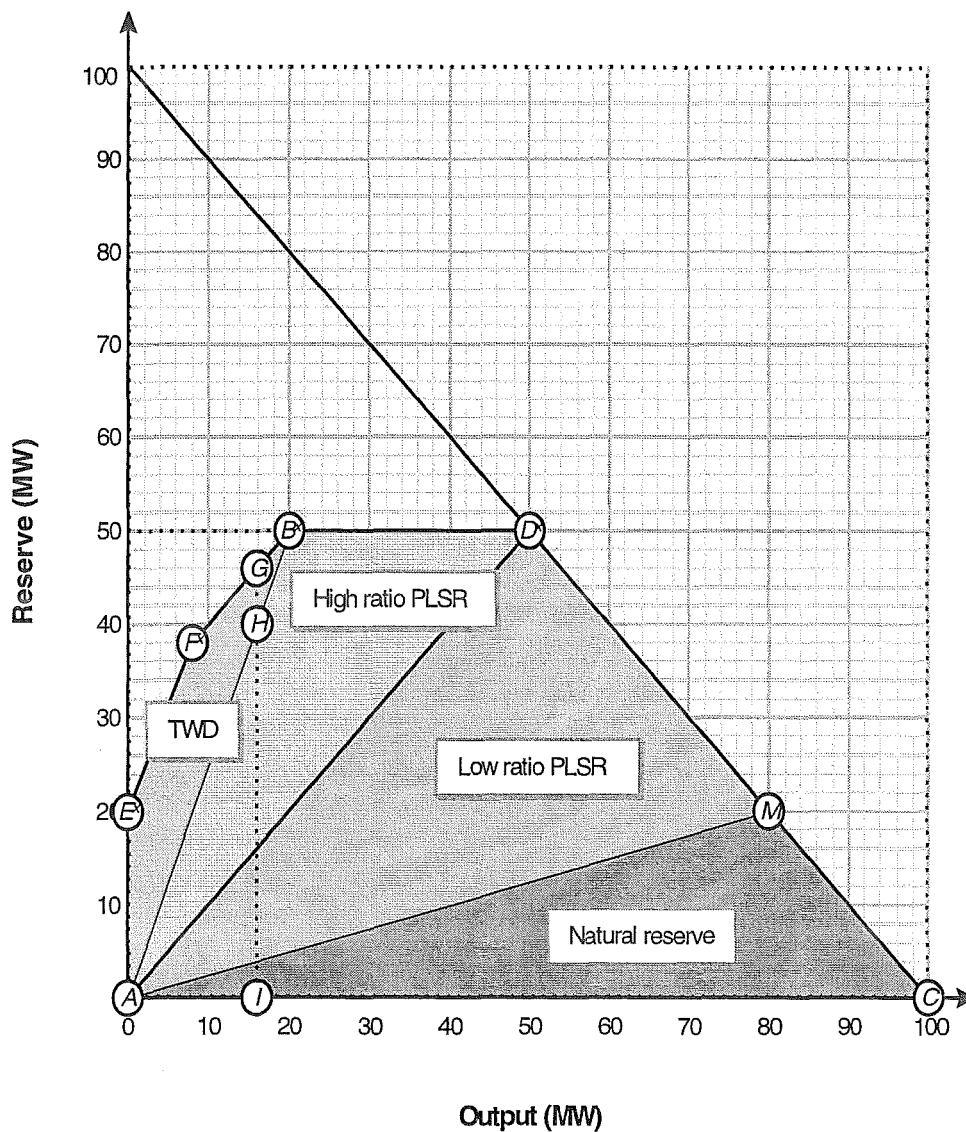


Figure 6-4: The feasible region with PLSR and TWD reserves when reserve availability is time limited

Point D^x in Figure 6-4 represents the x -second limit on PLSR shown earlier. The points labelled E^x and F^x show how the points, E and F in Figure 6-3 are projected onto the time limited reserve feasible region. Point E moves down and represents the fact that, even when all depressible units are committed, they provide only a fraction of their capacity in the x -second timeframe. This ratio of capacity committed to TWD to that available in x -seconds is the Capacity Utilisation Factor (CUF) for TWD. Beyond E , if the remaining non-depressible units are constrained on to provide PLSR, the amount of reserve delivered increases linearly from point E^x , parallel to the top of last 'high ratio'

PLSR wedge, until all units are committed (at point F^x). Thus, the line segment $E^x F^x$ is the projection of the EF limit, which is a result of the overall limit of TWD capability.

From this point, units on PLSR must replace TWD units. This produces the line $F^x B^x$ up to the maximum x -second reserve-line, $E^x D^x$. Thus, the line segment $F^x B^x$ is a projection of the FB segment of the 45-degree line representing the limit on the capacity available for combined energy/reserve service down onto the feasible region for delivered energy. This follows from the earlier observation that the ‘cap’ $B^x D^x$ can equally be thought of as the projection of the BD section of the combined capacity limit.

Referring to Figure 6-4, suppose that given the energy market clearing price found by the LP, the generator would have wanted to operate at full output C , but was constrained off to I to provide GI reserves then it would need to be paid at least:

- the energy price for AI
- plus compensation for all profitable generation foregone IC ;
- plus its nominated fees for the PLSR commitments of HI ;
- plus its nominated fee for the TWD commitment of GH

D.3 Calculating a Reserve Offer

The *IRP* and *CUF* parts of a reserve offer may appear complex, but they have a natural physical interpretation and they can be calculated directly from raw unit/station data. This section explains the relationship between the spare capacity and delivered reserve representations, and shows how to derive the *IRP* and *CUF* values for an example hydro station.

Raw Data					
Observation	Energy	Reserve	Slope	IRP	CUF
Maximum Output	100	0			
MELP	80	20	25.00%	25.00%	1
Intermediate	50	40	80.00%	55.00%	1.363636
MULP	20	40	200.00%	120.00%	2.5
TWD	60	20			3

Table 6-81: Example observations of output and reserve

Consider the example of a 100 MW hydro station with 60 MW of TWD capability, and a minimum loading point of 20 MW for PLSR. A x -second instantaneous reserves offer can be calculated by taking some raw observations of:

1. The reserves provided by various amounts of TWD.
2. The reserves provided at various loading points with no TWD.

Table 6-81 shows a set of three PLSR points at 80, 50 and 20 MW which made available 20, 40 and 40 MW of reserve in x -seconds respectively. It also shows a single TWD observation in which the whole 60 MW of TWD was committed and 20 MW of reserve was available in x -seconds. The *IRP* and *CUF* can be calculated for three PLSR wedges and one TWD block using the formulae in Figure 6-5.

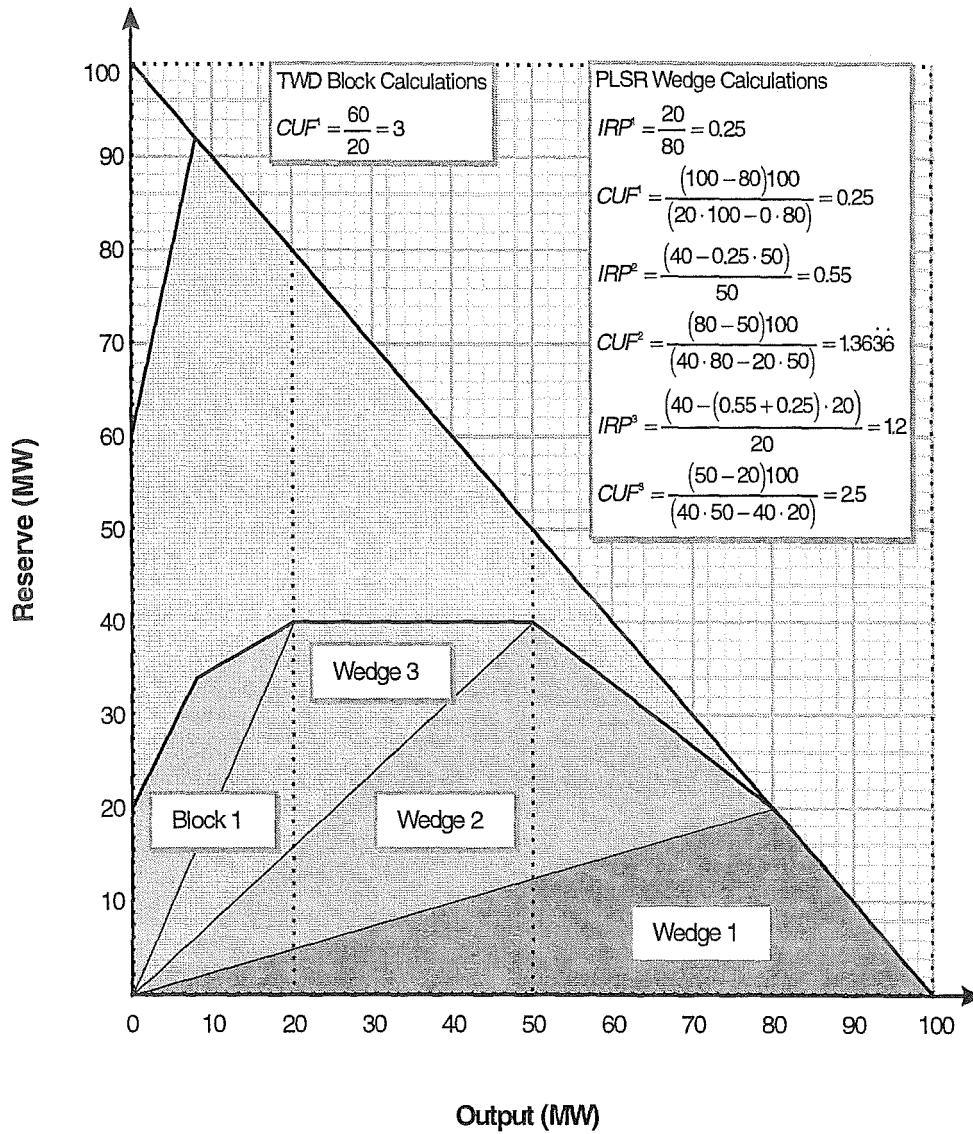


Figure 6-5: Calculating CUF values for a general piece-wise linear reserve curve

The CUF for a block of TWD is simply the ratio of the capacity committed to the reserve made available in x -seconds. The CUF for a PLSR wedge is more complex due to the interaction of the ray constraints, but it has the same interpretation.

In general, if the raw energy/PLSR observations are denoted (e^w, r^w) , and the station's capacity is \bar{o} , then the IRP and CUF values are given by Equation 6-22 and Equation 6-23. Note that $e^0 = \bar{o}$ and $r^0 = 0$.

$$IRP^1 = \frac{r^1}{e^1}$$

$$IRP^w = \frac{r^w - e^w \sum_{k=1}^{w-1} IRP^k}{e^w}$$

Equation 6-22

$$CUF^w = \frac{\bar{o}(e^{w-1} - e^w)}{(e^{w-1}r^w - e^wr^{w-1})}$$

Equation 6-23